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Unstable particle production

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An effective theory approach

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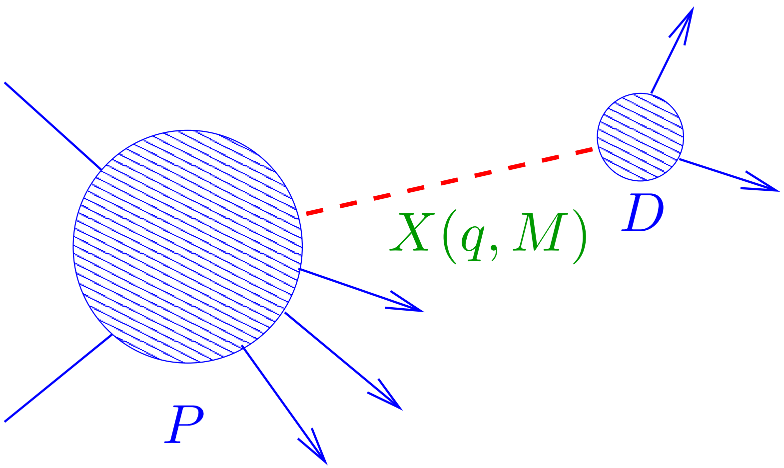
Unstable particles

Study of unstable particles $X \in \{W^{\pm}, Z, t, H, \dots\}$ close to resonance

Physical picture: separation of production, propagation and decay

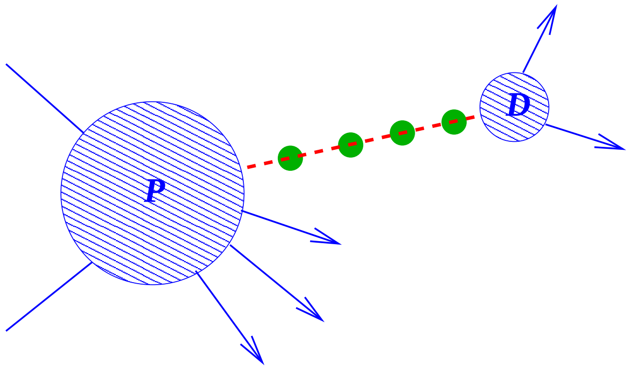


Amplitude:

$$\mathcal{A}^{(\mathrm{tree})}(q^2) = \mathcal{P}(q^2) \frac{i}{q^2 - M^2} \mathcal{D}(q^2)$$

▶ non-integrable singularity for resonant unstable particle, i.e. $q^2 \sim M^2$

The problem



$$\mathcal{A}^{(\text{“tree”})} = P(q^2)\frac{i}{q^2 - M^2 - \Pi(q^2)}D(q^2)$$

$\Pi \Rightarrow$ self energy

- Im$(\Pi) \neq 0$ (finite width) $\rightsquigarrow$ pole *off* the real axis
- Resummation: divergence $\rightsquigarrow$ resonance

However: not a strict order-by-order expansion ⚠

☛ The selection of only some *arbitrary* higher order corrections spoils properties valid order by order in PT ($\Rightarrow$ gauge invariance)! ☹

Various “standard” approaches

Theoretical approaches

- ✗ Fixed width scheme
- ✗ Running width scheme
- ✗ Overall-factor scheme
- ✗ Complex mass scheme
- ✗ Fermion loop scheme
- ✗ The pole approximation

Problems/drawbacks

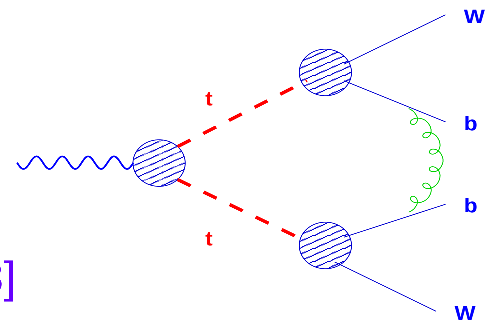
- ⚠ ad-hoc, no physical justification
- ⚠ predictions violate unitary
- ⚠ complex mass and weak mixing angles
- ⚠ unphysical effects off resonance
- ⚠ no hope to improve accuracy

$\Downarrow$

Altogether unsatisfactory theoretical situation!

Beyond DPA

- ✗ At Linear Collider need to go beyond DPA, e.g:
  $\Delta m_t \lesssim 100$ MeV [Heinemeyer et al. hep-ph/0306181,]
- ✗ Problem in Quantum Field Theory ! [Veltman 1963]



Two ways to go beyond

- higher order in α $\Longrightarrow$ beyond one loop
  standard PT expansion
- higher order in Γ/M $\Longrightarrow$ beyond the pole approximation
  how to expand?

Features

- Since $\alpha \sim \Gamma/M$ the two approaches must be interlinked
- Since $\alpha \sim \Gamma/M \ll 1 \Rightarrow$ need a systematic double expansion

Physical scales

Process with unstable particles two scales:

✗ production/decay time	$t \sim 1/E$	$\Lambda_1 \sim E$
✗ propagation time	$\tau_\phi \sim 1/\Gamma \cdot E/M$	$\Lambda_2 \sim M\Gamma/E$

$\Lambda_2 \ll \Lambda_1$ ▶ effective theory

- ▶ need to identify relevant modes
- ▶ "integrate out" unwanted modes

Split radiative corrections

Hard	$\tau_h \sim 1/\Lambda_1 \ll \tau_\phi$	$\Longrightarrow$	factorizable	$\Longrightarrow$	integrated out
Soft/Res.	$\tau_s \sim 1/\Lambda_2 \sim \tau_\phi$	$\Longrightarrow$	non factorizable	$\Longrightarrow$	dynamical modes

underlying theory

$\mathcal{L}(\phi_h, \phi_{r/c}, \phi_s)$

dynamical modes:

hard, resonant/collinear, soft

integrate out hard modes

effective theory

factorizable corrections

non-factorizable corrections

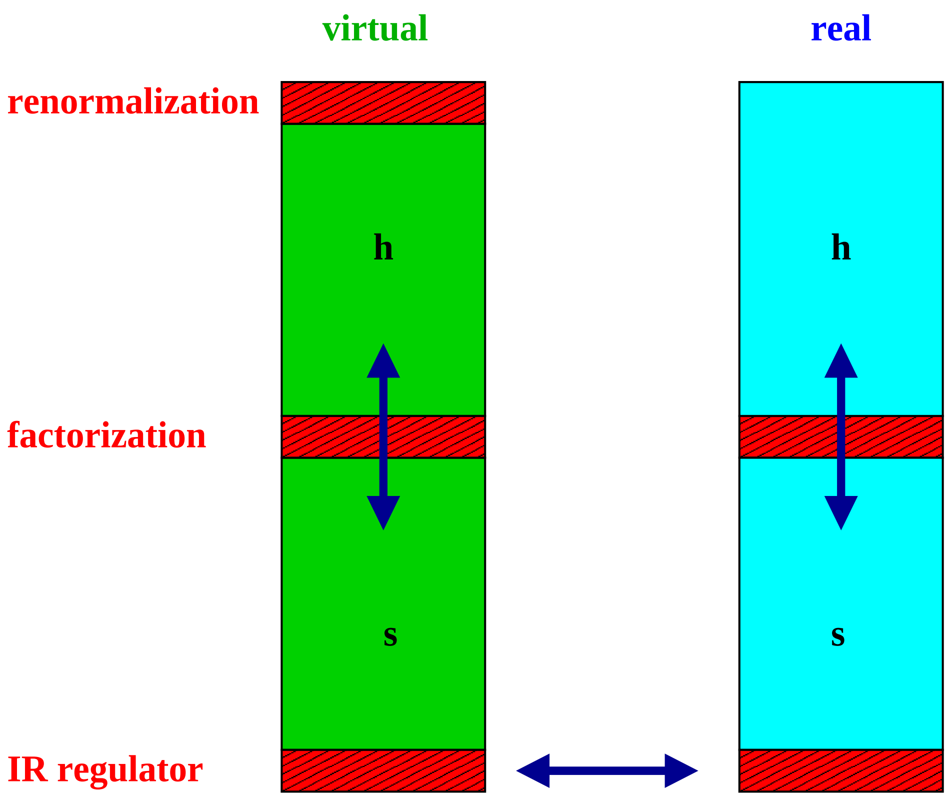
$\mathcal{L}_{\text{eff}} = \sum_n c_n(\text{h})\, O_n(\phi_{r/c}, \phi_s)$

dynamical modes:

resonant/collinear, soft

- While the α expansion is standard, we construct an effective theory to formulate the Γ/M expansion

Singularities



- requires mode expansion of phase space integrals
- relate inclusive PS integrals to loop integrals

Our toy Model

The Lagrangian

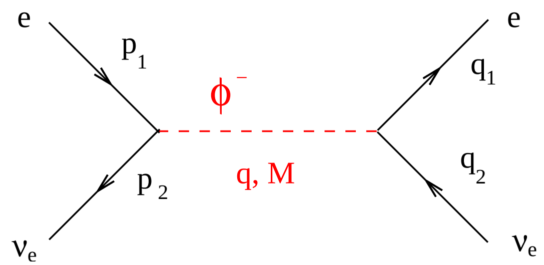
$$\mathcal{L} = (D_\mu\phi)^\dagger D^\mu\phi - M^2\phi^\dagger\phi + \overline{\psi} i \not{D}\psi + \overline{\chi} i \not{\partial}\chi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$+ \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + y\phi\overline{\psi}\chi + y^*\phi^\dagger\overline{\chi}\psi + \frac{\lambda}{4}(\phi^\dagger\phi)^2 + \mathcal{L}_{\rm ct}\ , \qquad D_\mu = \partial_\mu - igA_\mu$$

The process

$$e^-(p_1)\ \bar{\nu}_e(p_2) \quad \rightarrow \quad \phi^-(q, M) \rightarrow e^-(q_1)\ \bar{\nu}_e(q_2)$$

$$q^2 \equiv (p_1 + p_2)^2$$



Close to resonance: $\delta \equiv \dfrac{q^2 - M^2}{M^2} \sim \alpha \sim \Gamma/M \ll 1$

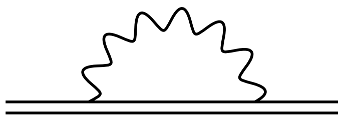
The couplings

$\alpha_g \equiv \dfrac{g^2}{4\pi}\ ,\ \alpha_y \equiv \dfrac{y^2}{4\pi}\ ,\ \alpha_\lambda \equiv \dfrac{\lambda}{4\pi}$ with $\alpha_g \sim \alpha_y \sim \alpha\ ,\ \alpha_\lambda \sim \dfrac{\alpha^2}{4\pi}$

The effective lagrangian

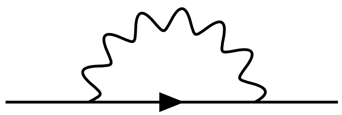
$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm HSET} + \mathcal{L}_{\rm SCET} + \mathcal{L}_{\rm int}$$

Heavy Scalar Effective Theory (HSET)



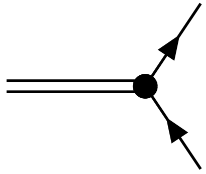
- propagation of the Heavy Scalar and it’s interaction with soft fields

Soft Collinear Effective Theory (SCET)



- propagation of energetic fermions and their interaction with SC fields

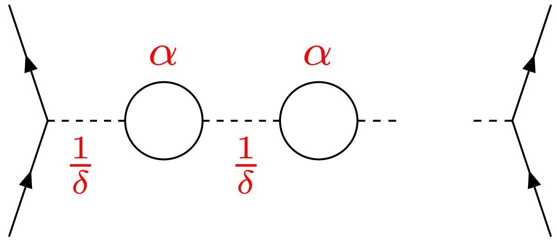
Interaction: production/decay vertices



- interaction terms between heavy scalar and energetic fermions

The amplitude

Defi ne: $\alpha = [\alpha_y, \alpha_g]$; $\quad \delta \equiv \dfrac{q^2 - M^2}{M^2}$



Close to resonance: $\dfrac{\alpha}{\delta} = \mathcal{O}(1) \;\Rightarrow$ double expansion and resummation

$$\begin{aligned}
\mathcal{A}^{\rm tree} &= \left(\frac{\alpha}{\delta}\right) c_{0,0} + \left(\frac{\alpha}{\delta}\right) \delta\, c_{0,1} + \left(\frac{\alpha}{\delta}\right) \delta^2\, c_{0,2} + \dots \\
\mathcal{A}^{\rm 1loop} &= \left(\frac{\alpha}{\delta}\right)^2 c_{1,0} + \left(\frac{\alpha}{\delta}\right)^2 \delta\, c_{1,1} + \left(\frac{\alpha}{\delta}\right)^2 \delta^2\, c_{1,2} + \dots \\
\mathcal{A}^{\rm 2loop} &= \left(\frac{\alpha}{\delta}\right)^3 c_{2,0} + \left(\frac{\alpha}{\delta}\right)^3 \delta\, c_{2,1} + \left(\frac{\alpha}{\delta}\right)^3 \delta^2\, c_{2,2} + \dots
\end{aligned}$$

$$\mathcal{A}^{(0)} \quad + \quad \mathcal{A}^{(1)} \quad + \quad \mathcal{A}^{(2)} \quad + \dots$$

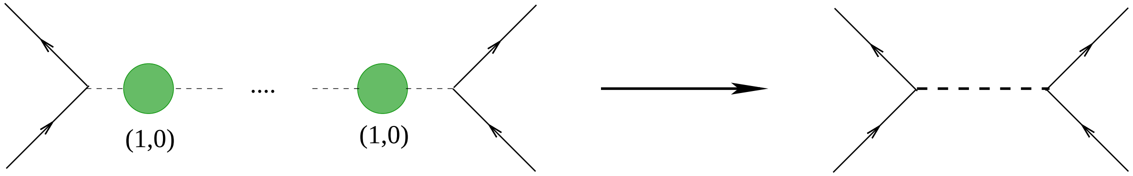
LO NLO NNLO

☞ $N^j LO$ resums all (and only) terms suppressed by j powers of α or δ with respect to LO

$\mathcal{A}^{(j)}$ gauge invariant and well-defi ned for $\delta \to 0$

The amplitude

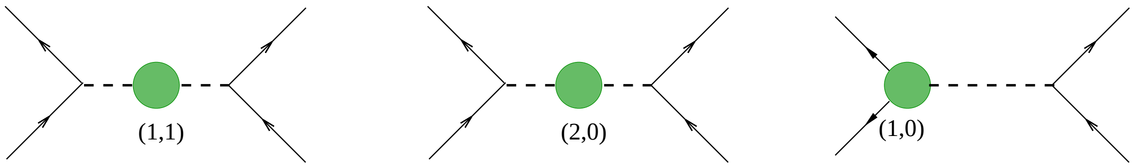
At LO:



$$\mathcal{A}^{(0)} = -4\pi\alpha_y \langle q_2 \,|\, q_1\rangle \langle p_2 \,|\, p_1\rangle \frac{i}{q^2 - M^2 + \Pi_{\phi,\phi}^{h(1,0)}}$$

- strict order by order expansion, no resummation of subleading terms

At NLO:



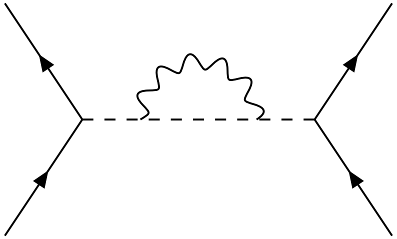
$$\mathcal{A}^{h(1)} = -4\pi\alpha_y \langle q_2 \,|\, q_1\rangle \langle p_2 \,|\, p_1\rangle \left(\frac{i(\Pi_{\phi,\phi}^{h(1,1)} + \Pi_{\phi,\phi}^{h(2,0)})}{(q^2 - M^2 + \Pi_{\phi,\phi}^{h(1,0)})^2} + \frac{i2\Psi_{\phi\phi\chi}^{h(1,0)}}{q^2 - M^2 + \Pi_{\phi,\phi}^{h(1,0)}} \right)$$

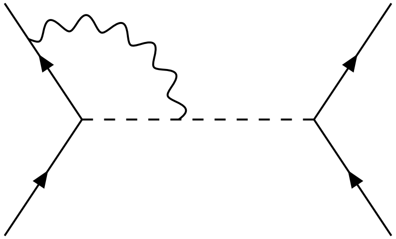
- again strict expansion, $\mathcal{A}^{(1)}$ resums *only* terms $\mathcal{O}\left\{\left(\frac{\alpha}{\delta}\right)^n \delta\right\}$
- but at NLO dynamical soft modes enter

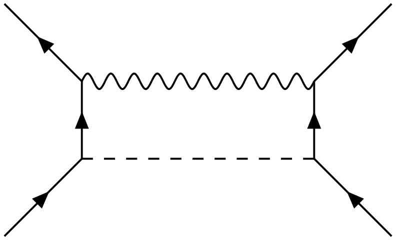
Notation: $X^{h(n,m)}$ term of order $\alpha^n \delta^m$ of *the hard part of* X

Soft contributions to the amplitude at NLO

Examples of $\mathcal{O}\left(\alpha^2\right)$ gauge independent / dependent contributions







	LO + NLO + NNLO	NLO + NNLO	NLO + NNLO
hard	$\alpha^2\delta^{-2} + \alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^0 + \ldots$
soft	$+ \alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$	$\alpha^2\delta^{-1} + \alpha^2\delta^0 + \ldots$

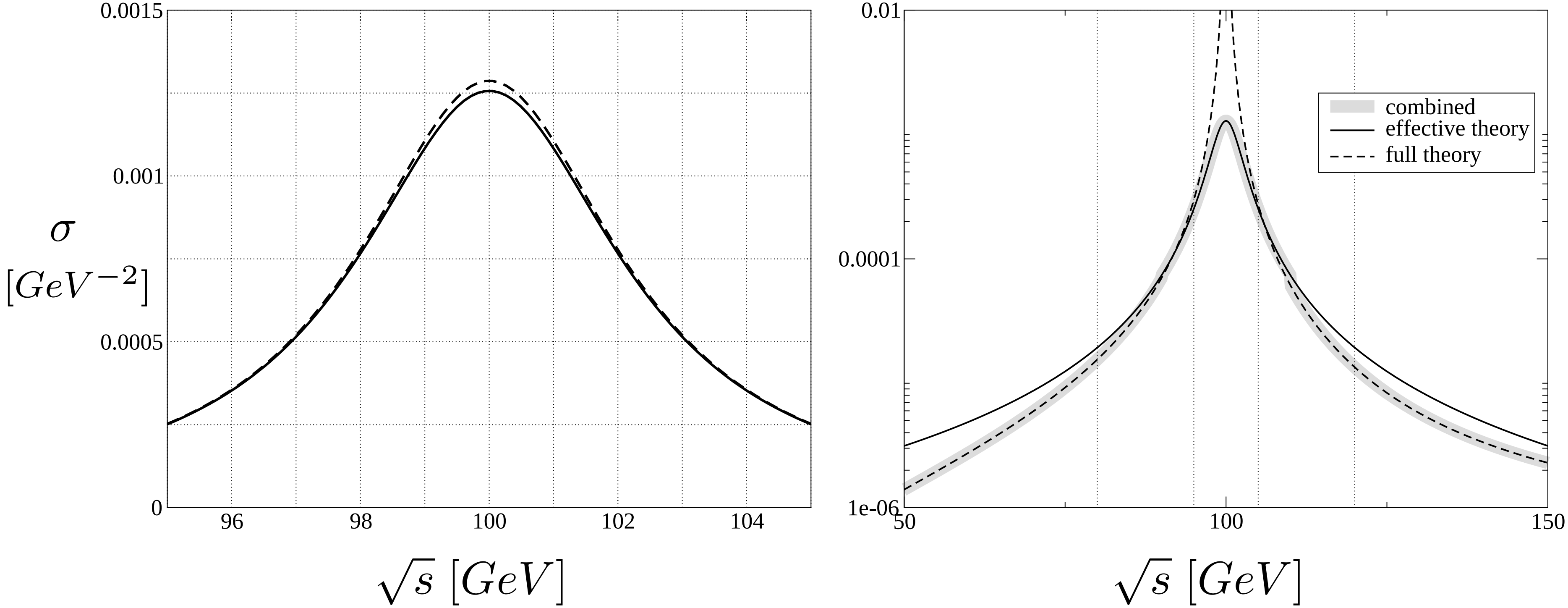
- Soft emissions (dynamical modes of the eff. theory) enter at NLO
- non trivial gauge cancellations at NLO
- systematic generalization to higher orders possible

Plots: Inclusive line-shape

Inputs: $M_{Pole} = 100$GeV, $\alpha_y(M_{pole}) = 0.1$, $\alpha_g(M_{pole}) = 0.1$, $\alpha_\lambda(M_{pole}) = 0.1^2/(4\pi)$

LO line-shape

Dashed: Pole Scheme, Solid: $\overline{MS}$ Scheme ($M^{(1)}_{\overline{MS}} = 98.8$ GeV)

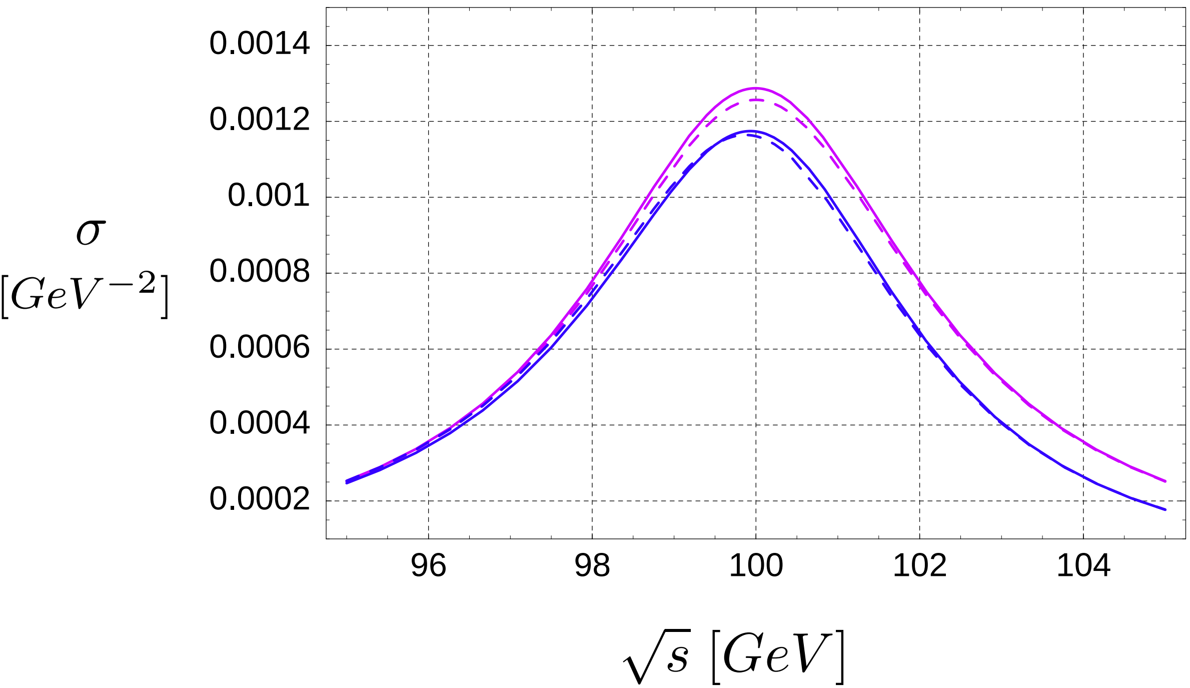


- effective theory result valid where $\sqrt{s} - M \sim \alpha M$ (corrections $+\mathcal{O}(\alpha, \delta)$)
- matching of full and effective theory in intermediate region $\delta \sim \sqrt{s} - M$ needed

Plots: Inclusive line-shape

Inputs: $M_{Pole} = 100$GeV, $\alpha_y(M_{pole}) = 0.1, \ \alpha_g(M_{pole}) = 0.1, \ \alpha_\lambda(M_{pole}) = 0.1^2/(4\pi)$

NLO line-shape



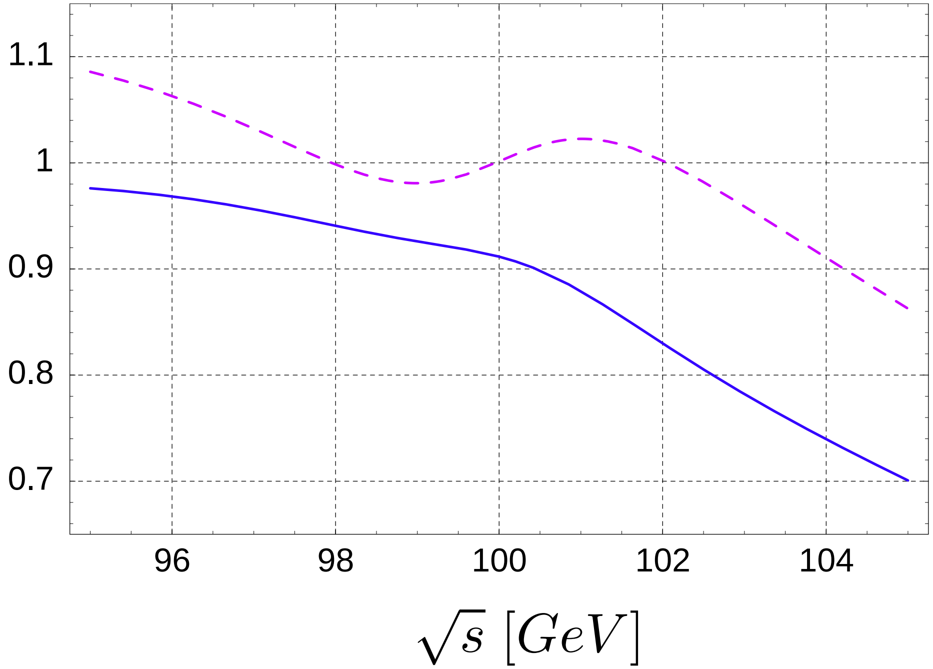
Dashed: Pole Scheme
Solid: $\overline{MS}$ Scheme
($M_{\overline{MS}}^{(2)} = 99.1$ GeV)
Blue: NLO
Magenta: LO

- scheme dependence effect very small
- NLO correction $\sim -10\%$ at the peak and up to -30% in the above range

Plots: Inclusive line-shape

Inputs: $M_{Pole} = 100$GeV, $\alpha_y(M_{pole}) = 0.1, \; \alpha_g(M_{pole}) = 0.1, \; \alpha_\lambda(M_{pole}) = 0.1^2/(4\pi)$

Ratio of NLO line-shapes



Solid: σ_{NLO}/σ_{LO}

Dashed: σ_{NLO}/σ_{BW}

- deviation from Breit-Wigner up to 15%
- output mass parameter of the Breit-Wigner fi t differs from the input $M = 100$GeV by $\delta M = 160$MeV
  - ➥ data should be fi tted to theoretically predicted line-shapes rather than to BW-fi ts!

Summary

- ✗ Perturbative treatment of unstable particles requires partial summation of PT series
- ✗ however the guiding principle of resummation was not understood
- ✗ breakdown of weak coupling PT related to the appearance of a second small parameter ($\alpha, \Gamma/M$)
- ✗ we take the attitude that $\Gamma \ll M$ is *the* characteristic feature
- ✗ other issues (resummation, gauge invariance . . .) follow automatically in a theory that formulates the expansion correctly
- ✗ Two-scale problem $\rightsquigarrow$ effective field theory (H”Q”ET + SCET)
- ✗ mode expansion $\Longrightarrow$ strategy of regions

☛ References

M. Beneke, A. Chapovsky, A. Signer, GZ, hep-ph/0312331 and hep-ph/0401002

Advantages of field theory approach

- calculations split in well-defined pieces (matching, matrix elements, loops)
  $\Rightarrow$ calculation efficient and transparent
- power counting scheme in the small parameters (α, δ)
  $\Rightarrow$ identification of terms required to achieve a certain accuracy
- Feynman rules needed to compute the minimal set of terms required
  $\Rightarrow$ calculations are as simple as possible
- can be extended to any accuracy in α, δ
  (at the price of performing complicated, but standard loop integrals)
- gauge invariance is automatic

What next?

- more realistic cases $\Longrightarrow$ $t\bar{t}$ at the LC [pair-production]
- exclusive observables [real radiation $\leftrightarrow$ phase-space integrals]