Difference of $\tilde{\epsilon}$ and ϵ in fitting the parameters of CKM matrix

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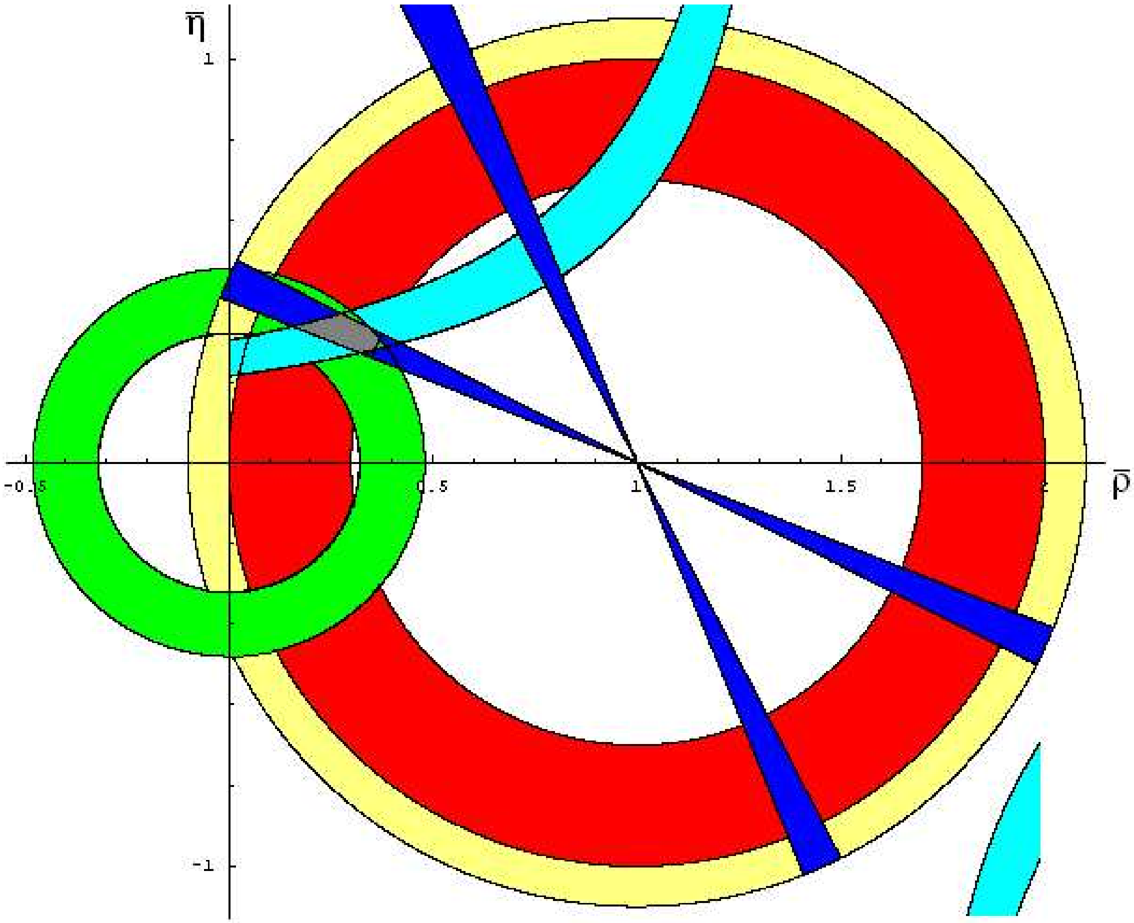
- Motivation
- $K^0 - \bar{K}^0$ mixing. Parameter $\tilde{\epsilon}$
- Neutral $K \to \pi\pi$ decays. Parameters ϵ and ϵ'
- Estimation of difference between ϵ and $\tilde{\epsilon}$
  - Omitting the contribution from EW penguins
  - Taking the result of explicit calculation of ImA_0
- Fit of the parameters of CKM matrix
- Conclusions

Motivation

- CKM matrix does explain large amount of experimental data
- the precision of experimental measurements is continuously improving
- fitting the CKM matrix parameters is an important tool in searching for New Physics
- the two less precisely known CKM parameters $\bar{\rho}$ and $\bar{\eta}$ are constrained from different sources (see figure)



Domains in $(\bar{\rho}, \bar{\eta})$ plane defined by V_{ub}, Δm_{B_d}, $\tilde{\epsilon}$ and $\sin 2\beta$ measurements

Motivation (cont.)

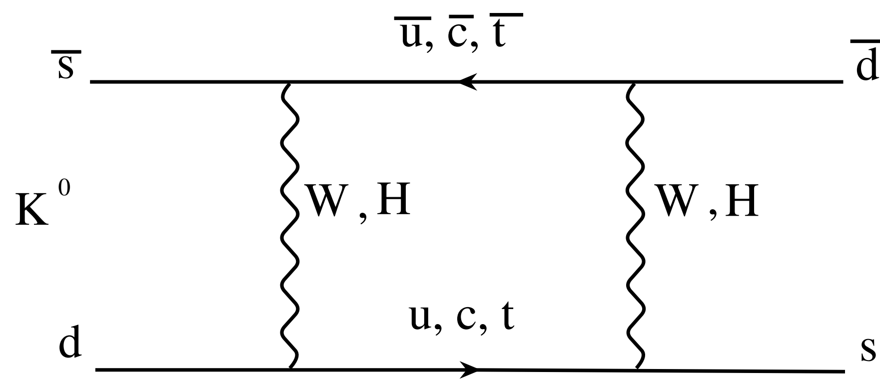
- theoretically computed parameter $\tilde{\epsilon}$ gives a constraint in the $\bar{\rho} - \bar{\eta}$ plane
- experimentally measured parameter is ϵ
- in the literature it is claimed that the difference between $\tilde{\epsilon}$ and ϵ is about 2% and could be neglected while fitting CKM matrix parameters
- our aim is to estimate the difference between $\tilde{\epsilon}$ and ϵ more accurately
- the difference appeared to be about $5 - 10\%$ and should be taken into account in view of increasing experimental and theoretical precision

$K^0 - \bar{K}^0$ mixing. Parameter $\tilde{\epsilon}$

$$K_L = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}\left[\frac{K^0+\bar{K}^0}{\sqrt{2}} + \tilde{\epsilon}\frac{K^0-\bar{K}^0}{\sqrt{2}}\right] \qquad K_S = \frac{1}{\sqrt{1+|\tilde{\epsilon}|^2}}\left[\frac{K^0-\bar{K}^0}{\sqrt{2}} + \tilde{\epsilon}\frac{K^0+\bar{K}}{\sqrt{2}}\right.$$

Within the SM the transition $K^0 - \bar{K}^0$ originates from "box" diagrams:

$$\tilde{\epsilon} \approx -\frac{iImM_{K^0\bar{K}^0}/2m_K}{(\Delta m_{LS} - \frac{i}{2}(\Gamma_L - \Gamma_S))}$$



The theoretical expression for $\tilde{\epsilon}$ is:

$$|\tilde{\epsilon}| \approx \frac{G_F^2 m_W^2 m_K f_K^2}{12\sqrt{2}\pi^2 \Delta m_K} B_K \Big(\eta_{cc} S(x_c, x_c) Im[(V_{cs}V_{cd}^*)^2] + \eta_{tt} S(x_t, x_t) Im[(V_{ts}V_{td}^*)^2] +$$

$$+ 2\eta_{ct} S(x_c, x_t) Im[V_{cs}V_{cd}^* V_{ts}V_{td}^*]\Big)$$

Neutral $K \to \pi\pi$ decays. Parameters ϵ and ϵ'

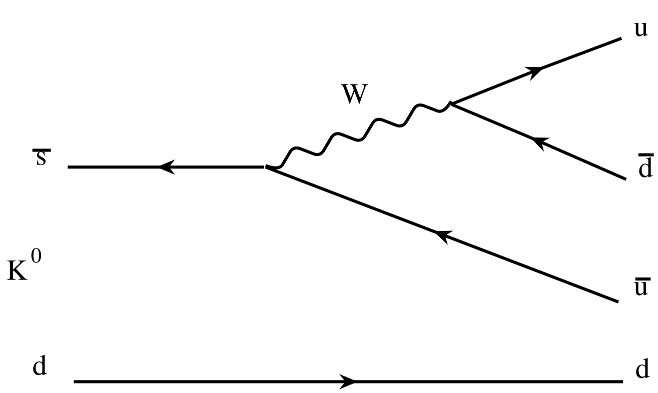
The experimentally measured quantities are:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \qquad \eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}.$$

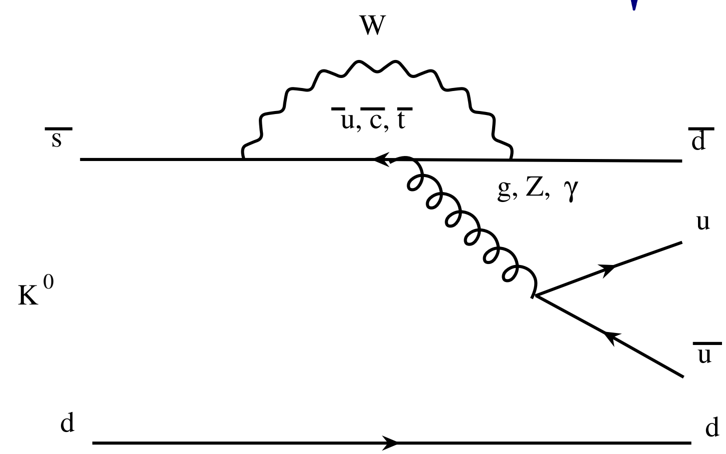
Decay amplitudes into final $(\pi\pi)$ states with definite isospin:

$$A(K^0 \to (\pi\pi)_I) = A_I e^{i\delta_I} \qquad A(\bar{K}^0 \to (\pi\pi)_I) = A_I^* e^{i\delta_I} \qquad I = 0, 2$$

$$A(K^0 \to \pi^+\pi^-) = \frac{1}{\sqrt{3}} A_2 e^{i\delta_2} + \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} \qquad A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_2 e^{i\delta_2} - \frac{1}{\sqrt{3}} A_0 e$$



"Tree"



"Penguin"

Three relations between the decay amplitudes into states with definite isospin hold:

1). $ImA_I << ReA_I$ 2). $ImA_2 << ImA_0$ 3). $w = \frac{|A_2|}{|A_0|} \approx \frac{1}{22.2}$

1). from the smallness of imaginary part of V_{cd} and smallness of t - quark contrib

2). from the dominance of QCD penguins in ImA_0 and EW penguins in ImA_2

3). from experiment

Quantities η_{00} and η_{+-} are expressed in terms of ϵ and ϵ':

$$\eta_{+-} = \epsilon + \epsilon' \qquad \eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon' = \frac{i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \frac{1}{ReA_0} \left[ImA_2 - wImA_0\right] \qquad \epsilon = \tilde{\epsilon} + i\frac{ImA_0}{ReA_0}$$

Estimation of difference between ϵ and $\tilde{\epsilon}$

Relation between absolute values of $\tilde{\epsilon}$ and ϵ:

$$|\tilde{\epsilon}| \approx |\epsilon| - \frac{1}{\sqrt{2}} \frac{ImA_0}{ReA_0}$$

$|\epsilon^{exp}| = 2.28(2) \times 10^{-3}$ experimental value from PDG

Omitting the contribution from EW penguins

$$ImA_2 = 0$$

$$|\epsilon'| = \frac{1}{\sqrt{2}}\frac{1}{ReA_0}\left[ImA_2 - wImA_0\right] \quad \Longrightarrow \quad |\epsilon'| \approx -\frac{w}{\sqrt{2}}\frac{ImA_0}{ReA_0}$$

Note that as $ImA_2 < 0$, in general $|\epsilon'| < -\frac{w}{\sqrt{2}}\frac{ImA_0}{ReA_0}$.

Substituting experimental numbers we get:

$$\frac{ImA_0}{ReA_0} \approx -\frac{\sqrt{2}|\epsilon'|}{w} = -(1.3 \pm 0.3) \times 10^{-4}$$

This leads to 4% difference between ϵ and $\tilde{\epsilon}$:

$$|\tilde{\epsilon}| = 2.37(2) \times 10^{-3}$$

This value must be considered as a lower bound.

Taking the result of explicit calculation of ImA_0

In order to explain the observed value of ϵ' people calculate ImA_0 and ImA_2.

A standard parametrization of ImA_0 is:

$$ImA_0 = -\frac{G_F}{\sqrt{2}} Im(V_{td}V_{ts}^*) \sum_i y_i \langle (\pi\pi)_{I=0} | Q_i | K^0 \rangle (1 - \Omega_{IB})$$

$$ReA_0 = 3.33 \times 10^{-7} GeV \qquad \text{(experiment)}$$

$$\frac{ImA_0}{ReA_0} = (-2.23 \pm 0.66) \times 10^{-4}$$ [T.Hambye, S.Peris, E.de Rafael,hep-ph/030510

This leads to $5 - 10\%$ difference between ϵ and $\tilde{\epsilon}$:

$$|\tilde{\epsilon}| = (2.44 \pm 0.04) \times 10^{-3}$$

Fit of the parameters of CKM matrix

We perform a simple fit of CKM matrix parameters.

Experimentally measured data are assumed to be normally distributed, as well as theoretical uncertainties.

Input: V_{us}, V_{cb}, V_{ub}, V_{ud}, V_{cd}, V_{cs}, $\underline{\underline{|\tilde{\epsilon}|}}$, Δm_{B_d}, Δm_{B_s}, $sin2\beta$.

The result of the fit with $|\tilde{\epsilon}| = (2.44 \pm 0.04) \times 10^{-3}$:

$$A = 0.83 \pm 0.04$$
$$\lambda = 0.223 \pm 0.002$$
$$\bar{\rho} = 0.2 \pm 0.08$$
$$\bar{\eta} = 0.35 \pm 0.04$$

Conclusions

- We estimate the shift of $\tilde{\epsilon}$ relative to ϵ and find it to be about $5-10\%$.
- So it should be taken into account while fitting the CKM matrix parameters as the experimental and theoretical precision increases.