# Leptogenesis
# neutrino mass bound

Pasquale Di Bari
(IFAE, Universitat Autònoma de Barcelona)

Reference paper: W. Buchmüller, PDB, M. Plümacher, hep-ph/0401240

# Baryon asymmetry of the Universe

- CMB + cosmic rays exclude a baryon symmetric universe with matter- anti matter domains, on scales as large as the whole horizon

  (Cohen, De Rujula, Glashow, '98)

- from CMB acoustic peaks (WMAP) + large scale structure (SLOAN):

$$\eta_B^{CMB} = (6.3 \pm 0.3) \times 10^{-10}$$

  (Tegmark et al. 2003)

- . . . in very good agreement with the determination from SBBN + primordial Deuterium measurements :

$$\eta_B^{SBBN} = (6.1 \pm 0.5) \times 10^{-10}$$

  (Cyburt et al. 2001, Kirkman et al. 2003)

# Models of Baryogenesis

- at the Planck scale
- from phase transitions
  - Electroweak Baryogenesis
    (Carlos Wagner's talk)
    - in the Standard Model
    - in the MSSM
    - $\cdots$
  - $\cdots$
- Affleck-Dine
  - at preheating
  - Q-Balls
- from black holes evaporation
- spontaneous baryogenesis
- $\cdots$
- $\boxed{\text{from heavy particle decays}}$
  - GUT baryogenesis
  - leptogenesis

- Heavy particle decays
- Seesaw+neutrino mixing data $\Rightarrow$

  $\Rightarrow$ Leptogenesis

- Leptogenesis+CMB bound $\Rightarrow$

  constraints on seesaw parameters

  $\swarrow$

  (in particular on neutrino masses)

# Heavy particle decays

1. A baryogenesis toy model (Kolb,Turner)

$$X \xrightarrow{\Gamma} b\, b \qquad X \xrightarrow[\bar{\Gamma}]{} \bar{b}\, \bar{b}$$

$$(B_b = 1/2 \Rightarrow \Delta_{B-L} = +1)$$

2. Leptogenesis (Fukugita,Yanagida '86)

$$N_i \xrightarrow{\Gamma} l\, \bar{\Phi} \qquad N_i \xrightarrow[\bar{\Gamma}]{} \bar{l}\, \Phi$$

$$(L_l = 1 \Rightarrow \Delta_{B-L} = -1)$$

After sphaleron conversion

(Kuzmin,Rubakov,Shaposhnikov'85;Khlebnikov,Shaposhnikov'88;Harvey,Turner'90):

$$N_B^{\mathrm{f}} \simeq \frac{1}{3}\, N_{B-L}^{\mathrm{f}}, \qquad N_L^{\mathrm{f}} \simeq -\frac{2}{3}\, N_{B-L}^{\mathrm{f}}$$

- CP asymmetry parameter:

$$\varepsilon = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}\,, \qquad \tilde{\varepsilon} = \varepsilon\, \Delta_{B-L}$$

- Total decay parameter:

$$\Gamma_D = \Gamma + \bar{\Gamma} = \Gamma_D^{\mathrm{rest}} \left\langle \frac{1}{\gamma} \right\rangle$$

# Out of equilibrium decays

$$z = M_X/T, \qquad D = \Gamma_D/H\,z$$

$$\begin{aligned} \frac{dN_X}{dz} &= -D(z)\,N_X(z) \\ \frac{dN_{B-L}}{dz} &= -\tilde{\varepsilon}\,\frac{dN_X}{dz} \end{aligned}$$

$$N_X^{\rm eq}(z \ll 1) = 1$$

$$N_X(z) = N_X^{\rm i}\, e^{-\int_{z_{\rm i}}^{z} dz'\, D(z')}$$

- Efficiency factor:

$$N_{B-L}(z) = \cancel{N_{B-L}^{\rm i}} + \tilde{\varepsilon}\,\kappa(z)$$

$$\kappa(z) = N_X^{\rm i} - N_X(z) \Rightarrow \kappa_{\rm f} = N_X^{\rm i}$$

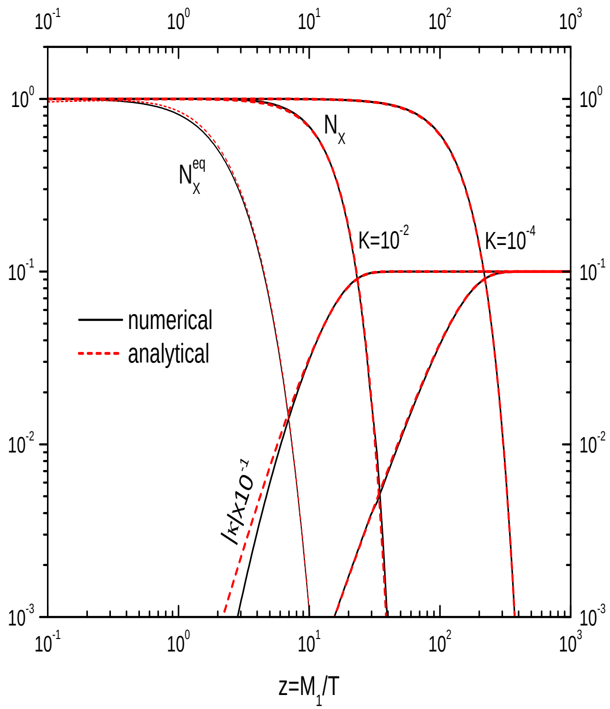
$$\boxed{N_{B-L}^{f} = \tilde{\varepsilon}\,\kappa_f}$$

$$\boxed{\eta_B^{\rm f} \simeq \frac{N_{B-L}^{\rm f}/3}{N_\gamma^{\rm rec}} \simeq 10^{-2}\;\tilde{\varepsilon}\;\kappa_f}$$

- Decay parameter:

$$\boxed{K = \frac{\Gamma_D^{\rm rest}}{H|_{z=1}}} \Rightarrow D = K\left\langle \frac{1}{\gamma}\right\rangle z$$

- Dilation factor: $\left\langle \frac{1}{\gamma}\right\rangle \simeq \frac{z}{z+\frac{15}{8}}$



# Decays and Inverse Decays

$$\boxed{\begin{aligned} \frac{dN_X}{dz} &= -D\,N_X + D\,N_X^{\rm eq} \\ \frac{dN_{B-L}}{dz} &= -\tilde{\varepsilon}\,\frac{dN_X}{dz} - W_{ID}\,N_{B-L} \end{aligned}}$$

$$W_{ID} = \beta\,\frac{N_X^{\rm eq}}{N_{b,l}^{\rm eq}}\,D \propto K$$

$$N_{B-L}(z) = N_{B-L}^{\rm in}\,e^{-\int_{z_{\rm i}}^{z} dz'\,W_{ID}(z')} + \tilde{\varepsilon}\,\kappa(z)$$

$$\kappa(z;K,z_{\rm i}) = -\int_{z_{\rm i}}^{z} dz'\,\left[\frac{dN_{N_1}}{dz'}\right]\,e^{-\int_{z'}^{z} dz''\,W_{ID}(z'')}$$

- Weak wash-out regime for $K \lesssim 1$

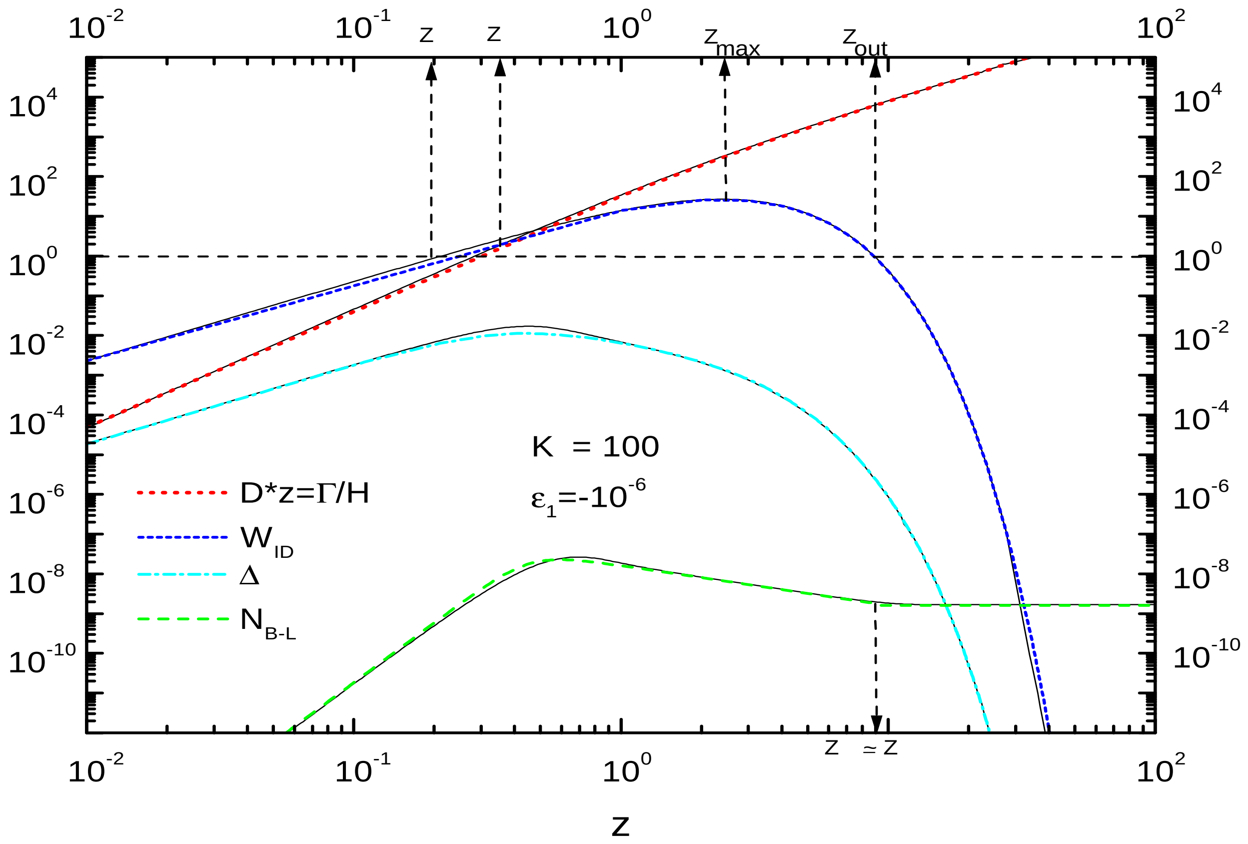
(the out of equilibrium decays picture has to be recovered asymptotically in the limit $K \to 0$)

- Strong wash-out regime for $K \gtrsim 1$

# Strong wash-out regime

$$\Delta(z) = N_{N_1}(z) - N_{N_1}^{\rm eq}(z)$$

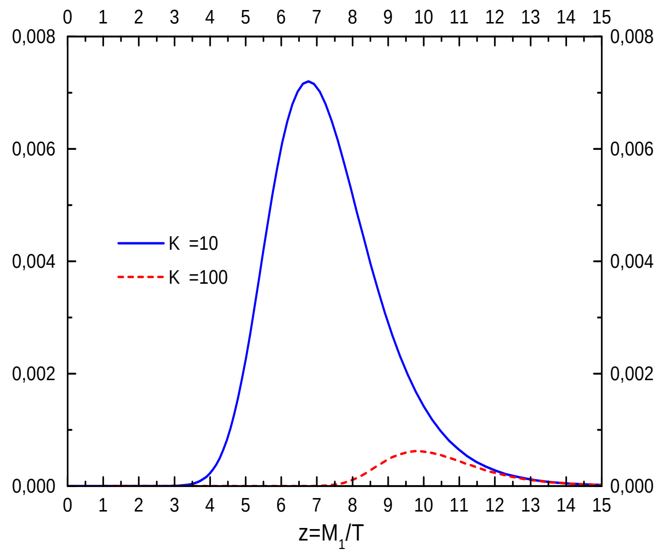
$$N_X^{\rm i} = N_X^{\rm eq} = 1$$



## Close-to-equilibrium approximation:

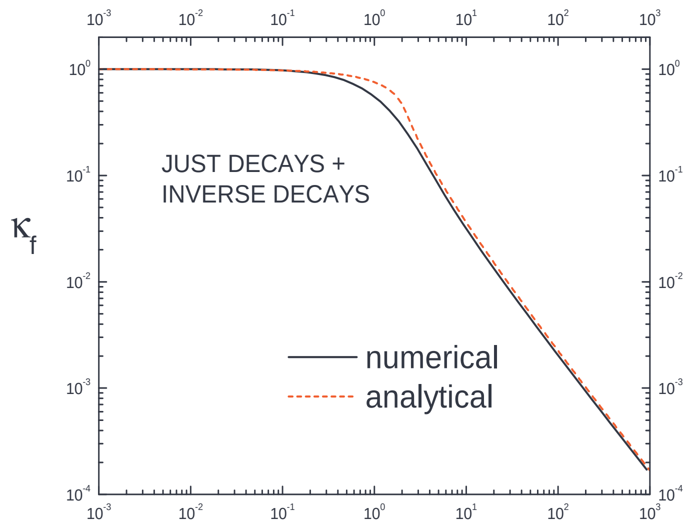
$$\boxed{\frac{dN_X}{dz} \simeq \frac{dN_X^{\rm eq}}{dz}} = -\frac{1}{2}\, z^2\, K_1(z)$$

## Baryogenesis temperature $T_B = \frac{M_X}{z_B}$

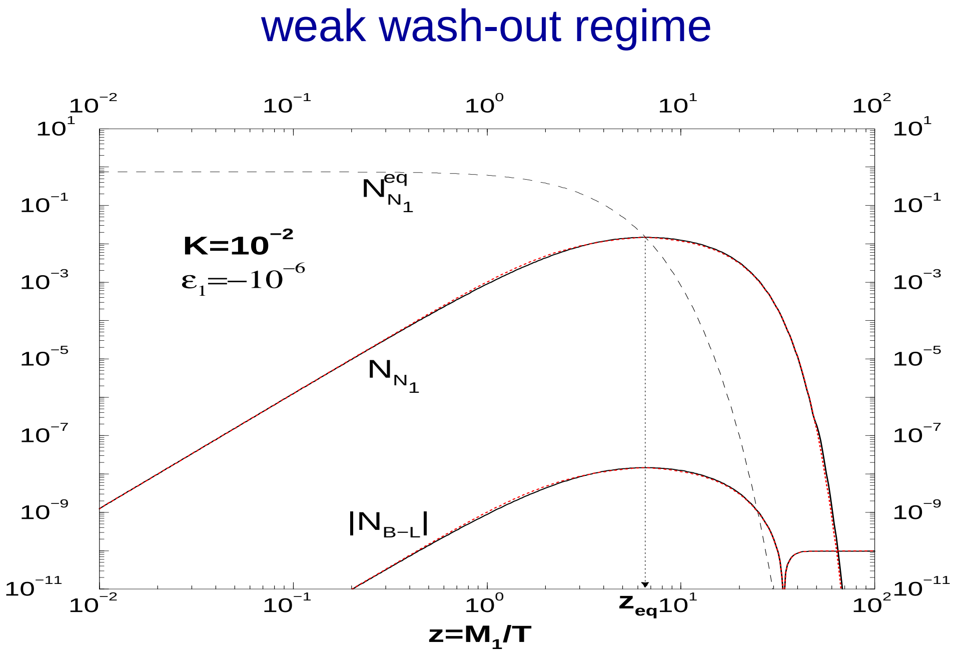


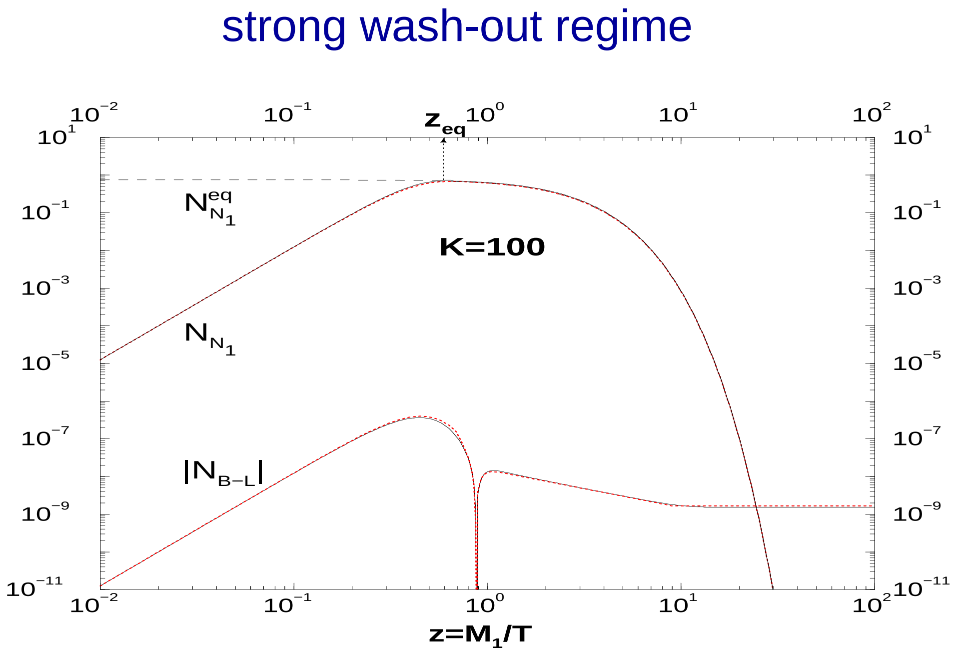

$$\begin{aligned}
\kappa_{\rm f} &= \frac{1}{2}\int_0^\infty dz'\, z'^2\, K_1(z')\; e^{-\frac{\beta\, K}{4}\int_{z'}^\infty dz''\, z''^3\, K_1(z'')} \\
&= \int_0^\infty dz'\; e^{-\psi(z',\infty)} \qquad \text{(Laplace integral)} \\
&\simeq \int_{z_B-\Delta z_B}^{z_B+\Delta z_B} dz'\; e^{-\psi(z',\infty)} \qquad \left(\left.\frac{d\psi}{dz'}\right|_{z=z_B} = 0\right) \\
&\simeq \frac{1}{2}\int_0^\infty dz'\, z'^2\, K_1(z')\; e^{-\frac{\beta\, K\, z_B}{4}\int_{z'}^\infty dz''\, z''^2\, K_1(z
\end{aligned}$$

$$\boxed{\kappa_{\rm f} \simeq \frac{2}{\beta\, K\, z_B}\left(1 - e^{-\frac{\beta\, K\, z_B}{2}}\right)}$$



# Neutrino production




$$\kappa^-(z) = \int_0^{z \le z_{\rm eq}} dz'\, e^{-\psi(z',z)}\,, \qquad \kappa^+(z) = \int_{z_{\rm eq}}^{z \ge z_{\rm eq}} dz'\, e^{-\psi(z',z)}$$

$$\boxed{\kappa(z) = -|\kappa^-(z)| + \kappa^+(z) \Leftrightarrow N_{B-L}(z) = -|N^-_{B-L}(z)| + N^+_{B-L}(z)}$$

# Final efficiency factor

(Buchmuller,PDB,Plumacher,04)

- Initial thermal abundance

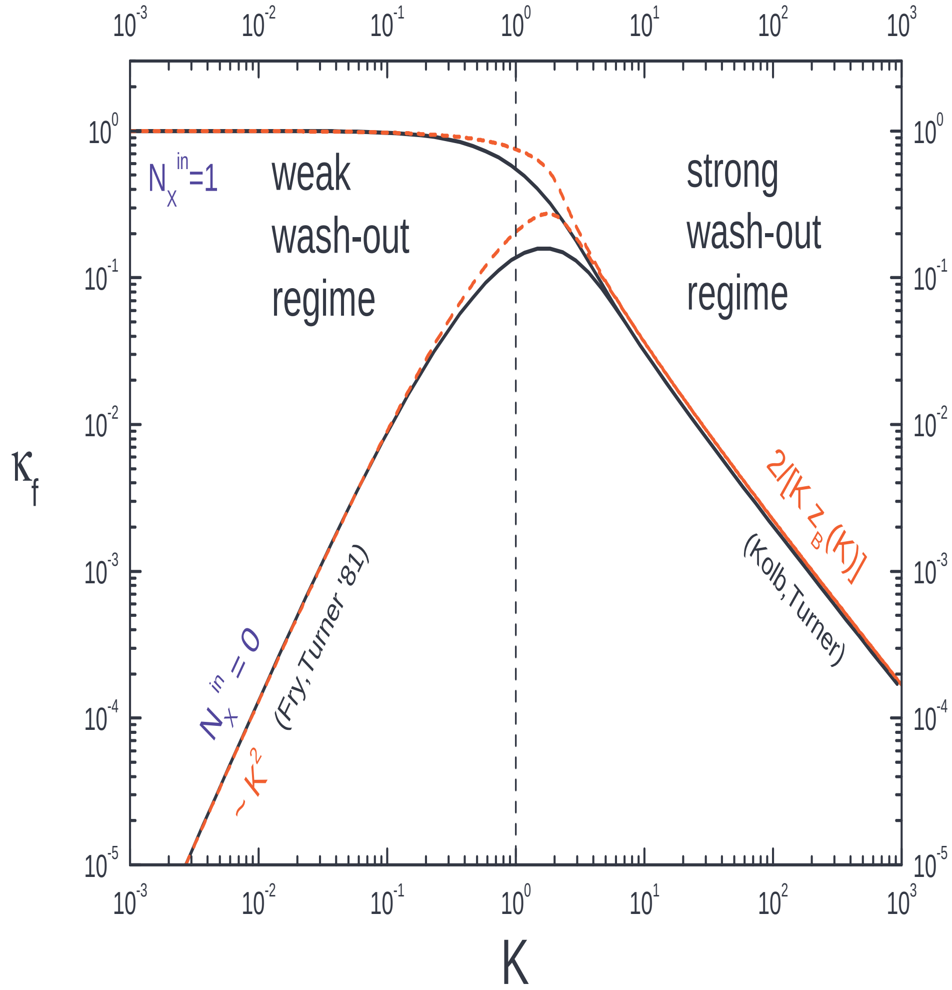
$$\kappa_{\rm f} \simeq \frac{2}{K\,z_B}\left(1 - e^{-\frac{K\,z_B}{2}}\right)$$

- Neutrino production

$$\kappa_{\rm f}^{-} = -2\,e^{-\frac{3\pi}{8}\,K}\left[e^{\frac{1}{2}\,K\,N_X(z_{\rm eq})} - 1\right]$$

$$\kappa_{\rm f}^{+} = \frac{2}{K\,z_B}\left[1 - e^{-\frac{1}{2}\,K\,z_B\,N_X(z_{\rm eq})}\right]$$

## DECAYS+INVERSE DECAYS




Negligible (strong) dependence on the initial conditions in the strong (weak) wash-out regime

# Seesaw $\Rightarrow$ Leptogenesis

- simple seesaw formula $\boxed{m_\nu = -m_D\,\frac{1}{M}\,m_D^T}$

$m_D$, $m_\nu$ and $M$ are complex matrices $\Rightarrow$ natural source of $CP$ violation

- heavy RH neutrino mass matrix $M$ $\Rightarrow$ three new heavy RH neutrinos

  $N_1$,$N_2$ and $N_3$ with masses $M_{\rm ew} \ll M_1 \leq M_2 \leq M_3$
- Dirac neutrino mass matrix $m_D \sim M_{\rm ew}$
- light neutrino mass matrix $m_\nu$ $\quad (m_\nu = m_D^2/M \sim 10^{-2}\,{\rm eV}$ for $M \sim 10^{15}\,{\rm GeV})$
- lightest RH neutrinos play the role of the decaying particles $X \equiv N_1$ (Fukugita,Yanagida '86)
- total decay rate $\quad \Gamma_D^{\rm rest} = \dfrac{\widetilde{m}_1\,M_1^2}{8\,\pi\,v^2}$
- effective neutrino mass (Plumacher '96) $\quad \boxed{\widetilde{m}_1 \equiv \dfrac{(m_D^\dagger\,m_D)_{11}}{M_1}}$
- decay parameter and equilibrium neutrino mass (Luty '92;Buchmuller,PDB,Plumacher '03;PDB'04)

$$K = \frac{\Gamma_D^{\rm rest}}{H|_{z=1}} = \frac{\widetilde{m}_1}{m_\star} \qquad \boxed{m_\star = \frac{v^2}{M_\star} \simeq 10^{-3}\,{\rm eV}} \qquad \left(M_\star = \frac{3\,\sqrt{5}}{16\,\pi^{5/2}}\,\frac{M_{Pl}}{\sqrt{g_\star}} \simeq 3\times 10^{16}\,{\rm GeV}\right)$$

# Range of $\widetilde{m}_1$

- A useful tool is given by (Casas, Ibarra '01):

$$\Omega = D_m^{-\frac{1}{2}}\, U^\dagger\, m_D\, D_M^{-\frac{1}{2}} \Rightarrow \boxed{\Omega^T\,\Omega = I}$$

$$-U^\dagger\, m_\nu\, U^\star = \mathrm{diag}(m_1, m_2, m_3) \equiv D_m$$

- $\widetilde{m}_1 = \dfrac{(m_D^\dagger\, m_D)_{11}}{M_1} = \sum_{j=1}^3\, m_j\, |\Omega_{j1}|^2 \geq m_1$

(Fujii,Hamaguchi,Yanagida '02):

- moreover barring strong cancellations:

$$\widetilde{m}_1 \leq m_3 \sum_i |\Omega_{j1}^2| \approx m_3 \left|\sum_i \Omega_{j1}^2\right| = m_3$$

- for fully hierarchical neutrinos ($m_1 \ll m_{\rm sol}$):

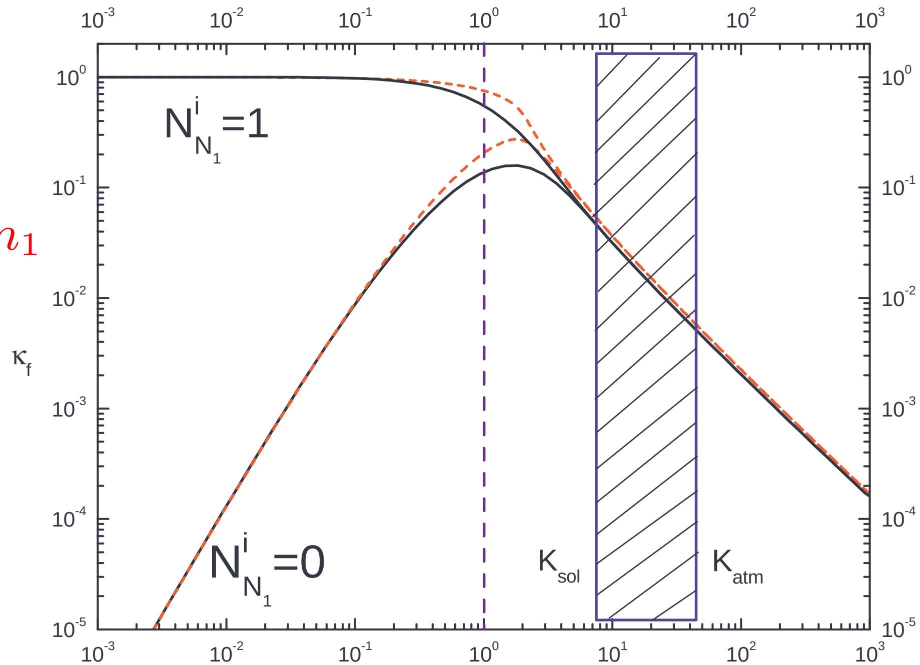
$$\mathcal{O}(m_{\rm sol} \simeq 0.008\,\mathrm{eV}) < \widetilde{m}_1 < \mathcal{O}(m_{\rm atm} \simeq 0.05\,\mathrm{eV})$$

$$m_{\rm atm,sol} \equiv \sqrt{\Delta m^2_{\rm atm,sol}}$$

# Leptogenesis K range

Translating $\widetilde{m}_1$ in terms of $K = \widetilde{m}_1/m_\star$:

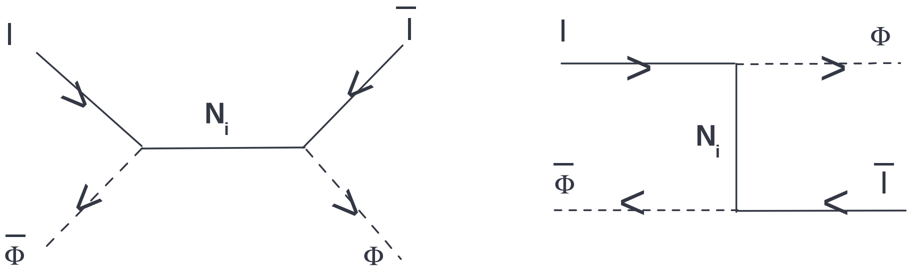
$$\boxed{8 \simeq K_{\rm sol} \lesssim K \lesssim K_{\rm atm} \simeq 50}$$




Neutrino mixing data favor leptogenesis

to lie in the strong wash-out regime

# $\Delta\, L = 2$ processes




$\Longrightarrow$ $W = W_{ID} + W_{\Delta L=2}$

One has to be careful not to double count
the corresponding on-shell processes:


$$W_{\Delta L=2}(z) = \cancel{W^{\rm res}_{\Delta L=2}(z)} + \Delta W(z)$$

$$W^{\rm res}_{\Delta L=2} = \frac{1}{2}\, W_{ID} \quad !!$$

(Buchmuller,PDB,Plumacher '02)

This term has been indeed found to be spurious

(Giudice et al '03;BDP04)

- $\Delta W$ is important only in the
  non relativistic regime

$$\Delta W(z \ll 1) \propto \frac{M_1\, \bar{m}^2}{z^2}$$

$$\boxed{\bar{m}^2 \equiv \sum_i m^2_{\nu_i}}$$

- contributes to the upper bound on
  the absolute neutrino mass scale

- negligible for

$$\boxed{M_1 \ll 10^{14}\,\mathrm{GeV}\ \frac{m^2_{\rm atm}}{\bar{m}^2}}$$

# Scatterings

(Luty'92;Plumacher'97;Buchmuller,PDB,Plumacher'02; Pilaftsis,Underwood'03;Giudice et. al.'03)

Two types:

1. involving the top quark (Higgs mediated):
   $N_1\, l \leftrightarrow t\, q + \ldots$
2. involving the g. bosons (Higgs mediated):
   $N_1\, A \leftrightarrow H + \bar{l} + \ldots$

$$\frac{dN_X}{dz} = -(D+S)\,(N_X - N_X^{\rm eq})$$

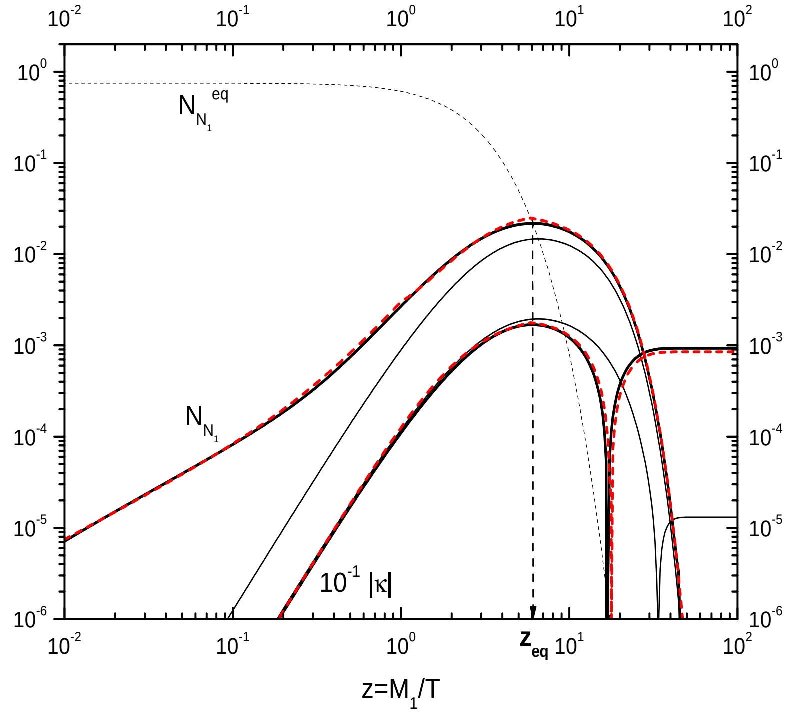
$$\frac{dN_{B-L}}{dz} = -\varepsilon\, D\,(N_X - N_X^{\rm eq}) - (W_{ID} + W_S)\, N_{B-L}$$

Two effects:

1. in the weak wash-out regime enhance the neutrino production
2. in the strong wash-out regime increase (sub-dominantly) the wash-out

Weak wash-out regime ($K \lesssim 1$):

example: $K = 10^{-2}$




strong sensitivity to the theoretical assumptions

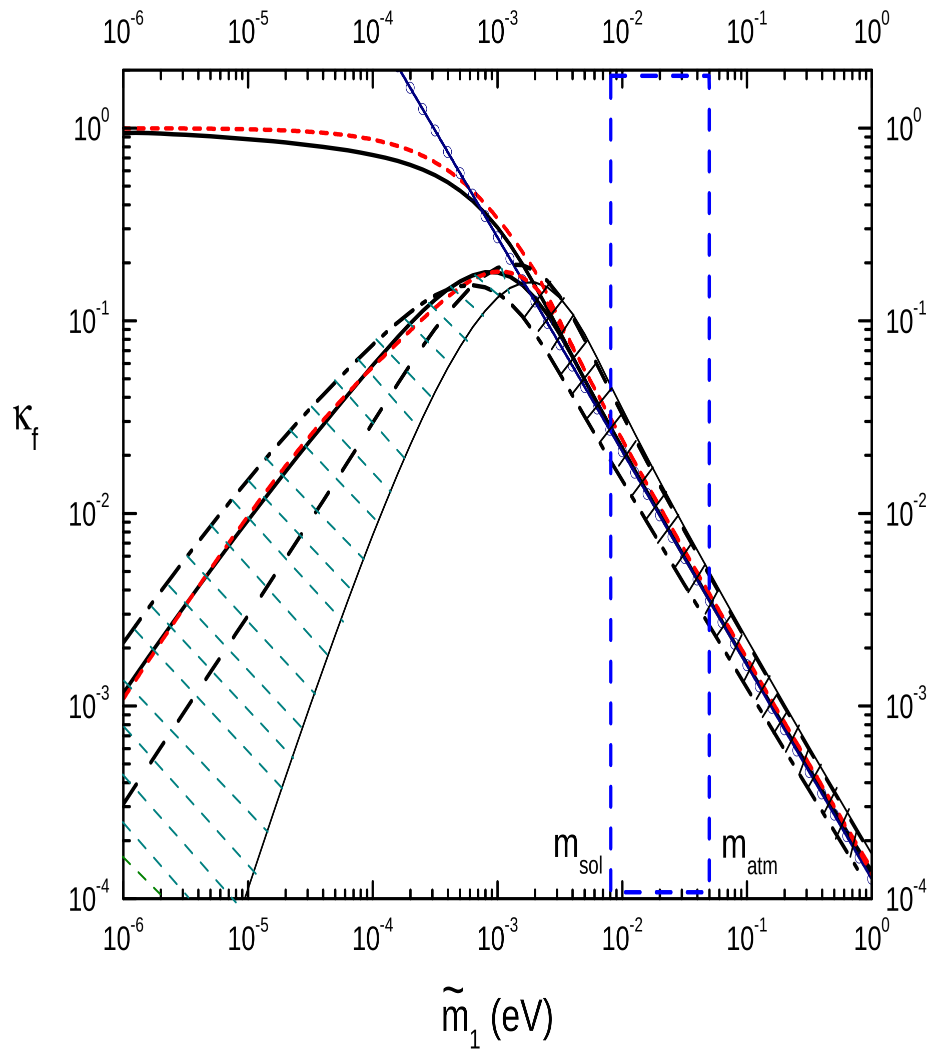
$\Rightarrow$ source of large theoretical uncertainties

# Theoretical uncertainties on $\kappa_{\rm f}$

Big changes in the weak wash-out regime:

- I.R. cut-off on the Higgs mass $\Rightarrow \kappa_{\rm f} \propto K$
  (Luty'92,Plumacher'96,Barbieri et. al.'00, Buchmuller,PDB,Plumacher '02,'03)
- thermal corrections to the Higgs mass+ running Yukawa coupling $\Rightarrow \kappa_{\rm f} \propto K^2$
  (Barbieri et. al.'03;Giudice et.al.'03)
- addition of scatterings involving gauge bosons $\Rightarrow \kappa_{\rm f} \propto K$
  (Pilaftsis,Underwood'03;Giudice et.al.'03)
- 'spectator processes' $\Rightarrow \mathcal{O}(1)$ factor suppression (Buchmuller,Plumacher'01)

$\sim 50\%$ uncertainties in the strong wash-out regime



# CP asymmetry

- Interference between tree level and (vertex + self energy) one-loop diagrams

$$\Rightarrow \varepsilon_1 \quad \simeq \quad \frac{1}{8\pi\, v^2\,(m_D\, m_D^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im}\left[(m_D\, m_D^\dagger)^2_{i1}\right] \times \left[f_V\left(\frac{M_i^2}{M_1^2}\right) + f_S\left(\frac{M_i^2}{M_1^2}\right)\right]$$

(Flanz,Paschos,Sarkar'95; Covi,Roulet,Vissani'96; Buchmüller,Plümacher'98)

barring strong RH neutrino degeneracies (Davidson,Ibarra'02) and phase cancellations (Hambye et.al.'03):

- Maximum CP asymmetry and effective leptogenesis phase

$$\boxed{\varepsilon_1 = \varepsilon_1^{\max} \sin\,\delta_L}$$

$$\varepsilon_1^{\max}(m_1, \widetilde{m_1}, M_1) \simeq 10^{-6}\, \frac{M_1}{10^{10}\,\mathrm{GeV}}\, \frac{m_{\mathrm{atm}}}{m_3 + m_1}\, f(m_1, \widetilde{m}_1)$$

(Asaka et al.'99; Hamaguchi,Murayama,Yanagida '01; Davidson, Ibarra '02; Buchmüller,PDB,Plümacher '03)

$f \leq 1$ and $f|_{m_1=0} = 1 \Rightarrow$ It is maximum for fully hierarchical neutrinos ($m_1 = 0$)

# CMB bound and the leptogenesis mountain

$$\eta_B^{\max}(m_1,\widetilde{m}_1,M_1) \simeq 10^{-2}\,\varepsilon_1^{\max}(m_1,\widetilde{m_1},M_1)\,\kappa_{\rm f}(\widetilde{m_1},M_1\bar{m}^2)$$

$$\eta_B = \eta_B^{\max}\,\sin\delta_L = \eta_B^{CMB}$$

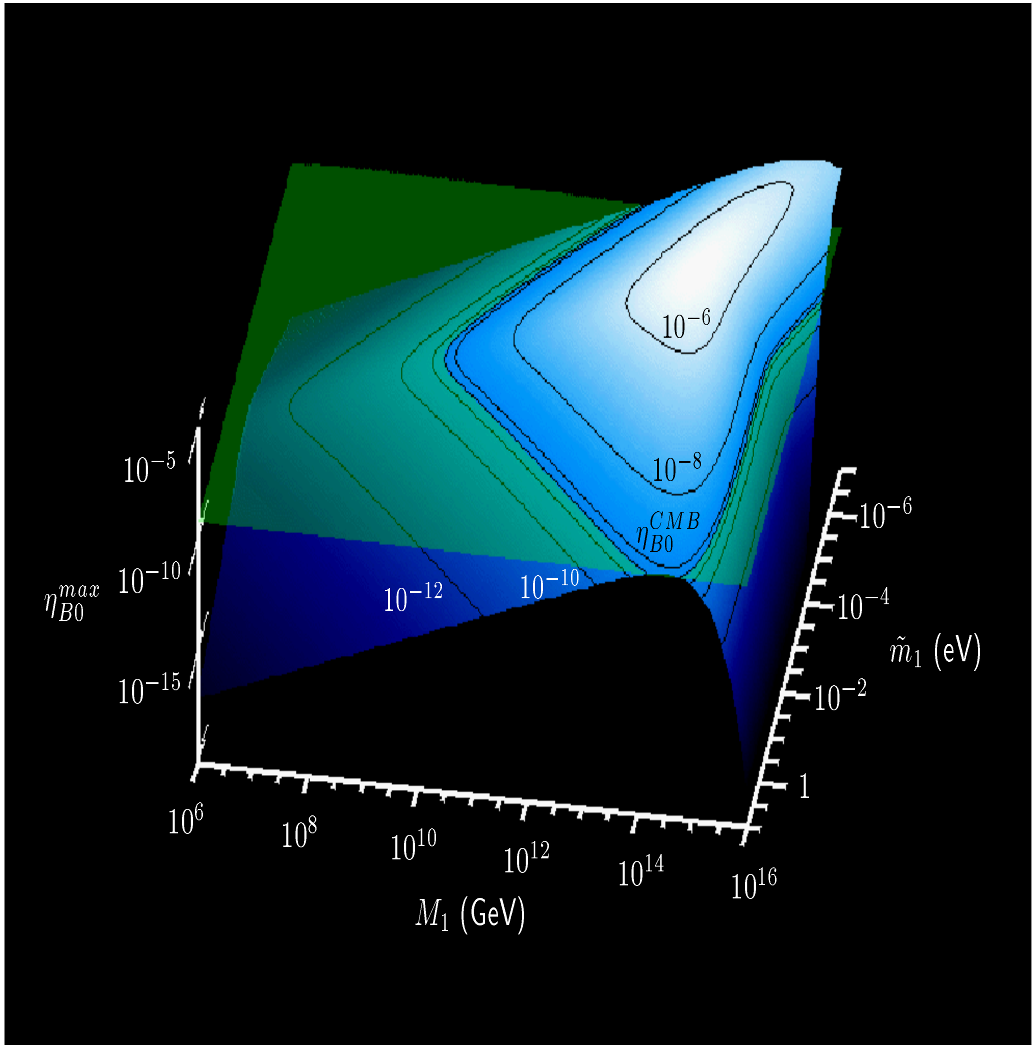
CMB bound:

$$\boxed{\eta_B^{\max}(m_1,\widetilde{m}_1,M_1) \geq \eta_B^{CMB}}$$

This is further maximized for $m_1 = 0$:

$$\boxed{\eta_B^{\max}(\widetilde{m}_1,M_1)|_{m_1=0} \geq \eta_B^{CMB}}$$

$$\eta_B^{\max}(\widetilde{m}_1,M_1)|_{m_1=0} \propto M_1\,e^{-\frac{M_1}{10^{14}\,\mathrm{GeV}}}$$



# Lower bound on the lightest RH neutrino mass $M_1$

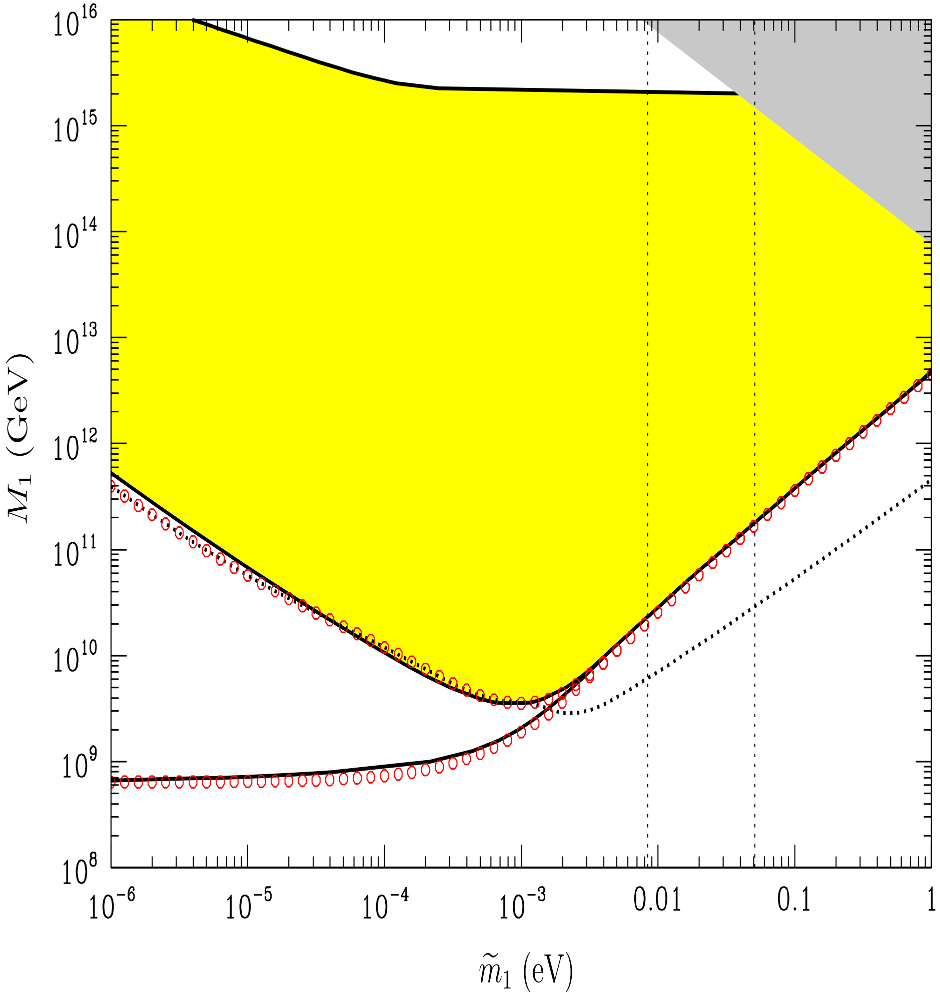
For $\widetilde{m}_1$ between $m_{\rm sol}$ and $m_{\rm atm}$:

$$\boxed{M_1 \gtrsim (1.5 \div 10)\, 10^{10}\,\mathrm{GeV}}$$

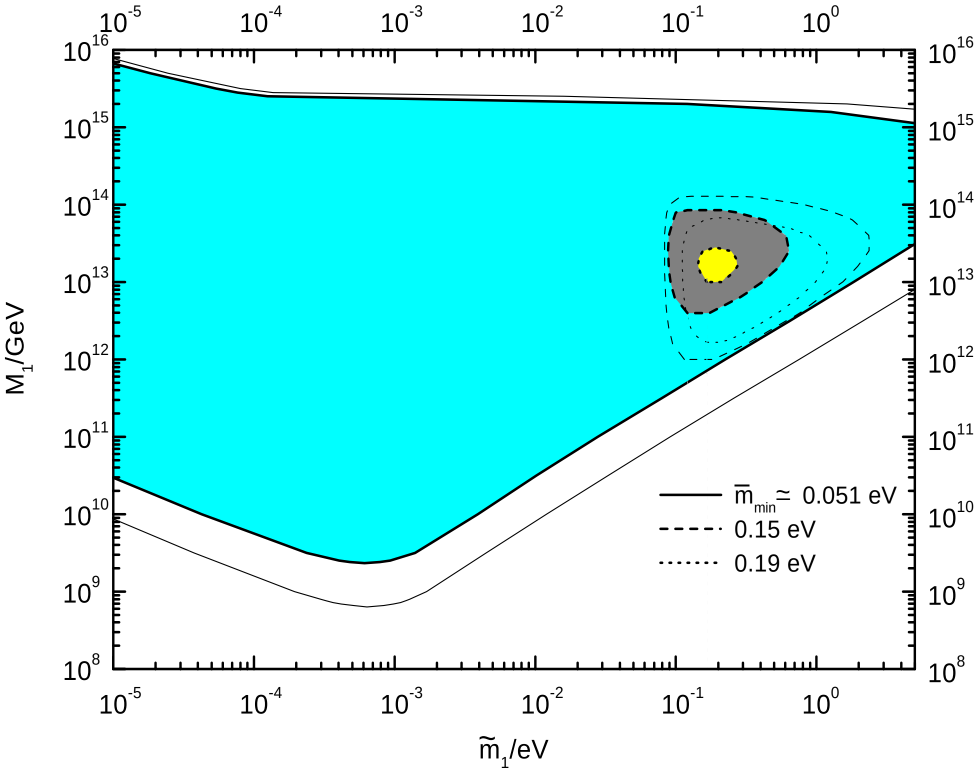
Bound on the 'initial' temperature $T_i$:

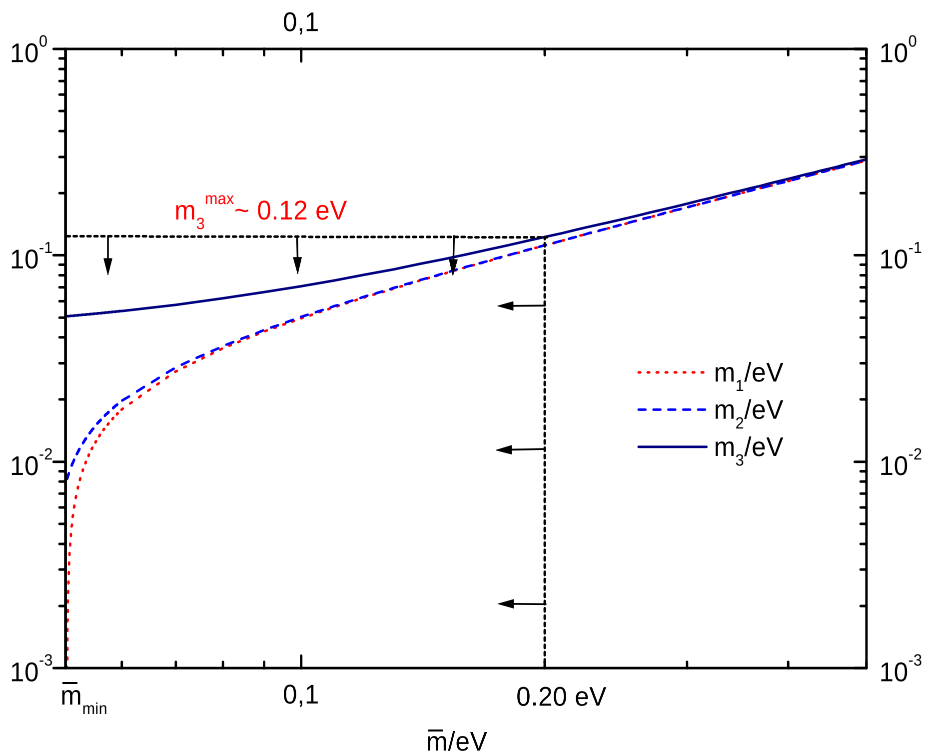
$$\boxed{T_i \gtrsim \frac{M_1^{\rm min}(\widetilde{m}_1)}{z_B(\widetilde{m}_1)-2} \simeq \frac{M_1^{\rm min}}{4\div 6}}$$

Within inflation $T_i = T_{\rm reheating}$ !!!



# Upper bound on the absolute neutrino mass scale




Analytically one can maximize

$$\eta_B^{max} \propto \frac{M_1}{\widetilde{m}_1} \exp\left\{ -\frac{w}{z_B^2} \frac{M_1}{10^{10}\,\text{GeV}} \frac{\bar{m}}{\text{eV}}^2 \right\} \Rightarrow \boxed{m_i < \left( \frac{10^{25}\, g_{\rm rec}\, a_{\rm sph}\, \pi^6\, v^4}{3^{9/2}\, e\, M_{\rm PL}^2\, \text{GeV}^2} \right)^{1/4} \simeq 0.121\,\text{eV}}$$

# Changing the bounds

$$\eta_B^{\max} \longrightarrow \eta_B^{\max}\,\xi$$

$$\Rightarrow M_1^{\min}, T_i^{\min} \longrightarrow \frac{M_1^{\min}, T_i^{\min}}{\xi}$$

$$\Rightarrow m_1^{\rm bound} \longrightarrow m_1^{\rm bound}\,\xi^{1/4}$$

# The supersymmetric case

Small changes:

$$\boxed{(M_1^{\min}, T_i^{\min})^{MSSM} \sim 0.7\,(M_1^{\min}, T_i^{\min})^{SM}}$$

The upper bound on the neutrino mass is practically the same as in the SM within the theoretical uncertainties

$$\boxed{m_i < 0.11\,\mathrm{eV}}$$

# Conclusions

- Leptogenesis is easy and can be solved analytically with good accuracy (within $10\%$) and important physical insight;
- Neutrino mixing data favor leptogenesis to work in a mildly strong wash-out regime with predictions independent on initial conditions and with minimal theoretical uncertainties (it resembles SBBN);
- potential well known problem with the $M_1$ and $T_{\rm reh}$ lower bounds
- prediction: thermal leptogenesis favors hierarchical neutrinos. Testable with absolute neutrino mass scale experiments (CMB+LSS,neutrinoless $\beta\beta$ decay, Tritium $\beta$ decay); if quasi degenerate neutrinos will be found:
  - thermal leptogenesis with strong $M_i$ degeneracies or phase cancellations
  - non thermal leptogenesis (Lazarides,Shafi'91;Murayama,Yanagida'94)
  - type II seesaw leptogenesis