Constraints on graviton mass and tidal charge with observations of the Galactic Center

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Figure 1. Comparisons between the orbit of S2 star in Newtonian gravity (red dashed line) and Yukawa gravity during 10 orbital periods (blue solid line) for $\Lambda = 2.59 \times 10^3$ AU. In the left panel $\delta = +1/3$, and in the right $\delta = -1/3$. 

Borka et al. (2013)
Figure 6. The reduced $\chi^2$ for $\delta=1/3$ as a function of $\Lambda$ in case of NTT/VLT alone (left) and combined NTT/VLT+Keck (right) observations.

Figure 7. The maps of reduced $\chi^2$ over the $\Lambda - \delta$ parameter space in case of NTT/VLT observations. The left panel corresponds to $\delta \in [0,1]$, and the right panel to the extended range of $\delta \in [0.01, 10^{6}]$. The shades of gray color represent the values of the reduced $\chi^2$ which are less than the corresponding value in the case of Keplerian orbit, and three contours (from inner to outer) enclose the confidence regions in which the difference between the current and minimum reduced $\chi^2$ is less than 0.0005, 0.005 and 0.05, respectively.
Graviton Mass Estimates from Trajectories of Bright Stars near the Galactic Center

We use a modification of the Newtonian potential corresponding to a massive graviton case (Visser, 1998; Will, 1998, 2014):

\[ V(r) = -\frac{GM}{(1 + \delta)r} \left[ 1 + \delta e^{-\left(\frac{r}{\lambda}\right)} \right], \]  

(1)

where \( \delta \) is a universal constant (we put \( \delta = 1 \)). In our previous studies we found constraints on parameters of Yukawa gravity. As it was described in we used observational data from NTT/VLT. If we wish to find a limiting value for \( \lambda_x \), so that \( \lambda > \lambda_x \) with a probability \( P = 1 - \alpha \) (where we select \( \alpha = 0.1 \))
normalized $\chi^2$ depending on $\lambda_x$ has to be equal to the threshold depending on degree of freedom $\nu$ and parameter $\alpha$ or in other words, $\chi^2(\lambda_x) = \chi^2_{\nu,\alpha}$. Computing these quantities we obtain $\lambda_x = 2900$ AU $\approx 4.3 \times 10^{11}$ km. Now we obtain the upper limit on a graviton mass and we could claim that with a probability $P = 0.9$, a graviton mass should be less than $m_g = 2.9 \times 10^{-21}$ eV (since $m_g = h c / \lambda_x$) in the case of $\delta = 1$. 
Zakharov et al. (2016)
stars are probing space-time in a higher potential and around a central body much more massive than in the other experiments. This is highlighted in the right panel of Fig. 2, where $\lambda$ is expressed in terms of the gravitational radius of the central body. Furthermore, short-period stars probe the space-time around a SMBH, which is conceptually different from Solar System tests where the space-time curvature is generated by weakly gravitating bodies. Specifically, some nonperturbative effects may arise around strongly gravitating bodies (see, e.g., Ref. [76]). In addition, in models of gravity exhibiting screening mechanisms, deviations from GR may be screened in the Solar System (see, e.g., Ref. [77]). In this context, searches for alternative theories of gravitation in other environments are important.

A specific theoretical model covered by the fifth force framework is a massive graviton. In that context, we found a 90% confidence limit $\lambda > 5000$ A.U. for $\alpha = 1$, which can be interpreted as a lower limit on the graviton's Compton wavelength $\lambda_g > 7.5 \times 10^{11}$ km or, equivalently, as an upper bound on the graviton's mass $m_g < 1.6 \times 10^{-21}$ eV/$c^2$ (see also Ref. [36]). This constraint is one expected for various observational scenarios: the dashed green (light) line corresponds to the data used in this analysis, the continuous orange (light) line corresponds to data that will be available by the end of 2018. The two red (dark) lines include 16 additional years of observations with two astrometric observations and one spectroscopic observation per year with the following astrometric (spectroscopic) accuracy for an S0-2-like star: current Keck accuracy, 0.5 mas (30 km/s); TMT-like improved accuracy, 15 μas (5 km/s).

Hees et al. (2017)
Orbital precession due to general central-force perturbations

A general expression for apocenter shifts for Newtonian potential and small perturbing potential is given as a solution of problem 3 in Section 15 in the classical Landau & Lifshitz (L & L) textbook [Mechanics]. Orbital precession $\Delta \phi$ per orbital period, induced by small perturbations to the Newtonian gravitational potential $\Phi_N(r) = -\frac{GM}{r}$ could be evaluated as:

$$\Delta \phi^{rad} = \frac{-2L}{GMe^2} \int_{-1}^{1} \frac{z \cdot dz}{\sqrt{1 - z^2}} \frac{dV(z)}{dz},$$

(2)
while in the textbook it was given in the form

\[
\Delta \varphi^{rad} = \frac{-2L}{GM_e} \int_0^\pi \cos \varphi r^2 \frac{dV(r)}{dr} d\varphi, \quad (3)
\]

where \( V(z) \) is the perturbing potential, \( r \) is related to \( z \) via: \( r = \frac{L}{1 + ez} \) in Eq. (2) (and \( r = \frac{L}{1 + e \cos \varphi} \) in Eq. (3)), and \( L \) being the semilatus rectum of the orbital ellipse with semi-major axis \( a \) and eccentricity \( e \):

\[
L = a \left( 1 - e^2 \right), \quad (4)
\]

while \( \Delta \varphi \) represents true precession in the orbital plane, and the corresponding apparent value \( \Delta s \), as seen from Earth at distance \( R_0 \),
is (assuming that stellar orbit is perpendicular to line of sight and taking into account an inclination of orbit one has to write an additional factor which is slightly less than 1 in the following expression):

\[ \Delta s \approx \frac{a(1 + e)}{R_0} \Delta \varphi. \] (5)

In order to compare the orbital precession of S-stars in both GR and Yukawa gravity, we applied the same procedure as described in to perform the two-body simulations of the stellar orbits in the framework of these two theories.
Orbital precession in General Relativity

In the case of GR: $\beta = 1$ and $\gamma = 1$, and then this PPN correction induces the well known expression for Schwarzschild precession:

$$\Delta \varphi^{rad} \approx \frac{2\pi GM}{c^2 a(1 - e^2)} (2 + 2\gamma - \beta)$$

or

$$\Delta \varphi^{rad}_{GR} \approx \frac{6\pi GM}{c^2 a(1 - e^2)}. \quad (6)$$

The corresponding formula for apparent precession $\Delta s_{GR}$, as seen from Earth at distance $R_0$, could be calculated according to

$$\Delta s_{GR} \approx \frac{6\pi GM}{c^2 (1 - e) R_0}.$$
which does not depend on $a$ in the case of GR.
In order to simulate orbits of S-stars in Yukawa gravity we assumed the following gravitational potential (Borka et al. 2013):

\[
\Phi_Y(r) = -\frac{GM}{(1 + \delta)r} \left[ 1 + \delta e^{-\frac{r}{\Lambda}} \right],
\]

where \( \Lambda \) is the range of Yukawa interaction and \( \delta \) is a universal constant. Here we will assume that \( \delta > 0 \), as indicated by data analysis of astronomical observations. Yukawa gravity induces a perturbation to the Newtonian
gravitational potential described by the following perturbing potential:

\[ V_Y(r) = \Phi_Y(r) + \frac{GM}{r} = \frac{\delta}{1 + \delta} \frac{GM}{r} \left[ 1 - e^{-\frac{r}{\Lambda}} \right] \]  \hspace{1cm} (10)

The exact analytical expression for orbital precession in the case of the above perturbing potential could be presented in the integral form Eqs. (2) and (3), but we will calculate the approximate expression for \( \Delta \varphi \) using power series expansion of \( V_Y(r) \), assuming that \( r \ll \Lambda \):

\[ V_Y(r) \approx -\frac{\delta GMr}{2(1 + \delta)\Lambda^2} \left[ 1 - \frac{r}{3\Lambda} + \frac{r^2}{12\Lambda^2} - \cdots \right], \quad r \ll \Lambda, \]  \hspace{1cm} (11)

where we neglected the constant term since it does not affect \( \Delta \varphi \). By substituting the above expression into (2) we obtain the following
approximation for the angle of orbital precession in Yukawa gravity:

\[
\Delta \varphi_{Y}^{rad} \approx \frac{\pi \delta \sqrt{1 - e^2}}{1 + \delta} \left( \frac{a^2}{\Lambda^2} - \frac{a^3}{\Lambda^3} + \frac{4 + e^2}{8} \frac{a^4}{\Lambda^4} - \ldots \right). \tag{12}
\]

The right-hand side in Eq. (12) could be presented as series of Gauss's hypergeometric function \(2F_1\) with different arguments. Since \(r \ll \Lambda\) also implies that \(a \ll \Lambda\), we can neglect higher order terms in the above expansion. The first order term then yields the following approximate formula for orbital precession:

\[
\Delta \varphi_{Y}^{rad} \approx \frac{\pi \delta \sqrt{1 - e^2}}{1 + \delta} \frac{a^2}{\Lambda^2}, \quad a \ll \Lambda. \tag{13}
\]

Both, \(\Delta \varphi_{GR}\) and \(\Delta \varphi_{Y}\) represent the angles of orbital precession per orbital period in the orbital plane (i.e. true precession). The corresponding
apparent values in Yukawa case $\Delta s_Y$, as seen from Earth at distance $R_0$ is (for $\delta = 1$) according to (Weinberg et al. 2005):

\[
\Delta s_Y \approx \frac{a(1 + e)}{R_0} \Delta \varphi_Y^{rad} \approx 0.5\pi \frac{a^3}{R_0 \Lambda^2} (1 + e) \sqrt{1 - e^2}.
\] (14)

If one believes that a gravitational field at the Galactic Center is described with a Yukawa potential, then the maximal $\Delta s_Y$ value corresponds to $e = 1/2$ when function $(1 + e) \sqrt{1 - e^2}$ has its maximal value (assuming that all other parameters are fixed).
Expectations to constrain the range of Yukawa gravity with future observations

We assume that in future GR predictions about precession angles for bright star orbits around the Galactic Center will be successfully confirmed, therefore, for each star we have a constraint on $\Lambda$ which can be obtained from the condition for $\Lambda$, so that Yukawa gravity induces the same orbital precession as GR. This constraint can be obtained directly from (7) and (13), assuming that $\Delta \varphi_Y = \Delta \varphi_{GR}$. In this way we obtain that:

$$\Lambda \approx \sqrt{\frac{\delta c^2 (a \sqrt{1 - e^2})^3}{6(1 + \delta)GM}}.$$  \hspace{1cm} (15)

As it can be seen from the above expression, taking into account that $\delta$ is universal constant, the corresponding values of $\Lambda$ in the case of all S-stars
depend only on the semi-major axes and eccentricities of their orbits. In order to stay in accordance with (Zakharov et al., 2016), here we will also assume that \( \delta = 1 \), in which case formula (15) reduces to:

\[
\Lambda \approx \frac{c}{2} \sqrt{\frac{(a\sqrt{1-e^2})^3}{3GM}} \approx \sqrt{\frac{(a\sqrt{1-e^2})^3}{6R_s}},
\]  

Using Kepler law we could write the previous equation in the following form

\[
\Lambda \approx \frac{T}{T_0} \sqrt{\frac{(a_0\sqrt{1-e^2})^3}{6R_s}}.
\]
Constraints on (tidal) charge of the supermassive black hole at the Galactic Center with trajectories of bright stars

In paper (Dadhich et al. 2001) it was shown that the Reissner – Nordström metric with a tidal charge could arise in Randall – Sundrum model with an extra dimension. Astrophysical of braneworld black holes are considered assuming that they could substitute conventional black holes in astronomy, in particular, geodesics and shadows in Kerr – Newman braneworld metric are analyzed in (Schee and Stuchlik, 2009a), while profiles of emission lines generated by rings orbiting braneworld Kerr black hole are considered in (Schee and Stuchlik, 2009a). Later it was proposed to consider signatures of gravitational lensing assuming a presence of the Reissner – Nordström black hole with a tidal charge at the Galactic Center (Bin-Nun
2010a, 2010b, 2011). In paper (Zakharov, 2014) analytical expressions for shadow radius of Reissner – Nordström black hole have been derived while shadow sizes for Schwarzschild – de Sitter (Köttler) metric have been found in papers (Stuchlik 1983, Zakharov 2014). In the paper we derive analytical expressions for Reissner – Nordström – de-Sitter metric in post-Newtonian approximation and discuss constraints on (tidal) charge from current and future observations of bright stars near the Galactic Center.
Basic notations

We use a system of units where $G = c = 1$. The line element of the spherically symmetric Reissner – Nordström – de-Sitter metric is

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\theta^2 + r^2 \sin^2 \theta d\phi^2,$$

where function $f(r)$ is defined as

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{1}{3}\Lambda r^2.$$

Here $M$ is a black hole mass, $Q$ is its charge and $\Lambda$ is cosmological constant. In the case of a tidal charge (Dadnich et al. 2001), $Q^2$ could be negative.
Similarly to Carter (1973), geodesics could be obtained the Lagrangian

$$\mathcal{L} = -\frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda},$$

(20)

where \( g_{\mu\nu} \) are the components of metric (18). There are three constants of motion for geodesics which correspond metric (18), namely

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = m,$$

(21)

which is a test particle mass and two constants connected with an independence of the metric on \( \phi \) and \( t \) coordinates, respectively

$$g_{\phi\nu} \frac{dx^\nu}{d\lambda} = h,$$

(22)
and

\[ g_{t\nu} \frac{dx^\nu}{d\lambda} = E. \] (23)

For vanishing \( \Lambda \)-term these integrals of motion \((h \text{ and } E)\) could be interpreted as angular momentum and energy of a test particle, respectively. Geodesics for massive particles could be written in the following form

\[ r^4 \frac{dr^2}{d\lambda} = E^2 r^4 - \Delta (m^2 r^2 + h^2), \] (24)

where

\[ \Delta = \left(1 - \frac{1}{3} \Lambda r^2\right) r^2 - 2Mr + Q^2. \] (25)
or we could write Eq. (24) in the following form

\[ r^4 \left( \frac{dr}{d\tau} \right)^2 = (\hat{E}^2 - 1) r^4 + 2Mr^3 - Q^2 r^2 - \frac{1}{3} \Lambda r^6 - \hat{h}^2 (r^2 - \frac{\Lambda}{3} r^4 - 2Mr + Q^2), \]

(26)

where \( \hat{E} = \frac{E}{m} \) and \( \hat{h} = \frac{h}{m} \). We will omit symbol \( \wedge \) below. Since

\[ r^4 \left( \frac{d\phi}{d\tau} \right)^2 = \hat{h}^2, \]

(27)

one could obtain

\[ \left( \frac{dr}{d\phi} \right)^2 = \frac{1}{\hat{h}^2} (E^2 - 1) r^4 + \frac{2Mr^3}{\hat{h}^2} - \frac{Q^2 r^2}{\hat{h}^2} + \frac{1}{3} \Lambda r^6 - (r^2 - \frac{\Lambda}{3} r^4 - 2Mr + Q^2), \]

(28)
It is convenient to introduce new variable \( u = 1/r \). Since

\[
(\frac{du}{d\tau})^2 = (\frac{dr}{d\phi})^2 u^4,
\]

one obtains

\[
(\frac{du}{d\tau})^2 = \frac{1}{h^2}(E^2 - 1) + \frac{2M u}{h^2} - \frac{Q^2 u^2}{h^2} + \frac{\Lambda}{3h^2 u^2} - (u^2 - \frac{\Lambda}{3} - 2Mu^3 + Q^2 u^4),
\]

therefore,

\[
\frac{d^2u}{d\tau^2} + u = \frac{M}{h^2} + 3Mu^2 - \frac{Q^2 u}{h^2} - 2Q^2 u^3 - \frac{\Lambda}{3h^2 u^3},
\]

and as it is known the first term in the right hand side of Eq. (31) corresponds to the Newtonian case, the second term corresponds to the
GR correction from the Schwarzschild metric, while third and forth term correspond to a presence of $Q$ parameter in metric (18), the fifth term corresponds to a $\Lambda$-term presence in the metric. Assuming that second, third, forth and fifth terms in the right hand side of Eq. (31) are small in respect to the basic Newtonian solution, one could evaluate relativistic precession for each term and after that one has to calculate an algebraic sum of all shifts induced by different terms.
Relativistic precession evaluation

An expression for apocenter (pericenter) shifts for Newtonian potential plus small perturbing function is given as a solution in the classical (L \& L) textbook (Landau and Lifshitz, 1976) (see also applications of the expressions for calculations of stellar orbit precessions in presence of the the supermassive black hole and dark matter at the Galactic Center (Dokuchaev and Eroshenko 2015a, 2015b). In paper (Adkins and McDonnell 2007), the authors derived the expression which is equivalent to the (L \& L) relation and which can be used for our needs. According to the procedure proposed in (Adkins and McDonnell 2007) one could re-write Eq. (31) in the following form

\[
\frac{d^2u}{d\tau^2} + u = \frac{M}{\hbar^2} - \frac{g(u)}{\hbar^2},
\]

(32)

where \(g(u)\) is a perturbing function which is supposed to be small and it
could be presented as a conservative force in the following form

\[ g(u) = r^2 F(r)|_{r=1/u}, \quad F(r) = -\frac{dV}{dr}. \quad (33) \]

For potential \( V(r) = \frac{\alpha - (n+1)}{r^{-(n+1)}} \) (where \( n \) is a natural number) one obtains (Adkins and McDonnell 2007)

\[ \Delta \theta(-(n+1)) = \frac{-\pi \alpha_{-(n+1)} \chi_n^2(e)}{ML^n}, \quad (34) \]

where

\[ \chi_n^2(e) = n(n+1)_{2F_1}\left(\frac{1}{2} - \frac{n}{2},\frac{1}{2} - \frac{n}{2},2,e^2\right), \quad (35) \]

\( 2F_1 \) is the Gauss hypergeometrical function, \( L \) is the semilatus rectum \( (L = h^2/M) \) and we have \( L = a(1 - e^2) \) (\( a \) is semi-major axis and \( e \) is eccentricity).
Adkins and McDonnell (2007) obtained orbital precessions for positive powers of perturbing function

$$\Delta \theta(n) = \frac{-\pi \alpha_n a^{n+1} \sqrt{1 - e^2 \chi_n^2(e)}}{M}.$$  

(36)

For GR term in Eq. (31) the perturbing potential is $V_{GR}(r) = -\frac{M h^2}{r^3}$ and one obtains the well-known result $n = 2$ (see, for instance (Adkins and McDonnell 2007) and textbooks on GR)

$$\Delta \theta(GR) := \Delta \theta(-(3)) = \frac{6\pi M}{L}.$$  

(37)

For the third term in Eq. (31) one has potential $V_{RN1}(r) = \frac{Q^2}{2r^2} (\alpha_{-2} = \frac{Q^2}{2}$
and $n = 1$), therefore, one obtains

$$\Delta \theta(RN1) := \Delta \theta(-(2))_{RN1} = -\frac{\pi Q^2}{ML}. \quad (38)$$

For the forth term in Eq. (31) one has potential $V_{RN2}(r) = \frac{h^2 Q^2}{2r^4}$

($\alpha_{-4} = \frac{h^2 Q^2}{2}$ and $n = 3$), therefore, one obtains

$$\Delta \theta(RN2) := \Delta \theta(-(4))_{RN2} = -\frac{3\pi Q^2(4 + e^2)}{2L^2}. \quad (39)$$

Since according to our assumptions $M \ll L$, one has $\frac{Q^2}{L^2} \ll \frac{Q^2}{ML}$ and we ignore the apocenter (pericenter) shift which is described with Eq. (39). For the fifth (de-Sitter or anti-de-Sitter) term in Eq. (31) one has potential
\[ V_{dS}(r) = -\frac{\Lambda r^2}{6} \]  \( \alpha_2 = -\frac{\Lambda}{6} \) and one has the corresponding apocenter (pericenter) shift (Adkins and McDonnell 2007) (see also, (Kerr et al. 2003, Sereno and Jetzer 2006)

\[
\Delta \theta(\Lambda) := \Delta \theta(2)_{dS} = \frac{\pi \Lambda a^3 \sqrt{1 - e^2}}{M}.
\]  

(40)

Therefore, a total shift of a pericenter is

\[
\Delta \theta(\text{total}) := \frac{6\pi M}{L} - \frac{\pi Q^2}{ML} + \frac{\pi \Lambda a^3 \sqrt{1 - e^2}}{M}.
\]  

(41)

and one has a relativistic advance for a tidal charge with \( Q^2 < 0 \) and apocenter shift dependences on eccentricity and semi-major axis are the same for GR and Reissner – Nordström advance but corresponding factors
$(6\pi M \text{ and } -\frac{\pi Q^2}{M})$ are different, therefore, it is very hard to distinguish a presence of a tidal charge and black hole mass evaluation uncertainties. For $Q^2 > 0$, there is an apocenter shift in the opposite direction in respect to GR advance.
Estimates

As it was noted by the astronomers of the Keck group (Hees et al. 2017), pericenter shift has not be found yet for S2 star, however, an upper confidence limit on a linear drift is constrained

\[ |\dot{\omega}| < 1.7 \times 10^{-3}\, \text{rad/yr}. \]  \hfill (42)

at 95% C.L., while GR advance for the pericenter is (Hees et al. 2017)

\[ |\dot{\omega}_{GR}| = \frac{6\pi GM}{Pc^2(1 - e^2)} = 1.6 \times 10^{-4}\, \text{rad/yr}, \]  \hfill (43)

where \( P \) is the orbital period for S2 star (in this section we use dimensional constants \( G \) and \( c \) instead of geometrical units). Based on such estimates
one could constrain alternative theories of gravity following the approach used in (Hees et al. 2017).
Estimates of (tidal) charge constraints

Assuming $\Lambda = 0$ we consider constraints on $Q^2$ parameter from previous and future observations of S2 star. One could re-write orbital precession in dimensional form

$$
\dot{\omega}_{RN} = \frac{\pi Q^2}{P G M L},
$$

(44)

where $P$ is an orbital period. Taking into account a sign of pericenter shift for a tidal charge with $Q^2 < 0$, one has

$$
\dot{\omega}_{RN} < 1.54 \times 10^{-3} \text{rad/yr} \approx 9.625 \dot{\omega}_{GR},
$$

(45)

therefore,

$$
-57.75 M^2 < Q^2 < 0,
$$

(46)
with 95% C. L. For $Q^2 > 0$, one has

$$|\dot{\omega}_{RN}| < 1.86 \times 10^{-3} \text{rad/yr} \approx 11.625 \dot{\omega}_{GR},$$

(47)

therefore,

$$0 < |Q| < 8.3516M,$$

(48)

with 95% C. L. As it was noted in (Hees et al. 2017) in 2018 after the pericenter passage of S2 star the current uncertainties of $|\dot{\omega}|$ will be improved by a factor 2, so for a tidal charge with $Q^2 < 0$, one has

$$\dot{\omega}_{RN} < 6.9 \times 10^{-4} \text{rad/yr} \approx 4.31 \dot{\omega}_{GR},$$

(49)

$$-25.875M^2 < Q^2 < 0,$$

(50)

For $Q^2 > 0$, one has

$$|\dot{\omega}_{RN}| < 9.1 \times 10^{-4} \text{rad/yr} \approx 5.69 \dot{\omega}_{GR},$$

(51)
therefore,

\[ 0 < |Q| < 5.80M, \]  \hspace{1cm} (52)

One could expect that subsequent observations with VLT, Keck, GRAVITY, E-ELT and TMT will significantly improve an observational constraint on \(|\dot{\omega}|\), therefore, one could expect that a range of possible values of \(Q\) parameter would be essentially reduced.

As it was noted in paper (Hees et al. 2017), currently Keck astrometric uncertainty is around \(\sigma = 0.16\) mas, therefore, an angle \(\delta = 2\sigma\) (or two standard deviations) is measurable with around 95% C.L. In this case \(\Delta \theta(GR)_{S2} = 2.59\delta\) for S2 star where we adopt \(\Delta \theta(GR)_{S2} \approx 0.83\). Assuming that GR predictions about orbital precession will be confirmed in the next 16 years with \(\delta\) accuracy (or \(\left| \frac{\pi Q^2}{ML} \right| \lesssim \delta\)), one could constrain \(Q\) parameter

\[ |Q^2| \lesssim 2.32M^2, \]  \hspace{1cm} (53)
where we wrote absolute value of $Q^2$ since for a tidal charge $Q^2$ could be negative.

If we adopt for TMT-like scenario uncertainty $\sigma_{TMT} = 0.015$ mas as it was used in (Hees et al. 2017) ($\delta_{TMT} = 2\sigma_{TMT}$) or in this case $\Delta \theta(GR)_{S2} = 27.67\delta_{TMT}$ for S2 star and assuming again that GR predictions about orbital precession of S2 star will be confirmed with $\delta_{TMT}$ accuracy (or $\left| \frac{\pi Q^2}{ML} \right| \lesssim \delta_{TMT}$), one could conclude that

$$|Q^2| \lesssim 0.216 M^2,$$

or based on results of future observations one could expect to reduce significantly a possible range of $Q^2$ parameter in comparison with a possible hypothetical range of $Q^2$ parameter which was discussed in (Bin-Nun 2010a, 2010b).
Estimates of $\Lambda$-term constraints

In this subsection we assume that $Q = 0$. One could re-write orbital precession in dimensional form

$$\dot{\omega}_\Lambda = \frac{\pi \Lambda c^2 a^3 \sqrt{1 - e^2}}{PGM},$$

(55)

Dependences of functions $\dot{\omega}_\Lambda$ and $\dot{\omega}_\text{GR}$ on eccentricity and semi-major axis are different and orbits with higher semi-major axis and smaller eccentricity could provide a better estimate of $\Lambda$-term (the S2 star orbit has a rather high eccentricity). However, we use observational constraints for S2 star. For positive $\Lambda$, one has relativistic advance and

$$\dot{\omega}_\Lambda < 1.54 \times 10^{-3}\text{rad/yr} \approx 9.625 \dot{\omega}_\text{GR},$$

(56)
or

$$0 < \Lambda < 3.9 \times 10^{-39} \text{cm}^{-2},$$ \hspace{1cm} (57)

for $\Lambda < 0$ one has

$$0 < -\Lambda < 4.68 \times 10^{-39} \text{cm}^{-2},$$ \hspace{1cm} (58)

if we use current accuracy of Keck astrometric measurements $\sigma = 0.16$ mas and monitor S2 star for 16 years and assume that additional apocenter shift ($2\sigma$) could be caused by a presence of $\Lambda$-term, one obtains

$$|\Lambda| < 1.56 \times 10^{-40} \text{cm}^{-2},$$ \hspace{1cm} (59)

while for TMT-like accuracy $\delta_{TMT} = 0.015$ mas one has

$$|\Lambda| < 1.46 \times 10^{-41} \text{cm}^{-2}.$$ \hspace{1cm} (60)
As one can see, constraints on cosmological constant from orbital precession of bright stars near the Galactic Center are much weaker than not only its cosmological estimates but also than its estimates from Solar system data.
Conclusions

We consider the first relativistic corrections for apocenter shifts in post-Newtonian approximation for the case of Reissner – Nordström – de-Sitter metric. Among different theoretical models have been proposed for the Galactic Center different black hole models are rather natural. Perhaps, assumptions about spherical symmetry and a presence of electric charge in the metric do not look very realistic because a space media is usually quasi-neutral, but the charged black holes are discussed in the literature. Moreover, a Reissner – Nordström metric could arise in a natural way in alternative theories of gravity like Reissner – Nordström solutions with a tidal charge in Randall–Sundrum model (Dadnich et al. 2001) (such an approach is widely discussed in the literature). Recently, it was found that Reissner – Nordström metric is a rather natural solution in Horndeski gravity (Babichev et al. 2017) and in this case $Q^2$ parameter reflects an
interaction with a scalar field and it could be also negative similarly to a tidal charge. In paper (Babichev et al. 2017) it was expressed an opinion that the hairy black hole solutions look rather realistic and these objects could exist in centers of galaxies and if such objects (hairy black holes in Horndeski gravity) exist in nature, in particular in the Galactic Center, current and future advanced facilities such as GRAVITY, E-ELT, TMT etc. may be very useful to detect signatures of black hole hairs of an additional dimension. Therefore, non-vanishing (positive or negative) $Q^2$ parameter is arisen due to a presence of extra dimension or in Hordeski gravity for black holes with a scalar hair. We outline a procedure to constrain $Q^2$ parameter with current and future observations of bright stars at the Galactic Center.
Conclusions

1. Range of Yukawa gravity $\Lambda$ can be constrained in such a way to induce the same orbital precession of stellar orbits as in GR;

2. Orbits with small eccentricities provide better constraints on graviton mass (see Eq. (17));

3. There is a linear dependence of $\Lambda$ constraint on orbital periods of bright stars (perhaps, monitoring bright stars for more than 50 – 100 years looks rather problematic, but people monitor comets for centuries);

4. Such a precession of stellar orbits around GC, if observed, could provide strong constraints on the mass of graviton, indicating that it is most
likely $\approx 8 \times 10^{-23}$ eV for orbital periods around a several decades, (see, parameters for S9 star, for instance).

5. If we assume that the future telescopes give an opportunity to a bright star with a period around 50 years and small eccentricity then the similar procedure give a graviton mass constraint around $\approx (5 - 6) \times 10^{-23}$ eV, while pericenter advance for such a star will be $\Delta s_{GR} \approx 0.1$ mas which is in principle should be detectable in the future.
graviton MASS in graviton

Van Dam and Veltman (VANDAM 1970), Iwasaki (IWASAKI 1970), and Zakharov (ZAKHAROV 1970) almost simultaneously showed that... there is a discrete difference between the theory with zero-mass and a theory with finite mass, no matter how small as compared to all external momenta.” The resolution of this “DVZ discontinuity” has to do with whether the linear approximation is valid. De Rham et al. (DE-RHAM 2011) have shown that nonlinear effects not captured in their linear treatment can give rise to a screening mechanism, allowing for massive gravity theories. See also GOLDHABER 2010 and DE-RHAM 2017 and references therein. Experimental limits have been set based on a Yukawa potential or signal dispersion. $\lambda_{0}$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$.

The following conversions are useful: $1 \, \text{eV} \approx 1.783 \times 10^{-33} \text{g} = 1.957 \times 10^{-8} \text{m}_{\mu} \approx (1.973 \times 10^{-7} \text{ m}) \times (1 \, \text{eV}/m_{\mu})$.

<table>
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<th>VALUE (eV)</th>
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<td>1 CHOUHURY 2004</td>
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<td>22 star orbit</td>
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<td>$2\gamma$ decay</td>
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</table>

1 CHOUHURY 2004 concludes from a study of weak-lensing data that masses heavier than about the inverse of 100 Mpc seem to be ruled out if the gravitational field has the Yukawa form.

2 DESAI 2016 limit based on dynamical mass models of galaxy cluster Abell 1689.

3 RANA 2018 limit, 86% CL, obtained using weak lensing mass profiles out to the radius at which the cluster density falls to 200 times the critical density of the Universe. Limit is based on the fractional change between Newtonian and Yukawa accelerations for the 50 most massive galaxy clusters in the Local Cluster Substructure Survey. Limits for other CL’s and other density cuts are also given.

4 RANA 2018 limit, 86% CL, obtained using mass measurements via the SZ effect out to the radius at which the cluster density falls to 500 times the critical density of the Universe for 128 optically confirmed galaxy clusters in an All-sky Cosmology Telescope survey. Limits for other CL’s and other density cuts are also given.

5 ABBOTT 2016 and ABBOTT 2017 assumed a dispersion relation for gravitational waves modified relative to GR.

6 ZAKHAROV 2016 constrains range of Yukawa gravity interaction from 62 star orbit about black hole at Galactic center. The limit is $< 2.9 \times 10^{-51} \text{eV}^2$ for $f = 100$.  

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Thank you very much for your kind attention