CPV in the kaon system: ε'/ε vs $K \to 3\pi$

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$K \to 3\pi$ in collaboration with E. Gamiz, J. Prades (U. Granada), JHEP 0310:042,2003

Contents of the talk

- A brief introduction to $|\Delta S| = 1$ processes

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- ε'/ε: status and problems
- Why to study CPV in $K \longrightarrow 3\pi$
- Newest results on $K \longrightarrow 3\pi$
- Conclusions

SM and Eff. theories

En. scale	Fields	Eff. Theory

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	$\downarrow$ $\mathbb{OPE}$	
$\lesssim m_c$	$\gamma, g, \mu, e, \nu_\mu, \nu_e, s, d, u$	$\mathcal{L}_{QCD}^{(n_f=3)}$
	$\downarrow$ Large N_C, Lattice,. . .	
M_K	$\gamma, \mu, e, \nu_\mu, \nu_e, \pi, K, \eta$	CHPT

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ChPT for $|\Delta S| = 1$

The e.m. and octet part at lowest order for $|\Delta S| = 1$

$$\mathcal{L}^{(2)} = C\, F_0^4 \left\{ e^2 F_0^2 G_E \left(u^\dagger Q u\right) + G_8 \left(u_\mu u^\mu\right) + G_8' \left(\chi_+\right) + G_{27} (..) \right\}_{32}$$

At order p^4 other operators appear. The octet combinations are

$\tilde{K}_1$	$G_8 \left(N_5^r - 2N_7^r + 2N_8^r + N_9^r\right) + G_{27}\left(-\frac{1}{2} D_6^r\right)$
$\tilde{K}_2$	$G_8 \left(N_1^r + N_2^r\right) + G_{27}\left(\frac{1}{3} D_{26}^r - \frac{4}{3} D_{28}^r\right)$
$\tilde{K}_3$	$G_8 \left(N_3^r\right) + G_{27}\left(\frac{2}{3} D_{27}^r + \frac{2}{3} D_{28}^r\right)$
$\tilde{K}_8$	$G_8 \left(2N_5^r + 4N_7^r + N_8^r - 2N_{10}^r - 4N_{11}^r - 2N_{12}^r\right) - \frac{2}{3} G_{27} \left(D_1^r - D_6^r\right)$
$\tilde{K}_9$	$G_8 \left(N_5^r + N_8^r + N_9^r\right) + G_{27}\left(-\frac{1}{6} D_6^r\right)$

Bijnens, Dhonte, Persson, N.P.B648:317,2003.

ε'/ε: status and unsolved problems

$$\text{Re}\ \frac{\varepsilon'}{\varepsilon} = \frac{\omega}{\sqrt{2}\ |\varepsilon|}\left[\frac{\text{Im}\ A_2}{\text{Re}\ A_2} - (1-\Omega_{\text{eff}})\frac{\text{Im}\ A_0}{\text{Re}\ A_0}\right]$$

Experimental Status (M. Sozzi):

$$\text{Re}\ \varepsilon'/\varepsilon = (1.63 \pm 0.23)\cdot 10^{-3} \qquad \text{W.A.}$$

Theoretical Status:

- ★ General agreement on the OPE part (Munich, Rome).
- ★ Matrix elements and input parameters
- ★ FSI

Hadronic Matrix Elements

- ☆ Lattice calculations: CP-PACS, SPQ$_{CD}$R, UKQCD
- ☆ QCD Sum Rules: Pich, de Rafael
- ☆ Large N_c: within different treatments of the low-energy physics
  - Vacuum Sat. and improvements: Bardeen et al.; Hambye et al.
  - Nambu–Jona-Lasinio like models: Bijnens and Prades
  - Minimal Hadronic Approximation: Knecht et al.
  - Ladder Resummation Approximation: Bijnens,Gámiz,Lipartia,Prades
- ☆ *Dispersive Methods:* Cirigliano et al.;Narison; Bijnens et al.

LO Chiral couplings

Authors,method	Im G_8/Im τ	e^2Im G_E/Im τ
Large N_c	1.9	-2.9
Bijnens, Gamiz, Lipartia, Prades	4.4 ± 2.2	
Hambye, Peris, de Rafael	~ 6	$-(6.7 \pm 2.0)$
Bijnens, Gamiz, Prades;Narison; Cirigliano, Donoghue, Golowich, Maltman(τ decays)		$-(4.0 \pm 0.9)$
Lattice	..	$-(3.2 \pm 0.3)$

Matrix elements and input parameters: news and old problems

- $\Omega_{IB}^{\pi_0\eta} = 0.163 \pm 0.045$ (Ecker, Neufeld, Pich) updated with e.m. corrections (Cirigliano, Ecker, Neufeld, Pich) $\Omega_{\rm eff} = 0.06 \pm 0.077$
- Im $\tau \equiv -$Im $(V_{td}V_{ts}^*/(V_{ud}V_{us}^*)) \sim -(6.05 \pm 0.50)10^{-4}$. Note: if ε_{th} is used in the formula for ε'/ε the dependence of the final result on Im τ is almost canceled. In this case the final result depends on the value of B_K (This is better in Large N_c).
- Strange quark mass. A big source of error in Large N_c. $m_s(2\mathrm{GeV}) \sim (110 \pm 25)\mathrm{MeV}$.$\leftrightarrow$ This dependence traded with quark condensates via GMOR relation.

NLO chiral couplings, $\widetilde{K}_i$?

Not much is known. Using factorization one needs the counterterms from strong chiral Lagrangian of order p^6...
A naive assumption

$$\frac{\text{Im } \widetilde{K}_i}{\text{Re } \widetilde{K}_i} \simeq \frac{\text{Im } G_8}{\text{Re } G_8} \simeq \frac{\text{Im } G_8'}{\text{Re } G_8'} \simeq (0.9 \pm 0.3) \text{ Im } \tau \,,$$

		Re $\widetilde{K}_i(M_\rho)$	Im $\widetilde{K}_i(M_\rho)$
8-et	$\widetilde{K}_2(M_\rho)$	0.35 ± 0.02	$[0.31 \pm 0.11]$ Im τ
8-et	$\widetilde{K}_3(M_\rho)$	0.03 ± 0.01	$[0.023 \pm 0.011]$ Im τ
27-et	$\widetilde{K}_5(M_\rho)$	$-(0.02 \pm 0.01)$	0
27-et	$\widetilde{K}_6(M_\rho)$	$-(0.08 \pm 0.05)$	0
27-et	$\widetilde{K}_7(M_\rho)$	0.06 ± 0.02	0

Re $\widetilde{K}_i(M_\rho)$ from Bijnens, Dhonte, Persson

Final State interaction

- FSI have been shown to be an important ingredient for ε'/ε (Pallante, Pich, S.).
- The degeneracy of $I = 0$ and $I = 2$ amplitude is removed by FSI and Ω_{IB}.
- PPS have included FSI using an Omnés dispersion relation.

Some conclusion from ε'/ε and $K \to 3\pi$

- ♢ FSI and IB effects are getting under control and/or are better checked
- ♢ The main uncertainty of ε'/ε come from the determination of the imaginary part of the couplings of the chiral Lagrangian.
- ♢ The same chiral Lagrangian describes CPV also in $K \to 3\pi$. Recent proposal by NA48 (CERN), KLOE(Frascati), OKA (Protvino). New precision 10^{-4} (Improvement of 2 orders of magnitude). Why not to check better?

✍ Conflicting results in the literature ($10^{-3} - 10^{-6}$)

Some History of $K \to 3\pi$

CP conserving observables

J. Kambor, J. Missimer, D. Wyler, NP **B 346** ('90) 17, PL **B 261** ('91) 496.
J. Kambor et al.,PRL **68** ('92) 1818.
G. Esposito-Farese, ZP C **50** ('91) 255.
G. Ecker, J. Kambor, D. Wyler NP **B 394** ('93) 101.
J. Bijnens, P. Dhonte, F. Persson, **NP B 648:317,2003**.

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CPV observables

B. Grinstein, S.-J Rey, M. Wise, PR **D 33** ('86) 1485.
A. Bel'kov et al., IJMP **A 7** ('92) 4757,PL **B 300**('93) 283.
G. Isidori et al., NP **B 381** ('92) 522.
G. D'Ambrosio et al. , PR **D 50** ('94) 5767., ERR-ibid.**D 51** ('95) 3975.
E. Shabalin, NP **B 409** ('93) 87, PAN **61** ('98) 1372.

The Target

We want to provide a complete (8-et, 27-et, ew-octet) one–loop (NLO)evaluation of chiral correction for both CP conserving and CPV observables in $K \to 3\Pi$.

$$K^+ \to \pi^+\pi^+\pi^- \qquad K_{1,2} \to \pi^+\pi^-\pi^0$$
$$K^+ \to \pi^0\pi^0\pi^+ \qquad K_{1,2} \to \pi^0\pi^0\pi^0$$

Observables: Decay rates, Γ, and

$$\frac{|A_{K^+\to 3\pi}(s_1, s_2, s_3)|^2}{|A_{K^+\to 3\pi}(s_0, s_0, s_0)|^2} = 1 + g\,y + h\,y^2 + k\,x^2 + \mathcal{O}(yx^2, y^3)$$

$$x \equiv \frac{s_1 - s_2}{m^2_{\pi^+}} \quad \& \quad y \equiv \frac{s_3 - s_0}{m^2_{\pi^+}} \quad \text{and} \quad s_i \equiv (k - p_i)^2, \quad 3s_0 \equiv m_K^2 + \sum_{i=1,2,3} m^2_{\pi(i)}\,.$$

Status of 1-loop in ChPT for $K \to 3\pi$

CP conserving part

- The 8-et and 27-et done also by Bijnens, Dhonte, Persson. We fully agree. They provide also a fit of the Re $\tilde{K}_i$. We checked Γ, *g*, *h*, *k*
  Re $G_8 = 6.8 \pm 0.6$ and Re $G_{27} = 0.48 \pm 0.06$

CP violating part

- We included e.m. penguin contribution (all decays, orders $e^2 p^0$ and $e^2 p^2$) and 2-loop imaginary part of the amplitudes, say FSI, using the optical theorem (for charged decays only, neutral decays are in progress)
- All results are analytical

A check on the CP conserving part

$K^\pm \to \pi^\pm\pi^\pm\pi^\mp$	g_C	$\Gamma_C(10^{-18}$ GeV$)$
LO	-0.16 ± 0.02	1.2 ± 0.2
NLO, $\widetilde{K}_i(M_\rho)$ from BDP	-0.22 ± 0.02	3.1 ± 0.6
NLO, $\widetilde{K}_i(M_\rho) = 0$	-0.28 ± 0.03	1.3 ± 0.4
PDG02	-0.2154 ± 0.0035	2.97 ± 0.02
$K^\pm \to \pi^0\pi^0\pi^\pm$	g_N	$\Gamma_N(10^{-18}$ GeV$)$
LO	0.55 ± 0.04	0.37 ± 0.07
NLO, $\widetilde{K}_i(M_\rho)$ from BDP	0.61 ± 0.05	0.95 ± 0.20
NLO, $\widetilde{K}_i(M_\rho) = 0$	0.80 ± 0.05	0.41 ± 0.12
PDG02	0.652 ± 0.031	0.92 ± 0.02
ISTRA+	0.627 ± 0.011	–
KLOE	0.585 ± 0.016	0.95 ± 0.01

Counterterms relevant for $\Gamma_i,\ h_i,\ k_i$

CP violating asymmetries

Definitions: Slopes

$$\Delta g_C \equiv \frac{g[K^+ \to \pi^+\pi^+\pi^-] - g[K^- \to \pi^-\pi^-\pi^+]}{g[K^+ \to \pi^+\pi^+\pi^-] + g[K^- \to \pi^-\pi^-\pi^+]}$$

and $$\Delta g_N \equiv \frac{g[K^+ \to \pi^0\pi^0\pi^+] - g[K^- \to \pi^0\pi^0\pi^-]}{g[K^+ \to \pi^0\pi^0\pi^+] + g[K^- \to \pi^0\pi^0\pi^-]}.$$

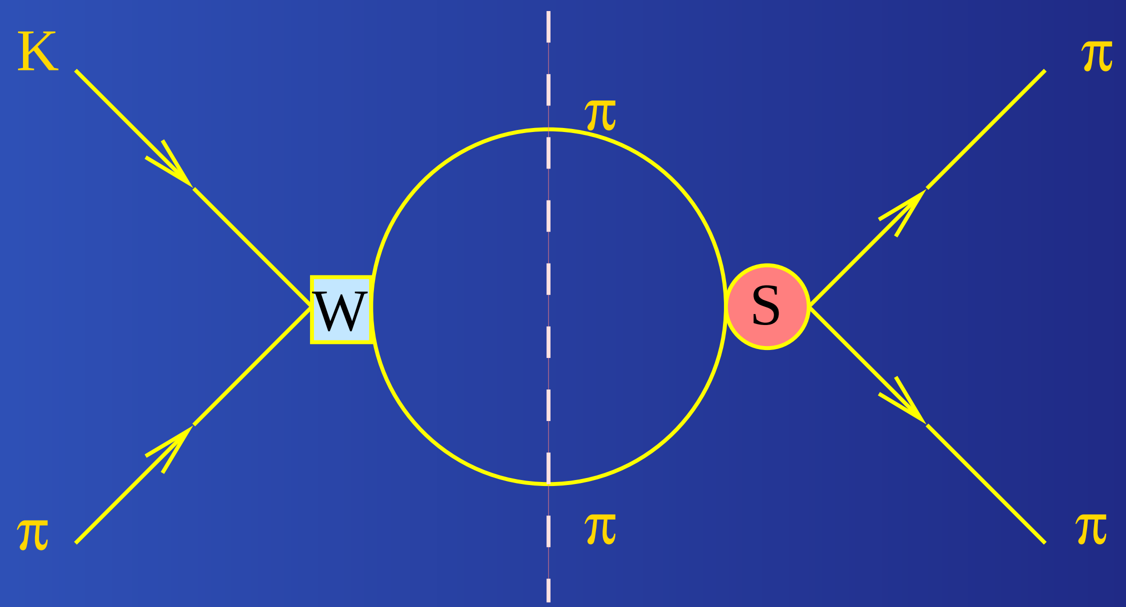
and the same for Decay Rates with $g \to \Gamma$.

LO Results

$$\Delta g_C^{\rm LO} \simeq [1.16\,\text{Im}\,G_8 - 0.12\,\text{Im}\,(e^2 G_E)] \times 10^{-2}\,,$$
$$\Delta g_N^{\rm LO} \simeq -\left[0.52\,\text{Im}\,G_8 + 0.063\,\text{Im}\,(e^2 G_E)\right] \times 10^{-2}\,.$$

	$\Delta g_C^{\rm LO}$ (10^{-5})	$\Delta\Gamma_C^{\rm LO}$ (10^{-6})	$\Delta g_N^{\rm LO}$ (10^{-5})	$\Delta\Gamma_N^{\rm LO}$ (10^{-6})
Large N_c	-1.5	-0.2	0.5	0.8
BGLP	-3.4 ± 2.1	-0.6 ± 0.4	1.2 ± 0.8	2.0 ± 1.3

NLO results: graphics for FSI



For Im$A \sim \mathcal{O}(p^4)$ (LO) one needs W, S$\sim \mathcal{O}(p^2)$.
For Im$A \sim \mathcal{O}(p^6)$ (NLO) one needs W$\sim \mathcal{O}(p^2)$ and S$\sim \mathcal{O}(p^4)$ and viceversa.

NLO results: FSI in the asymmetries

$$|A(K^{\pm} \to 3\,\pi)|^2 = A_0^{\pm} + y\,A_y^{\pm} + \mathcal{O}(x, y^2)$$

$$\Delta g = \frac{A_y^+ A_0^- - A_0^+ A_y^-}{A_y^+ A_0^- + A_0^+ A_y^-} \,.$$

- ✗ The sum $A_y^+ A_0^- + A_0^+ A_y^-$ does NOT contain FSI (i.e. $\mathcal{O}(p^6)$) at NLO (they would be part of the NNLO)
- ✗ The difference $A_y^+ A_0^- - A_0^+ A_y^- \sim$ Im A: to have it at NLO we must take into account FSI phases $\to$ FSI at NLO only in imaginary parts (in other words: Re $A \sim \mathcal{O}(p^2) + \mathcal{O}(p^4) + ..$ while Im $A \sim \mathcal{O}(p^4) + \mathcal{O}(p^6) + ..$)
- ✗ The calculation of the imaginary part can be done in ChPT using the optical theorem

Results for the asymmetries

NLO

$$\frac{\Delta g_C^{\rm NLO}}{10^{-2}} \simeq 0.66\,{\rm Im}G_8 + 4.33\,{\rm Im}\widetilde{K}_2 - 18.11\,{\rm Im}\widetilde{K}_3 - 0.07\,{\rm Im}(e^2 G_E)\,,$$

	$\Delta g_C^{\rm NLO}$ (10^{-5})	$\Delta\Gamma_C^{\rm NLO}$ (10^{-6})	$\Delta g_N^{\rm NLO}$ (10^{-5})	$\Delta\Gamma_N^{\rm NLO}$ (10^{-6})
$\widetilde{K}_i(M_\rho)$,BDP	-2.4 ± 1.2	$[-11, 9]$	1.1 ± 0.8	$[-9, 11]$
$\widetilde{K}_i(M_\rho) = 0$	-2.4 ± 1.3	1.0 ± 0.7	0.9 ± 0.5	4.0 ± 3.2

Comments on the charged $K \to 3\pi$ as.

- Δg_C is dominated by Im G_8. Ch-NLO on Δg_C give effects of about 20-30$\%$. The final error is due mainly to Im G_8.
  - consistency with ε'/ε
- Δg_N and $\Delta\Gamma_{C,N}$ are dominated by $\mathcal{O}(p^4)$ counterterms, $\tilde{K}_i$
  - New important information on Im $\tilde{K}_i$
- The new experimental limit of 10^{-4} will put ChPT under stringent test and so check eventual NP effects. SM prefers values of $\Delta g_C < 0.5 \times 10^{-4}$. For consistency with ε'/ε $\Delta g_C \simeq 0.2 \times 10^{-4}$.

Conclusions

- The main problem for a good estimate of ε'/ε are still hadronic matrix elements. It would be extremely helpful to measure other CP-violating channels in hadronic kaon decays. $K \to 3\pi$ offers several chances.

- We have provided the first NLO in ChPT estimate of CP-violating asymmetries in charged $K \to 3\pi$. The results for the 8-et and 27-et part agree with BDP. We have included e.w. penguin contribution (up to $\mathcal{O}(e^2p^2)$) and imaginary part of the amplitudes up to $\mathcal{O}(p^6)$ (FSI). Neutral channels are in progress.

- Forthcoming experiments on hadronic kaon decays have still the possibility to give many surprises.