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Gauge-Higgs Unification in Six Dimensions

Andrea Wulzer

SISSA, Trieste

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with C.A. Scrucca, M. Serone and L. Silvestrini

In spite of its phenomenological success, the Standard Model of Electroweak interactions (SM) suffers of an Hierarchy Problem in the Higgs sector.

Namely, the Higgs mass (and then the EW symmetry breaking (EWSB) scale) grows linearly with the cut-off Λ.

Therefore, in an effective field theory approach to the SM, new physics is expected at the TeV scale to stabilize the EWSB scale.

Standard solution: Supersymmetry (SUSY)

The phenomenological success of the SM is pushing up lower bounds on the scale of new physics (Barbieri, Strumia (1999))

$$\Rightarrow \text{little hierarchy problem}$$

The little hierarchy problem also affects SUSY and motivates the search for alternative mechanisms leading to a stable EWSB scale.

The Gauge-Higgs Unification (GHU) models (Manton (1979); Fairlie (1979); Forgacs and Manton (1980)) may solve the little hierarchy problem

We consider non-supersymmetric, extra-dimensional gauge theories compactified to 4D.

$$A_M \rightarrow \begin{cases} A_\mu = \{\text{massless 4D vector bosons + K.K. towers}\} \\ A_i = \{\text{massless 4D scalars + K.K. towers}\} \end{cases}$$

We want to make

EW bosons to arise from (the zero-modes of) A_μ,

the Higgs from A_i .

The short-distance (UV) dynamics of the Higgs, being insensitive to the compactness of the extra-space, is strongly constrained by extra-dimensional gauge invariance.

A dynamically generated Higgs mass-term, for instance, would be finite for smooth compactifications as it can only arise from non-local operators.

In the case of orbifold compactification, dangerous localized operators can arise at the fixed points and the stabilization of the EWSB scale is not guaranteed.

Anyway, we need orbifolds to obtain 4D chirality and Higgs not in the adjoint of the EW gauge group.

5-dimensional models are very interesting as in that case there are no gauge-invariant localized operators contributing to the Higgs potential (von Gersdorff, Irges, Quirós (2002))

Realistic GHU models in 5D (based on $SU(3)$ gauge group) have been studied (Scrucca, Serone, Silvestrini (2003)). An extra bonus of such models is a natural exponential hierarchy in fermion masses

Basic problem:

$$m_H/m_W \text{ is less than 1}$$

as the full Higgs potential is radiatively generated.

In 6D models, this problem may be solved, due to a tree-level quartic coupling for the Higgs (Arkani-Hamed, Cohen, Georgi (2001)).

• 6D Models

Consider $SU(3)$ gauge theory compactified on the $T^2/\mathbf{Z_N}$ orbifold (N=2,3,4,6)

$\mathbf{Z_N}$ acts as $z \to \tau z \quad (\tau = e^{\frac{2\pi i}{N}})$ on the complex coordinate z of T^2.

The gauge connection $A_M \equiv A_{M,\,A}\, t^A$ is periodic on T^2 and subjected to the b.c.

$$A_M(\tau z) = R_M^{\ N} P\, A_N(z)\, P^\dagger$$

with a gauge twist matrix

$$P = \begin{pmatrix} \tau & 0 & 0 \\ 0 & \tau & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow SU(3) \to SU(2) \times U(1)$$

The $\mathbf{Z_N}$ twists of each field are:

$$A_{\mu,\,a} : 1 \;,\; A_{\mu\,\pm i} : \tau^{\pm 1} \;,\; A_{z,a} : \tau^{-1} \;,\; A_{z,\pm i} : \tau^{-1\pm 1},$$

$$a = \{1,2,3,8\}\,, \quad \pm 1 = 4 \mp i5\,,\; \pm 2 = 6 \mp i7\,,$$

and $a \in \mathbf{3_0} \oplus \mathbf{1_0}\,,\; \pm i \in \mathbf{2}_{\pm\frac{1}{2}}$

We see that (for $N \neq 2$) just one doublet of scalar zero-modes is left by the projection.

We identify $A^0_{\mu,\,a}$ as the EW bosons, $A^0_{z,+i} \equiv H_i$ as the Higgs.

The zero-modes classical Lagrangian contains a potential for H_i

$$-\frac{1}{2}\mathrm{Tr}F^2 \;\longrightarrow\; -g^2\mathrm{Tr}[A^0_z, A^0_{\bar z}]^2 = \lambda(H^\dagger H)^2\,, \quad \lambda = \frac{g^2}{2}$$

Assuming that EWSB occurs, we can consider a quantum potential

$$V(H) = -\mu^2|H|^2 + \lambda|H|^4$$

From this potential we get $m_H = \sqrt{2}v\sqrt{\lambda}$ and $m_W = 1/2gv$ ($v = \mu/\sqrt{\lambda} = \sqrt{2}\langle|H|\rangle$).

Therefore, we find:

$$\frac{m_H}{m_W} = 2$$

We see how the mass ratio is increased in 1-Higgs 6D models with respect to the 5D ones.

Unfortunately, this is not the only new feature of 6D models. Indeed, being the gauge group restricted to $SU(2) \times U(1)$ at the fixed points, we can write the gauge invariant localized operator (tadpole) (Csaki, Grojean, Murayama; von Gersdorff, Irges, Qiurós (2003))

$$F^8_{z\bar{z}}\,\delta(z - z_f) = \left[\partial_z A^8_{\bar{z}} - \partial_{\bar{z}} A^8_z + g_6 f^{8bc} A_{z\,b} A_{\bar{z}\,c}\right]\,\delta(z - z_f)$$

It is quadratically divergent at 1 loop and contains a mass term for the Higgs

We have checked by direct computation that the tadpole is indeed generated already in the pure gauge theory.

One could hope that, adding a suitable bulk fermion content, an accidental 1 loop cancellation will take place.

We have computed the contribution to the tadpole from fermions and scalars in a generic representation of the gauge group. The result is that

The tadpole cannot be canceled at 1 loop

without introducing fundamental scalars

So, is it possible to obtain a stable hierarchy in 6D models?

To answer, consider in more detail the structure of the tadpole:

$$\mathcal{L}_{\text{tad}} = \sum_{\text{fix-points}} \mathcal{C}_i F^8_{z\bar{z}}\, \delta(z - z_i)$$

For $N = 3, 4, 6$ there are $1, 2, 3$ indipendent coefficents.

This is due to discrete symmetries of the $T^2/\mathbf{Z_N}$ orbifolds

Even if the tadpole cancellation cannot arise, the tadpole can be made globally vanishing ($\sum_i \mathcal{C}_i = 0$) by a suitable fermion content, in the $N = 4$ case.

This is enough for stabilization. Indeed, add the tadpole operator to the tree-level action:

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}F^2 + \mathcal{L}_{\mathrm{tad}}$$

Lorentz invariant background field configuration are obtained by minimizing

$$V = \frac{1}{2}\sum_{a=1}^{3}|F^a_{z\bar{z}}|^2 + |F^{-i}_{z\bar{z}}|^2 + \frac{1}{2}\Big|F^8_{z\bar{z}} - i\sum_i \mathcal{C}_i\delta^{(2)}(z - z_i)\Big|^2$$

We see that an absolute minimun of V is at $\langle A_z^{1,2,3}\rangle = \langle A_z^{\pm i}\rangle = 0$, if $\langle A_z^8(z)\rangle$ (which is neutral under $SU(2)\times U(1)$) satisfies the equation

$$F^8_{z\bar{z}}|_{A_z^{\pm i}=0} = (d\langle A^8\rangle)_{z\,\bar{z}} = i\sum_i \mathcal{C}_i\delta^{(2)}(z - z_i)$$

This admits solution for globally vanishing tadpole. Therefore

A globally vanishing tadpole does not induce EWSB

Let us now consider quantum fluctuations around this background. We have found that A_{z+i} admits still a zero mode that we interpret as the Higgs. This means that

A globally vanishing tadpole

does not contribute to the Higgs mass

• Conclusions

We have studied GHU models in 6D on $T^2/\mathbf{Z_N}$

W.r.t. the 5D case, 2 new features appear in 6D

- The Higgs mass is strongly increased due to a tree-level quartic coupling
- The stability of the EWSB scale is not a trivial issue due to a localized tadpole operator

We have shown that is possible to make the integrated tadpole vanish and that, under this condition, the EWSB scale is not destabilized.

This result is promising, but in order to build a realistic model one should find an anomaly-free bulk fermion content compatible with the condition of globally vanishing tadpole. This seems not a trivial issue.