Prospects from strings and branes

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References

Not-too-technical review paper, including numerous references:

Strings, Gravity and Particle Physics by Augusto Sagnotti and AS

In the proceedings of 37th Rencontres de Moriond on Electroweak Interactions and Unified Theories, 2002.

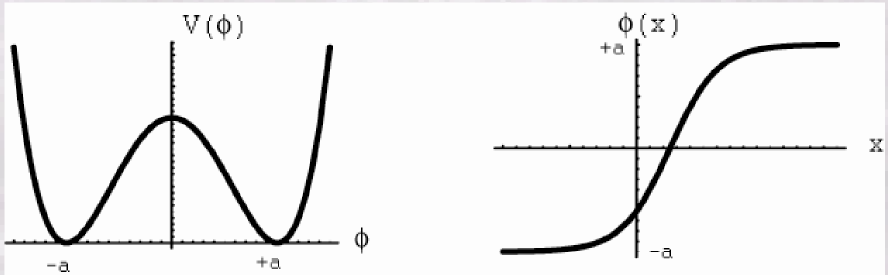
e-Print Archive: hep-ex/0209011

Contents

- Dirichlet-branes
- D-branes and gauge theories
  - Worldvolume point of view
  - AdS/CFT
- D-branes and black holes
- Cosmology
- Some conclusions

Branes

Solitons: solutions of the equations of motion with a finite energy(-density) and a mass inversely proportional to the coupling constant. E.g. Scalar field in d = 1 + 1: kink.



$$\text{mass} = \frac{12m^3}{\lambda}$$

Other example in d = 3 + 1: magnetic monopole:

$$\text{mass} \propto \frac{1}{g_{YM}^2}$$

Solitons in string theory: Dirichlet branes

Besides the “conventional” fields, such as e.g.,

$g_{\mu\nu}(x) = g_{\nu\mu}(x)$: metric = graviton

$\Phi(x)$: dilaton,

one has RR- potentials as well.

E.g. vector potential, A_μ :

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$A_\mu \to A_\mu - \partial_\mu f, \qquad F_{\mu\nu} \to F_{\mu\nu}.$$

Couples to particles:

$$\mathcal{S} = q \int d\tau\, \dot{x}^\mu(\tau) A_\mu(x(\tau)).$$

E.g. 2-form potential, $A_{\mu\nu} = -A_{\nu\mu}$:

$$F_{\mu\nu\rho} = \partial_\mu A_{\nu\rho} + \partial_\nu A_{\rho\mu} + \partial_\rho A_{\mu\nu},$$

$$A_{\mu\nu} \to A_{\mu\nu} - \partial_\mu f_\nu + \partial_\nu f_\mu, \qquad F_{\mu\nu\rho} \to F_{\mu\nu\rho}.$$

Couples to strings:

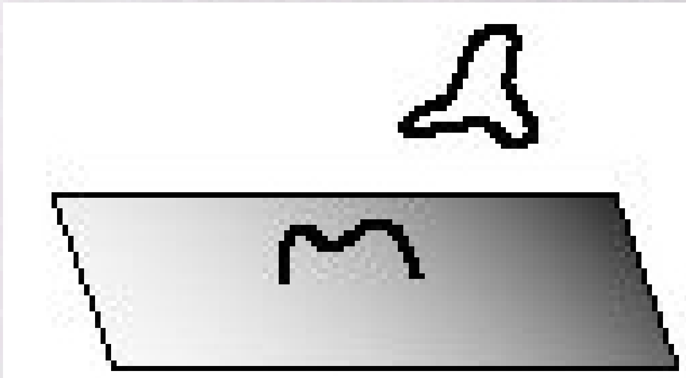
$$\mathcal{S} = q \int d\tau d\sigma \, \dot{x}(\tau,\sigma)^\mu x'(\tau,\sigma)^\nu A_{\mu\nu}(x(\tau,\sigma)).$$

Going on like this, one finds in type II string theory potentials coupling to p-dimensional objects with p= 0 (points), 1 (strings), 2 (membranes), 3 (blobs?), … They are called p-branes. What and where are they in string theory?

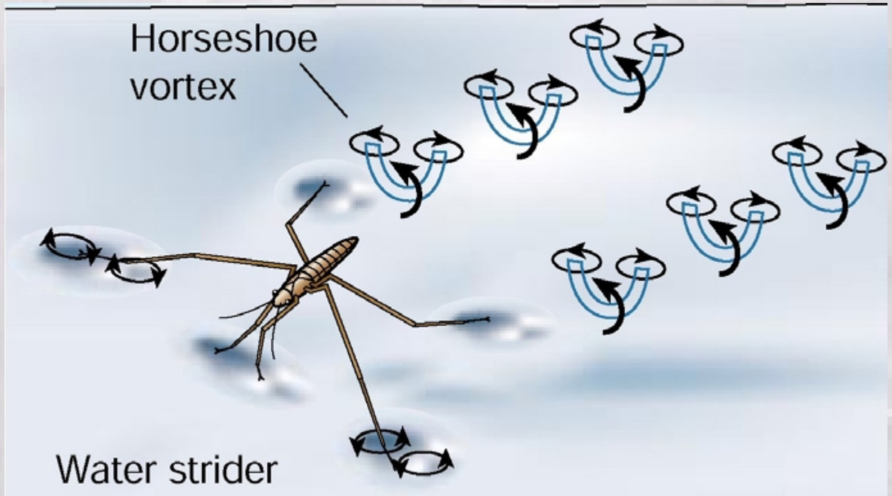
They are the Dirichlet-branes, objects on which open strings end. They are solitonic, tension:

$$T_p = (2\pi)^{-(p-1)/2}(2\pi\alpha')^{-(p+1)/2}\, g_S^{-1}$$

Open strings are “stuck” on Dp-branes, closed strings move freely in the bulk.



A nice metaphor: insects walking on water…



From the point of view of the worldvolume of the Dp-brane: (p+1)-dimensional effective field theory.

Degrees of freedom? Simple susy argument:

→ type II strings: 32 susy charges
→ open strings: 16 susy charges
→ insertion of a Dp-brane in type II

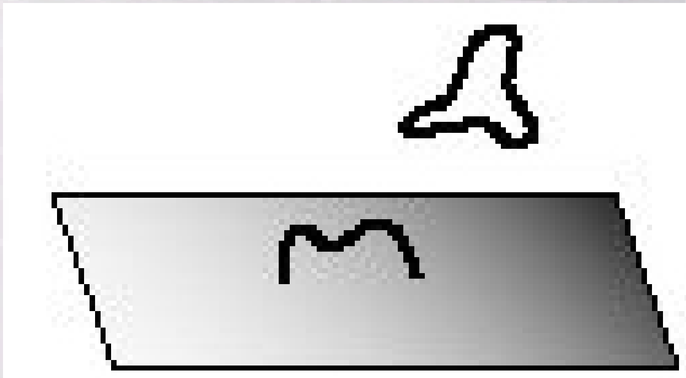
❾ 16 susy's broken ❾ 16 Goldstinos
❾ 8 fermionic propagating degrees of freedom.

SUSY ❾ 8 bosonic propagating degrees of freedom needed…

Dp-brane in 9 + 1 dimensional space-time:

9 – p transversal directions

- 9 – p scalar fields
- 8 – (9 – p) = p – 1 bosonic degrees of freedom missing
- vector field (U(1) gauge field) in p + 1dimensions.



In leading order described by a U(1) gauge theory.

Dp-brane worldvolume theory:

- U(1) gauge theory in p+1 dimensions.
- 9 - p scalar fields
- 16 fermions

Also: complicated couplings to bulk degrees of freedom!

For trivial bulk fields:

$$\mathcal{S} = \frac{1}{2\pi\alpha'^2 g^2} \int d^{p+1}x \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu \Phi^I \partial_\nu \Phi^I - 2\pi\alpha' F_{\mu\nu})}$$
$$+ \text{ derivative corrections}$$

$$\alpha' = \ell_{string}^2$$
$$g^2 = (2\pi)^{p-2} \alpha'^{(p-3)/2} g_S$$

Dirac-Born-Infeld action. Switch off the transversal scalars,

$$\mathcal{S} = \int d^{p+1}x \Big(-\frac{1}{4g^2} F_{\mu\nu}F^{\mu\nu} - \frac{(2\pi\alpha'^2)}{8g^2} F_{\mu\nu}F^{\nu\rho}F_{\rho\sigma}F^{\sigma\mu} + \frac{(2\pi\alpha'^2)}{32g^2}(F_{\mu\nu}F^{\mu\nu})^2 + \cdots \Big)$$

Born & Infeld: point source, $\rho = q\delta(\vec{r})$, has an ∞ energy in Maxwell. Modify Maxwell as above,

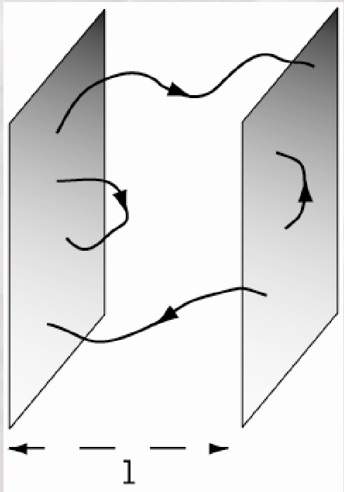
$$E_r = \frac{q}{4\pi\sqrt{r^4 + \left(\frac{2\pi\alpha' q}{4\pi}\right)^2}}$$

$$\text{Energy} \approx 0.349\, q^{3/2}(2\pi\alpha')^{-1/2}$$

D-branes and gauge theories

Worldvolume point of view

Mass of open strings ~ minimal distance between the branes it connects.



$$\ell \to 0 \Rightarrow U(1) \times U(1) \to U(2)$$

D-brane realization of Higgs mechanism, Higgs vev is ℓ .

N coinciding D-branes, in leading order in α', given by d=p+1 dimensional supersymmetric U(N) Yang-Mills. Higher order corrections are under investigation. Using orientifolds ⇓ SO(n) and Sp(2n) gauge groups as well.

$$A_\mu = \begin{pmatrix} A_\mu^{(1)} & W_\mu^+ \\ W_\mu^- & A_\mu^{(2)} \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \Phi^{(1)} & \Phi^+ \\ \Phi^- & \Phi^{(2)} \end{pmatrix}.$$

Similar for transversal scalars ⇓ introduces "fuzziness" in bulk geometry. Not too much known about it: active but very difficult domain of research!

D-branes provide geometric realization of gauge theories. E.g. Dirac monopole:

$$A_x^{(\pm)} = -\frac{m}{2}\frac{y}{r(z \pm r)},$$
$$A_y^{(\pm)} = +\frac{m}{2}\frac{x}{r(z \pm r)},$$
$$A_z^{(\pm)} = 0.$$

Where we need 2 patches, $A^{(+)}$ is defined on $\theta > -\varepsilon$ and $A^{(-)}$ on $\theta < \varepsilon$. Requiring them to coincide on the overlap gives the Dirac quantization condition, $m \in \mathbb{Z}$.

$$\vec{B} = \nabla \times \vec{A}, \qquad \nabla \cdot \vec{B} = 2\pi m \delta(\vec{x}).$$

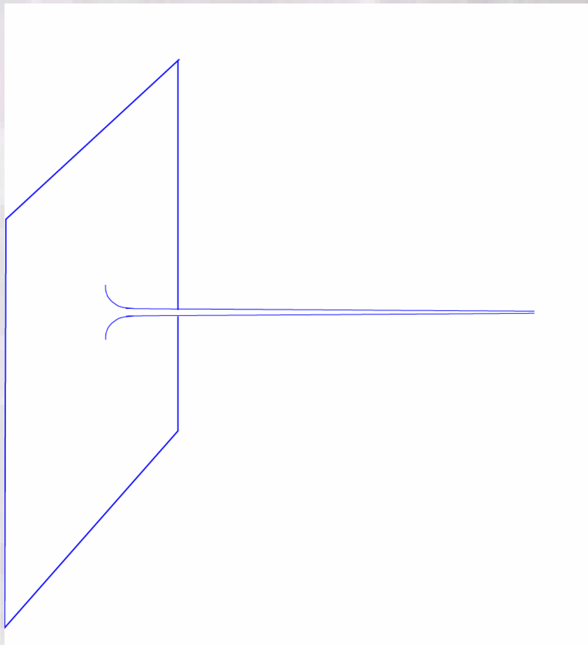
Rigorous definition through the introduction of a scalar field, Φ ,

$$\partial_a \Phi = -2\pi\alpha' B_a$$

with,

$$\Phi = \frac{\pi\alpha' m}{r}$$

View Φ as a coordinate transversal to a D3-brane ⇓ bound system of 1 D3-brane with m perpendicular D1-branes (called a BIon configuration).



Charges, energies, … all work out. Dual point of view as well possible: point of view of the m D1-branes (Myers effect) → relation with non-commutative geometry.

Similarly ‘t Hooft-Polyakov monopoles: m D1-branes stretched between two parallel D3-branes. Abelian limit easily understood.

❾ D-branes are a powerful tool for organizing the monopole zoo, understand the ADHM construction for instantons, explore gauge solitons in higher dimension (e.g. in octonionic analogue of Dirac and ‘t Hooft-Polyakov monopoles in d=7, octonionic instantons in d=8), …

AdS/CFT

Consider type IIB in flat d=9+1 space with N parallel D3-branes.

$$\mathcal{S} = \mathcal{S}_{branes} + \mathcal{S}_{bulk} + \mathcal{S}_{bulk/brane\ interactions}$$

Newton constant:

$$G_N^{(10)} = 8\pi^6 \alpha'^4 g_S^2$$

Take $\alpha' \to 0$ (low energy) , keeping g_S , N, … fixed ⇓

$\mathcal{S}_{bulk}$: non-interacting gravity theory

$\mathcal{S}_{bulk/brane\ interactions}$: vanishes

$\mathcal{S}_{branes}$: reduces to n=4, d=3+1, U(N) susy Yang-Mills

Take D3-brane solution of IIB supergravity, near horizon geometry (= low energy limit) is $AdS_5 \times S^5$

$$S^5 \text{ in } \mathbb{R}^6 : \quad X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 + X_6^2 = R^2$$

$$AdS_5 \text{ in } \mathbb{R}^{2,4} : \quad X_0^2 + X_1^2 - X_3^2 - X_4^2 - X_5^2 - X_6^2 = R^2$$

with $R^4 = 4\pi g_S \alpha'^2 N$.

Near horizon region decouples from bulk (free gravity theory).

Maldacena: string theory on $AdS_5 \times S^5$ is equivalent to d=3+1, n=4, U(N) susy Yang-Mills.

Yang-Mills in its perturbative region:

$$g_{YM}^2 N = \frac{g_S N}{2\pi} = \frac{R^4}{4\pi\alpha'^2} \ll 1$$

Supergravity description valid:

$$\frac{R^4}{\alpha'^2} = 4\pi g_s N = 8\pi^2 g_{YM}^2 N \gg 1$$

⇓ Supergravity calculations = SYM in deep non-perturbative regime…

→ Realization of ‘t Hooft’s holographic principle
→ Operator mappings known
→ Tested (anomalies, relevant + marginal deformations, …)
→ Other examples known

→ Full test difficult as string theory on $AdS_5 \times S^5$ is not tractable yet.

→ If conjecture accepted ⇓ powerful probe for certain non-perturbative aspects of gauge theories. E.g. leads to Dijkgraaf-Vafa correspondence…

→ Peculiar limit (pp-waves) can be studied as a string theory but corresponds to a very singular truncation of the gauge theory.

→ Remains very active field of research!

D-branes and black holes

Black holes are very simple objects characterized by a few parameters: their mass, angular momentum, various charges… All examples here for Reissner-Nordstrom.

$$ds^2 = \left(1 - \frac{2G_N M}{r\,c^2} + \frac{G_N Q^2}{r^2\,c^4}\right) dt^2 - \left(1 - \frac{2G_N M}{r\,c^2} + \frac{G_N Q^2}{r^2\,c^4}\right)^{-1} dr^2 - r^2 d\Omega^2,$$

Horizon:

$$r_H = c^{-2}\left(G_N M + \sqrt{(G_N M)^2 - G_N Q^2}\right).$$

Hawking radiation, thermal with temperature and entropy:

$$T_H = \frac{c^3\hbar\sqrt{(G_N M)^2 - G_N Q^2}}{2\pi k_B (G_N M + \sqrt{(G_N M)^2 - G_N Q^2}\,)^2},$$

$$\frac{S_H}{k_B} = \frac{\pi}{c\hbar G_N}(G_N M + \sqrt{(G_N M)^2 - G_N Q^2}\,)^2 = \frac{1}{4} A_H\, l_p^{-2}.$$

Boltzmann: $S = k_B \log \Sigma$.

Questions:

- Microscopic description?
- Information problem.

Branes provide answers, at least for the case of extremal and near-extremal black holes.

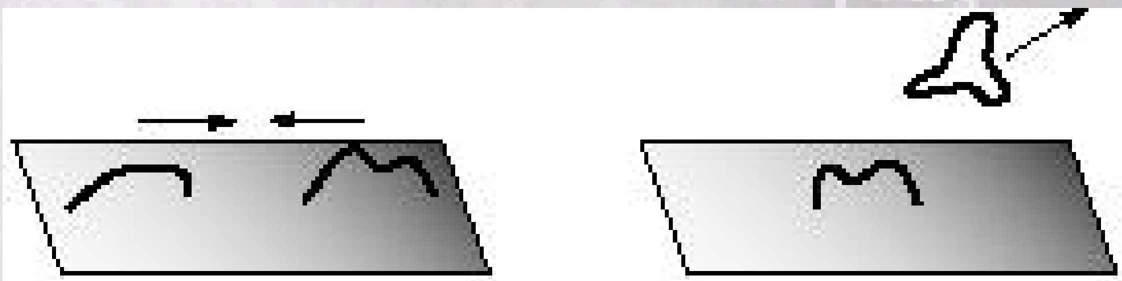
Extremal or supersymmetric black holes, e.g. for Reissner-Nordstrom:

$$M \to Q/\sqrt{G_N}$$

Behaves like an elementary particle.

Microstates (Strominger-Vafa): complicated composite of D-branes wrapped around the compact dimensions + strings ending on them. Macrostate characterized by a few quantum numbers, numerous microscopic realization. Bekenstein-Hawking reproduced!

Unitarity (Callan-Maldacena): black hole evaporation can be studied in the near-extremal case. Perfect agreement with Hawking (T and reaction rate), healthy quantum theory.



Cosmology

Observations (SN, WMAP, …) all point towards an accelerating universe. Can this be accommodated within string theory?

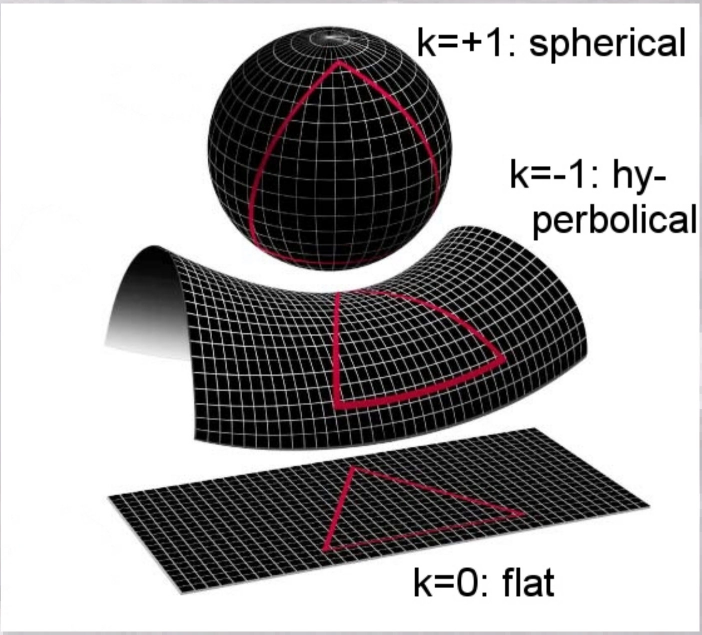
First some standard stuff…

FLRW model

$$d\tau^2 = dt^2 - R(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right).$$

Spatial geometry:

Observations: $k \simeq 0$



Determine $R(t)$ using Einstein's equations…

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda\, g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\dot{R}^2 + k = \frac{8\pi G}{3} R^2 \left(\rho + \frac{\lambda}{8\pi G} \right)$$

$$\dot{\rho} + (\rho + p) \frac{3\dot{R}}{R} = 0$$

Need a relation between energy density and pressure: equation of state. Simplest: linear, time-independent.

$$p(t) = \alpha\, \rho(t) \quad \Rightarrow \quad \rho = \rho_0 \frac{R_0^{3(\alpha+1)}}{R^{3(\alpha+1)}}$$

Radiation: $p = \rho/3 \quad \Rightarrow \quad \rho = \rho_0 \frac{R_0^4}{R^4}$

Non-relativistic matter: $p = 0 \quad \Rightarrow \quad \rho = \rho_0 \frac{R_0^3}{R^3}$

Positive cosmological constant: $p = -\rho \quad \Rightarrow \quad \rho = \rho_0$

Acceleration or deceleration?

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(1+3\alpha)\rho + \frac{\lambda}{3}$$

Acceleration if $\lambda > 0$ or more generally $\alpha < -\frac{1}{3}$.

Observation: $-1.62 < \alpha < -0.74$.

$\alpha < -1$: phantom matter, negative kinetic energies
⇓ instabilities.

Phantom Matter and the Cosmological Constant

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Abstract

Motivated by some recent speculative attempts to model the dark energy, scalar fields with negative kinetic energy coupled to gravity without a cosmological constant are considered. It is shown that in the presence of an ordinary fluid, any solution of the vacuum Einstein equations with cosmological constant is a solution provided $\rho - P = \frac{\Lambda}{4\pi G}$. The solutions can be interpreted as a steady state in which matter or entropy is being continuously created (or destroyed). The motion of the matter is not determined by the background Einstein spacetime, many different matter flows can be found giving rise to the same metric. Solutions without ordinary matter are also considered. Anti-gravitating multi-solutions and repulsive solutions which can chase ordinary matter or black holes are exhibited. These results may also have applications to gravity theories with higher derivatives.

1 Introduction

Desperate times evidently call for desperate measures. In order to model the observed acceleration of the scale factor $a(t)$ of our universe Caldwell [1] and others [2] have turned to matter with negative energy density. In particular resort has been made to scalar fields with negative kinetic energies [2]. Such ghost or phantom scalar fields have not been seen in cosmology since the days of the old Steady State theory with its negative kinetic energy creation field $C(x)$ [3]. The authors of that theory showed that the field equations, essentially gravity plus a pressure free fluid and massless scalar with negative kinetic energy, admit a de-Sitter solution with constant Hubble constant H with an exponential scale factor $a(t) = e^{Ht}$ and flat spatial cross sections satisfying the Perfect Cosmological Principle. Of course de-Sitter spacetime satisfies the Einstein equations with cosmological constant $\Lambda = 3H^2$. One of the aims of this paper is

Positive cosmological constant.

→ Why so small?

- Value: $\rho_{vac} \sim \left(10^{-3}\, eV\right)^4$

- Compare to Planck scale: $10^{19}\, GeV \Rightarrow \mathcal{O}(10^{124})$ mismatch

- Compare to susy scale: $10^{3}\, GeV \Rightarrow \mathcal{O}(10^{60})$ mismatch

→ Positive?

$$\alpha \neq -1 \qquad R(t) \propto t^{2/3(\alpha+1)}$$

$$\text{Radiation: } R \propto \sqrt{t}$$

$$\text{Dust: } R \propto t^{2/3}$$

$$\alpha = -1 \qquad R(t) \propto e^{\sqrt{\lambda} t/\sqrt{3}}$$

- Inflationary expansion
- Exponential evolution towards a de Sitter universe

Seems not possible in string or M theory….

$$\ddot{R}(t) = -R_{00} = -4\pi(T_{00} + g^{ij}T_{ij})$$

Strong energy condition (SEC): $R_{00} \geq 0$

d = 10, 11 supergravity satisfy SEC…

Gibbons; Maldacena-Nuñez: compactify on a smooth, compact manifold,

$$ds^2_{11,10} = \omega^2(y)ds^2_4(x) + ds^2_{7,6}(y)$$

$$R^{(11,10)}_{00} \geq 0 \Rightarrow R^{(4)}_{00} \geq 0.$$

Ways out?

→ It gets cured by higher order (α' , g_S) corrections. We cannot say anything sensible about this.

→ Reanalyze the premises of the no-go theorem and find a loophole. E.g. scalar field,

$$\ddot{\phi} = -V(\phi)' - 3H\dot{\phi}, \qquad H \equiv \dot{R}/R$$

$$\rho = \tfrac{1}{2}\dot{\phi}^2 + V, \qquad p = \tfrac{1}{2}\dot{\phi}^2 - V$$

String/supergravity: there is no stationary point with V > 0. V > 0 and not stationary ⇓ compact space is time dependent…

→ Two related approaches: hyperbolic compactifications and flux compactifications (e.g. d=11 supergravity on T7, 4-form flux).

Typical potential:

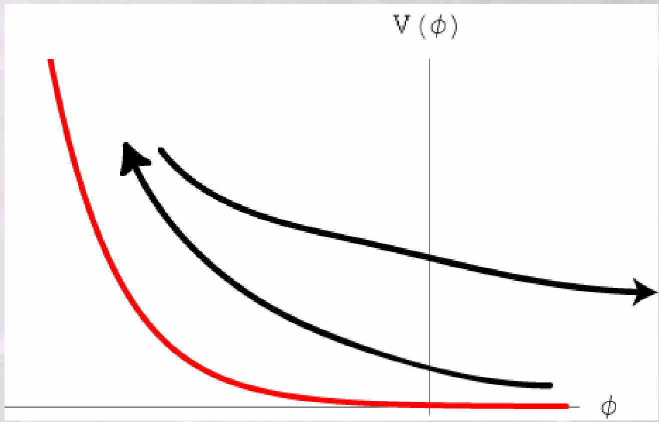
$$V(\phi) = b\, e^{-2a\phi}, \qquad b > 0$$

Hyperbolic: $\frac{1}{\sqrt{3}} < a < \sqrt{3}$

Flux: $a \geq \sqrt{3}$

Initially: $R \propto t^{1/3}, \qquad e^{\phi} \propto t^{-1/\sqrt{3}}$

Field rolls up the potential till $\dot{\phi} = 0$ which is a transitional period of acceleration. Field rolls back down ⇓ deceleration & decompactification.



Generic behaviour: eternal acceleration not possible.

Inflation?

While always big-bang singularity where,

$$R(t) \propto t^{1/3}$$

Previously discussed model not very well suited to describe initial inflation. Typically only a few e-foldings instead of 70 or more… Seems generic, e.g. systematic analysis of combined flux/hyperbolic scenarios (Chen-Ho-Neupane-Ohta-Wang; Wolfarth).

Supergravity/string based analysis (mainly at Stanford: Kachru, Kallosh, Linde, …) of inflation is encouraging (dilaton/volume stabilization, …).

Conclusions

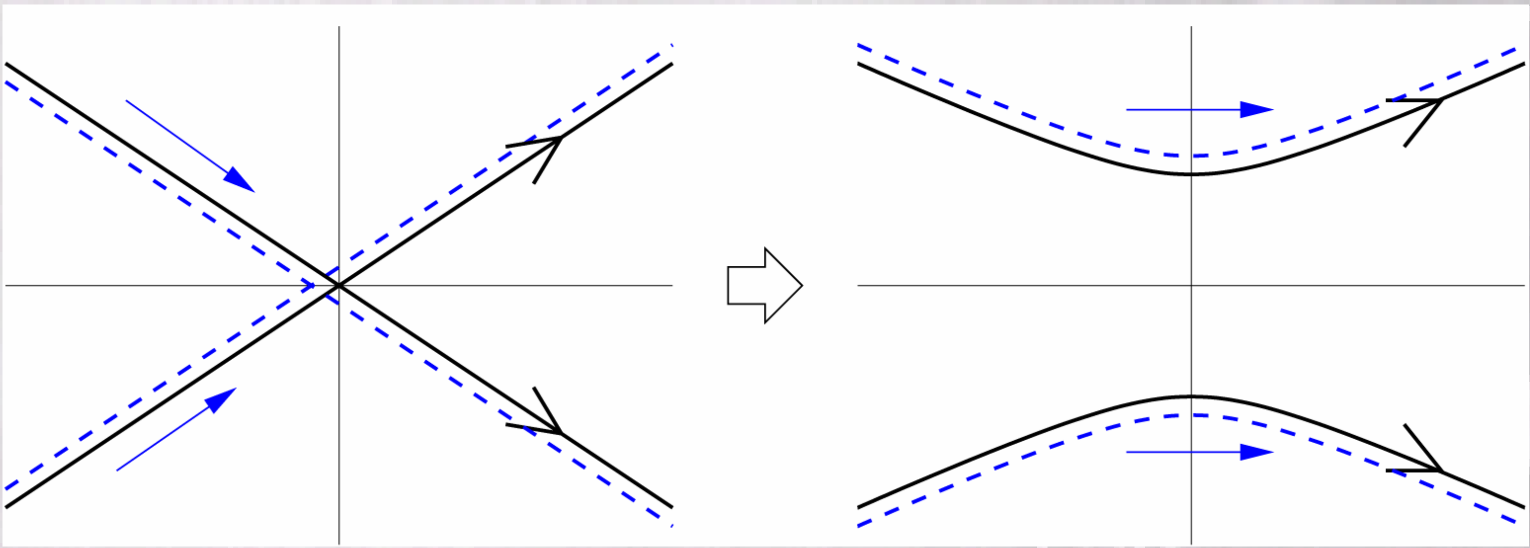
- Perfect tool for the study of diverse aspects of gauge theories.

- Cosmological applications:
  → time-dependent compactifications
  → study of the initial singularities
  → inflation

- Phenomenology: flux-compactifications
  → fix moduli
  → break susy
  → cosmological acceleration
  → but… no chiral fermions…
  ❾ more general fluxes are being studied…

- Phenomenology: intersecting brane-worlds (e.g. stacks of D6-branes intersecting in 3 dimensions)
  - → easy to get correct gauge group + 3 families of quarks and leptons
  - → many possibilities…
  - → generic features: right-handed neutrino, 2 or more Higgs
  - → models generically non-supersymmetric ⇓ stability (NS-tadpoles)?
  - → hierarchy problem (at least for toroidal and orbifold compactifications)
  - ➒ hunt for the MSSM (additional angle constraints)
  - ➒ construction of the low energy effective action ⇓ identification of generic qualitative features

Stability issue: e.g. 2 D1-branes at an angle will recombine and end up as 2 parallel D1-branes…



Picture: thanks to Taylor & Hashimoto.

New insights are urgently needed, e.g. concerning the lifting of the huge vacuum degeneracy…

1984 – 1985: anomaly cancellation, heterotic strings

1995: D-branes

2005: ???

New insights are urgently needed, e.g. concerning the lifting of the huge vacuum degeneracy…

Or on an LHC timescale:

1984 – 1985: anomaly cancellation, heterotic strings

1995: D-branes

2007: ???