Testing for Lorentz Invariance Violation through Birefringence Effects of Gravitational Waves

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Background

- GR & SM - two fundamental, experimentally supported theories
- Unified quantum theory of gravity at the Planck scale is still unknown

\[ S = \int d^4x \left( \mathcal{L}_{GR} + \mathcal{L}_{SM} + \mathcal{L}_{SME} \right) \]

*SME- Standard Model Extension (Framework)
**Working with Gravitational Waves**

First-order perturbation:

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

General Lagrange density:

\[ \mathcal{L}_{\mathcal{K}(d)} = \frac{1}{4} h_{\mu\nu} \mathcal{K}^{(d)\mu\nu\rho\sigma \varepsilon_1 \ldots \varepsilon_{d-2}} \partial_{\varepsilon_1} \ldots \partial_{\varepsilon_{d-2}} h_{\rho\sigma} \]

- Second order in \( h_{\mu\nu} \)
- Contains all Lorentz invariant and violating terms
- \( \mathcal{K}(d)_{\mu\nu\rho\sigma \varepsilon_1 \ldots \varepsilon_{d-2}} \): very small, unknown background fields, mass dimension 4-d; constrain experimentally

Ensure usual gauge symmetry of GR under the transformation: \( h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \) and derive modified dispersion relation (four momentum \( p^\mu = (\omega, p) \)):

\[ \omega = (1 - s^0) \pm \sqrt{(s^1)^2 + (s^2)^2 + (s^3)^2} \left| p \right| \]

- Depends on source sky location; given many events, one can map out the LV fields in the sky using spherical harmonics, \( Y_{jm}(\theta, \phi) \)

\[ \begin{align*}
    s^0 &= \sum_{djm} \omega^{d-4} Y_{jm}(\hat{n}) k_{(I)jm}^{(d)} \\
    s^1 \mp is^2 &= \sum_{djm} \omega^{d-4} Y_{jm}(\hat{n}) (k_{(E)jm}^{(d)} \pm ik_{(B)jm}^{(d)}) \\
    s^3 &= \sum_{djm} \omega^{d-4} Y_{jm}(\hat{n}) k_{(V)jm}^{(d)} \\
\end{align*} \]

\(|s| \leq j \leq d - 2\)

What could we see if there are Lorentz violating background fields?

• What could happen to the two polarizations of gravitational waves?

• **Dispersion relation**: can provide us with information on *phase shifts/time delays* between GW *polarizations* due to LV...
A possible, observable effect: **Birefringence**

\[ \Delta t \approx \sum_{d=5}^{\infty} 2\omega^{d-4} \int_0^z \frac{(1+z)^{d-4}}{H_z} \sum_{jm} Y_{jm} \mathcal{K}_{(d)}^{(v)jm} \]

- Lowest order Lorentz violating term, familiar GWs fields, quadratic in the Lagrange

- \( d = \) mass dimension, assumed constant and small; natural units \( h=c=1 \). Expect higher order fields \( \rightarrow \) smaller effects

- Birefringence effects: \( \mathcal{K}_{(d)}^{(v)jm}, d \geq 5 \) and is odd

- Redshift \( z \) \((0.01\sim0.67\text{ for LIGO events}) \rightarrow \text{distance here}\)
Question: What do we expect to see for $\Delta t$?
Answer: Depends...

A) Expected time delays should be smaller than the time between the peaks of GWs

- GW150914 [1]:
  - estimate $\Delta t \lesssim 0.003$ s
  - input parameter values within ranges for this LIGO event
    $\Rightarrow$ constrain Lorentz Violating coefficients (one at a time)

  GW150914 placed resulting constraints on coefficients, e.g.,:
  Kostelecky and Mewes 2016

- Likewise, PIRO:
  $0.0003 \lesssim \Delta t \lesssim 0.08$ s

( $\theta \approx 160^\circ, z \approx 0.09, \varphi \approx 120^\circ, f \approx 100Hz$ )
B) Can we calculate some values of $\Delta t$ given previous constraints?

- Coefficients: lowest order, birefringence effects, $d=5$
- Use same parameter values (GW150914[1]):

\[
\begin{align*}
\theta & \approx 160^\circ \\
z & \approx 0.09 \\
\varphi & \approx 120^\circ \\
f & \approx 100Hz
\end{align*}
\]

- Previous constraints’ order of magnitude ($\sim 10$) (Binary Pulsar, Kostelecky & Russell, 2019)

\[\Delta t \approx 10^{12} \text{ seconds}\]

- Previously constrained values: \textit{too large}
- The new constraint values: \textit{aim is to provide smaller constraints}

C) It is possible that the value of some coefficients are zero, we have just yet to show this. The values of coefficients may even be only visible on the Planck scale...
Many previous works...

Different tests for the coefficients provide different sensitivity levels. To see the current full list of constraints see: Rev. Mod. Phys. 83, 11 (2011); updated 2018.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Result</th>
<th>System</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^{ij}$</td>
<td>$(-20$ to $5) \times 10^{-25}$</td>
<td>Gravitational waves</td>
</tr>
<tr>
<td>$x^{00}$</td>
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<td></td>
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<tr>
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<td>$(-2$ to $10) \times 10^{-15}$</td>
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<td>$x^{0i}$</td>
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<td>$x^{00}$</td>
<td>$&gt; -3 \times 10^{-14}$</td>
<td>Cosmic ray</td>
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<td>$s_{XX} - s_{YY}$</td>
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<td>Gravimetry</td>
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<td>$s_{XY}$</td>
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<tr>
<td>$s_{XZ}$</td>
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<tr>
<td>$s_{YZ}$</td>
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<td>$s_{YZ}$</td>
<td>$1 \pm 1 \times 10^{-4}$</td>
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<tbody>
<tr>
<td>$\bar{s}_{XY}$</td>
<td>$(-4.6 \pm 7.5) \times 10^{-3}$</td>
<td>Combined</td>
</tr>
<tr>
<td>$\bar{s}<em>{XX} - \bar{s}</em>{YY}$</td>
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</tr>
<tr>
<td>$\bar{s}<em>{XY} - 2\bar{s}</em>{XZ}$</td>
<td>$(0.5 \pm 2.5) \times 10^{-11}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_{XY}$</td>
<td>$(-1.8 \pm 6.5) \times 10^{-11}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_{XX}$</td>
<td>$(-0.5 \pm 3.9) \times 10^{-11}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_{XY}$</td>
<td>$(-0.5 \pm 3.9) \times 10^{-11}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_{XX}$</td>
<td>$(-0.2 \pm 1.3) \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_{XX}$</td>
<td>$(0.8 \pm 2.3) \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_{XX}$</td>
<td>$(-0.8 \pm 5.5) \times 10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$\bar{s}_{XZ}$</td>
<td>$(-0.0 \pm 1.0) \times 10^{-8}$</td>
<td></td>
</tr>
</tbody>
</table>


See other Moriond talks:

**March 27**
Christophe Le Poncin-Lafitte - LLR recent results
Mike Tobar - Searching for New Physics with Precision Low Temperature Experiments

**March 28**
Christine Guerlin - Testing Lorentz symmetry with Microscope
Yuri Bonder – Lorentz violation in the gravitational sector
Examples of Previous Work: Gravity Sector

Lunar Laser Ranging (> 35 years of data)

• Modifies equation of motion - cause oscillations in Earth-moon distance

\[
\vec{F} = -\frac{G M m}{r^2} [\vec{r} - \vec{\bar{s}} \cdot \hat{r} + \frac{3}{2} (\hat{r} \cdot \vec{\bar{s}}) \hat{r}] + \ldots
\]

• Constraints placed on SME coefficients: \( d=4 \)

\[
\begin{align*}
\tilde{s}_{XX} &= (-0.9 \pm 1.0) \times 10^{-10} \\
\tilde{s}_{XY} &= (-5.7 \pm 7.7) \times 10^{-12} \\
\tilde{s}_{XZ} &= (-2.2 \pm 5.9) \times 10^{-12} \\
\tilde{s}_{YY} &= (0.6 \pm 4.2) \times 10^{-11} \\
\tilde{s}_{YZ} &= (6.2 \pm 7.9) \times 10^{-11} \\
\tilde{s}_{ZZ} &= (2.3 \pm 4.5) \times 10^{-11}
\end{align*}
\]


\[
\begin{align*}
\tilde{s}_{XY} &= (-0.5 \pm 3.6) \times 10^{-12} \\
\tilde{s}_{YZ} &= (2.1 \pm 3.0) \times 10^{-12} \\
\tilde{s}_{XZ} &= (0.2 \pm 1.1) \times 10^{-11} \\
\tilde{s}_{YY} &= (3.0 \pm 3.1) \times 10^{-12} \\
\tilde{s}_{ZZ} &= (-1.4 \pm 1.7) \times 10^{-8} \\
\tilde{s}_{ZZ} &= (-6.6 \pm 9.4) \times 10^{-9}
\end{align*}
\]

A. Bourgoin et al., Phys. Rev. Lett. 119, 201102 (2017); A. Bourgoin et al., in E. Aug e, J. Dumarchez, and J.T. Thanh V’an, eds., Proceedings of the 52nd Rencontres de Moriond on Gravitation, ARISF 2017

Binary Pulsars

- Modified Lagrange, Binary System, d=4, 5:

\[
L = \frac{1}{2} \left( m_a v_a^2 + m_b v_b^2 \right) + \frac{G m_a m_b}{r} \left( 1 + \frac{3}{2} \tilde{s}_{00} + \frac{1}{2} \tilde{s}_{jk} \hat{n}^j \hat{n}^k \right)
+ \frac{G m_a m_b}{2r} \left[ 3 \tilde{s}_{0j} \left( v_a^j + v_b^j \right) + \tilde{s}_{0j} \hat{n}^j \left( v_a^k + v_b^k \right) \hat{n}^k \right] - \frac{3G m_a m_b}{2r^2} v_{ab} \left( K_{jklm} \hat{n}^k \hat{n}^l \hat{n}^m - K_{jkl} \hat{n}^l \right)
\]

\[K_{jklm} = -\frac{1}{6} \left( q_{0j0m} + q_{00km} + q_{nijklm} + \text{permutations} \right)\]

- Constraints placed on SME coefficients d=4

\[
[s]^{TT} < 1.6 \times 10^{-5}
\]
\[
<s> \times 10^{-4}
\]
\[
-5.2 \text{ to } 5.3 \times 10^{-9}
\]
\[
-7.5 \text{ to } 8.5 \times 10^{-9}
\]
\[
-5.9 \text{ to } 5.8 \times 10^{-9}
\]
\[
-3.5 \text{ to } 3.6 \times 10^{-11}
\]
\[
-2.0 \text{ to } 2.0 \times 10^{-11}
\]
\[
-3.3 \text{ to } 3.3 \times 10^{-11}
\]
\[
-9.7 \text{ to } 10.1 \times 10^{-11}
\]
\[
-12.3 \text{ to } 12.2 \times 10^{-11}
\]


- Constraints placed on SME coefficients d=5

\[
\begin{array}{c|c|c|c}
\text{Symbol} & 1-\sigma \text{ limit [10}^8 \text{ m}] & K_{XXYY} & 6.6
\end{array}
\]
\[
\begin{array}{c|c|c|c|c|c}
K_{XXX} & K_{XXZ} & K_{XYY} & K_{XYZ} & K_{XYZ} & K_{YYYY}
\end{array}
\]

Binary Neutron Star Merger: LIGO GW170817

- Both light and GWs were emitted
- LV effects can modify their travel speeds; fractional difference:

$$-3 \times 10^{-15} \leq \frac{\Delta v}{v_{EM}} \leq 7 \times 10^{-16}$$

a) Light & GW emitted at same time (upper bound)

b) Light & GW arrive at same time (lower bound)

- Constraints placed on SME coefficients $d=4$

First Investigate the Impact of Lorentz Violations in the PIRO failed SN (LIGO-T1100093), which Has a Time Evolution

PIRO simulation:

- Failed SN -> BH mass=7
- Accretion disk strips mass = final mass is 0.3M
- D =15 Mpcs; this distance is within the optical trigger ranges for 01/02
- Orientation is “Face-on” to the LIGO detectors
- Semi-analytical GW waveform
- SNR > 400
- Feasible parameters used (not yet experimentally ruled out)
- Strong, energetic signal

Are Lorentz-violating effects observable in SN simulations?

How would they affect SN simulations?

**Methodology:**

- **PIRO Waveform:**
  \( h_+(t), h_\times(t) \)

- **LIGO Response:**
  \[
  F_{L,+}(\Theta, \varphi, \psi)h_+(t) + F_{L,\times}(\Theta, \varphi, \psi)h_\times(t) \\
  F_{H,+}(\Theta, \varphi, \psi)h_+(t) + F_{H,\times}(\Theta, \varphi, \psi)h_\times(t)
  \]

- **Model Strain**
  Insert artificial shifts into \( h_+(t) \):
  \[
  F_{H,+}(\Theta, \varphi, \psi)h_+(t + \Delta t) + F_{H,\times}(\Theta, \varphi, \psi)h_\times(t) \\
  F_{L,+}(\Theta, \varphi, \psi)h_+(t + \Delta t) + F_{L,\times}(\Theta, \varphi, \psi)h_\times(t)
  \]

- **Pseudo Data Strain:**
  + White, Gaussian Noise
  OR
  LIGO detector noise

- **\( \chi^2 \): Model Strain vs. Pseudo Data Strain**
  \( \rightarrow \chi^2_{\text{min}} \) describes where, if any, the time shift due to LV occurs in pseudo data strain

- **Histogram:** Hundreds of runs of \( \chi^2 \) with new generated noise each time
  \( \rightarrow \) Noise will also shift \( \chi^2_{\text{min}} \) values about the mean value

- **Checking Confidence in Resolving LV time shifts with \( \Delta t \) vs. SNR**
  \( \rightarrow \) Is the spread in the histograms comparable to the LV shift detected in the \( \chi^2_{\text{min}} \) values? What SNR values can one resolve certain \( \Delta t \) values?
Notes to Consider:

• Investigate the effect of background fields that break space-time anisotropy → a time delay between polarizations in a specific reference frame.

• Time delay between polarizations = physical effect: it does not depend upon the observer frame; it is an effect based on relative orientations of the GWs vs background LV field.

• Choice of $h_+$ and $h_\times$ is arbitrary; they are not gauge independent unless as a linear combination; assume optimal reference frame (LV field only affects $h_+$)

• LIGO event strain data does not distinguish between $h_+$ and $h_\times$ (LIGO orientations)

\[
V_L = F_{L+}(\theta, \phi, \psi) \ h_+(\theta, \phi, \psi, t, \tau, \Delta t) + F_{L\times}(\theta, \phi, \psi) \ h_\times(\theta, \phi, \psi, t, \tau)
\]

\[
V_H = F_{H+}(\theta, \phi, \psi) \ h_+(\theta, \phi, \psi, t, \tau, \Delta t) + F_{H\times}(\theta, \phi, \psi) \ h_\times(\theta, \phi, \psi, t, \tau)
\]

$\theta, \phi$: Source Location angles
$\psi$: GW frame rotation wrt detectors
$\tau$: Time delay between detectors
$\Delta t$: Time shift between plus and cross polarizations
Compare LIGO/VIRGO data to a model waveform with time delay, $\Delta t$.

- **Model:** $\text{PIRO waveform at 15Mpc}$
- **Pseudo Data Strain:** $\text{PIRO waveform at 15Mpc + Whitened Noise/LIGO Noise}$

**I can manipulate the magnitude of LV effects in the model then correlate to the pseudo data. The highest correlation is the $\chi^2$ min value.**

$\chi^2$ for Model vs Data

A histogram distribution of the $\chi^2$ min values provide constraints on SME coefficients

- **SNR~200**
- **H plus (in pseudo data) shifted by 0.305ms**

Where model best matches data

- **SNR~150**
- **H plus (in pseudo data) shifted by 0.305ms**
- **100 Noise Runs, taken from around GW150914**
- **180Mpc, BH=7M, eta=0.5**
Confidence in Resolving LV Time Delay

- Different possible shifts in the data & different SNR values ⇒ changes the ability to resolve $\Delta t$ in data:

  If $1\sigma < \Delta t_{in\,data}$, higher confidence in resolve $\Delta t_{LV}$
  If $1\sigma > \Delta t_{in\,data}$, less confidence in resolve $\Delta t_{LV}$

  
  \textit{We can apply higher confidence in attributing the time delay effect to LV fields rather than to, say, the uncertainty in other parameters such as noise}

- Inject PIRO for different SNR and different artificial data time shifts to check for regions of better $\Delta t$ resolution. If the shift is too small, it may not be resolvable with a $1\sigma$.
  
  \textit{Still, a constraint can be placed on the coefficients.}
- PIRO + Gaussian Noise

- PIRO + LIGO noise around GW150914

- Distance vs confidence in resolving time delay due to LV

<table>
<thead>
<tr>
<th>SNR (approx)</th>
<th>Distance [Mpcs]</th>
<th>$\sigma \approx$ Data Shift [ms]</th>
</tr>
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<tbody>
<tr>
<td>7000</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>1100</td>
<td>35</td>
<td>0.0699</td>
</tr>
<tr>
<td>770</td>
<td>50</td>
<td>0.309</td>
</tr>
<tr>
<td>130</td>
<td>300</td>
<td>0.758</td>
</tr>
<tr>
<td>70</td>
<td>500</td>
<td>0.881</td>
</tr>
<tr>
<td>7</td>
<td>5000</td>
<td>1.68</td>
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</table>
Summary

• SME framework provides a generic way to test breaking of underlying symmetries like Lorentz symmetry, which are built into GR

• Use gravitational wave signals to provide tighter constraints on LV coefficients, gravity sector

• In addition, we can provide a means to understand the confidence in attributing observed effects to Lorentz violations

• Coefficients depend on source sky location; given many events, one can map out the LV fields in the sky.

• Work in progress: apply similar and alternative techniques to LIGO events, and other SN simulations and waveforms
Acknowledgements