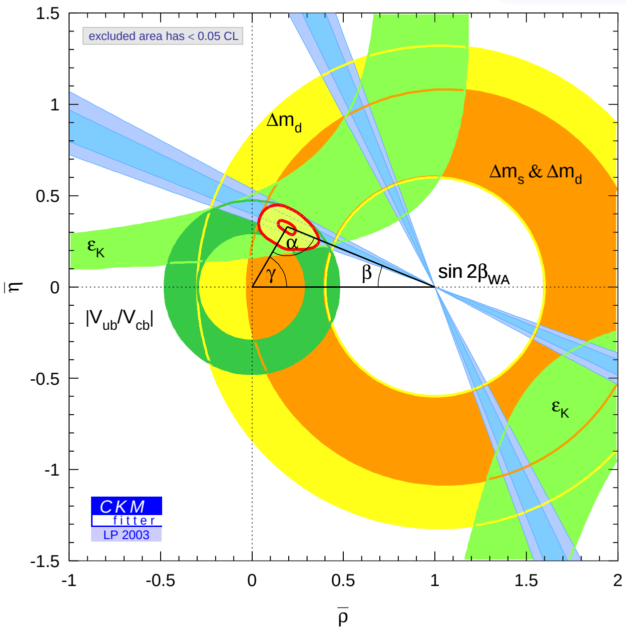
Theory Status of $\bar{B} \to X_s\gamma$

- Introduction
- Inclusive Decays
- Resummation of Logs
- $\bar{B} \to X_s\gamma$ and m_c
- Beyond NLO
- Conclusions

Moriond 2004

Tests of the Flavour Sector



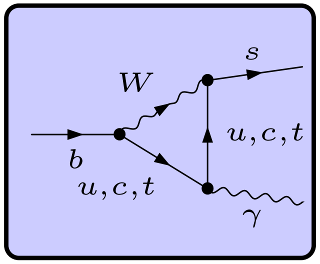
- Unitarity triangle fit [Höcker et al. '03] already constrains new sources of flavour and CP violation
- Only the $\Delta F = 2$ constraints test the quantum level, but they suffer from large hadronic uncertainties

Find theoretically clean decays

Test $\Delta F = 1$ decays up to quantum level

Inclusive $\bar{B} \to X_s \gamma$ decay

FCNC Decay $b \to s\gamma$



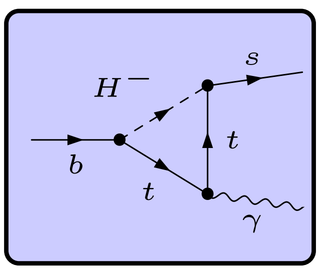
$$
\begin{array}{llll}
\text{top loop} & \propto V_{tb}V_{ts}^* = \mathcal{O}(\lambda^2) & \to -100\% \\
\text{charm loop} & \propto V_{cb}V_{cs}^* \approx -V_{tb}V_{ts}^* & \to +200\% \\
\text{up loop} & \propto V_{ub}V_{us}^* = \mathcal{O}(\lambda^4) & \to 0
\end{array}
$$

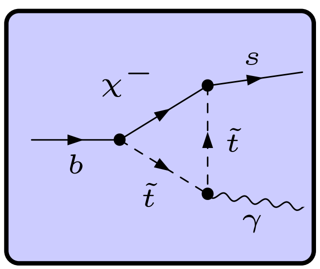
including LO QCD

In the SM forbidden at tree level & CKM suppressed

Precision test of the flavour sector

Enhanced sensitivity to new physics

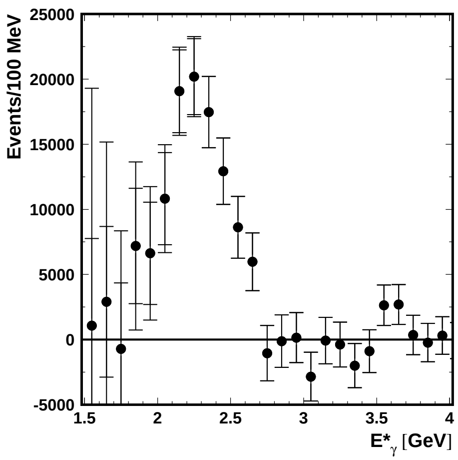




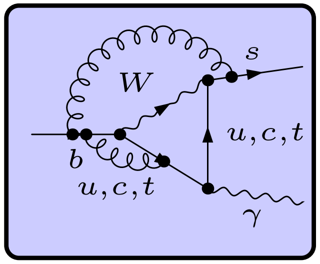
- Charged Higgs contribution enhance $b \to s\gamma$
- Different new physics contributions have to cancel

Two Problems: Bound States and Large Logs

- We can only observe decays of bound states $\Rightarrow$ decay at parton level may not approximate the hadronic decay
- Study inclusive decay:
  $\mathrm{BR}_\gamma = (3.34 \pm 0.38) \times 10^{-4}$



- New measurement by Belle:
  $\mathrm{BR}_\gamma = (3.59 \pm 0.32^{+0.30}_{-0.31}{}^{+0.11}_{-0.07}) \times 10^{-4}$
  used $E_\gamma > 1.8\mathrm{GeV}$



For n gluons we have

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^n \frac{m_b^2}{M_W^2} (LL)$$

$$\left(\frac{\alpha_s}{4\pi}\right)^n \log^{n-1} \frac{m_b^2}{M_W^2} (NLL)$$

- Large logs $\Rightarrow$ straightforward perturbation theory unreliable
- Leading Log enhance Branching Ratio by 200%
- Use renormalization group to resum leading and next-to-leading logs

Inclusive $\bar{B} \to X_s\gamma$ Decay

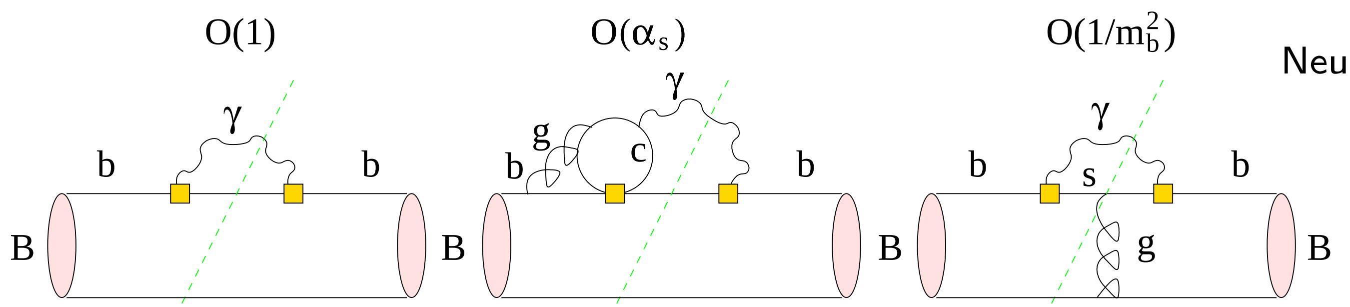
Sum over all X_s final states

$m_b \gg \Lambda_{QCD}$ hadron binding energy

Contribution of external states drops out

- For $m_b \to \infty$ is $\Gamma[\bar{B} \to X_s\gamma] \approx \Gamma[b \to s\gamma] + \Gamma[b \to s\gamma g]^\delta + \ldots$ [Chay et al. '90, Manohar et al. '93]

- $1/m_b^2$ and $1/m_c^2$ corrections can be added systematically [Falk et al. '93, Bigi '92, Voloshin '97, Ligeti et al. '97, Grant et al. '97, Buchalla et al. '98, Khodjamirian et al. '00]



Neubert '02

Effective Field Theories

At high scales $\mu_0 \sim M_W$ the full theory contains heavy $W, t, \ldots$ and light $g, b, \ldots$ fields:

$$\mathcal{L}_{\rm full} = \mathcal{L}_H(h, l) + \mathcal{L}(l).$$

At a low scale $\mu < \mu_0$ we obtain an effective Lagrangian:

$$\mathcal{L}_{\rm eff} = \mathcal{L}(l) + \delta\mathcal{L}(l)$$

The calculation takes three steps

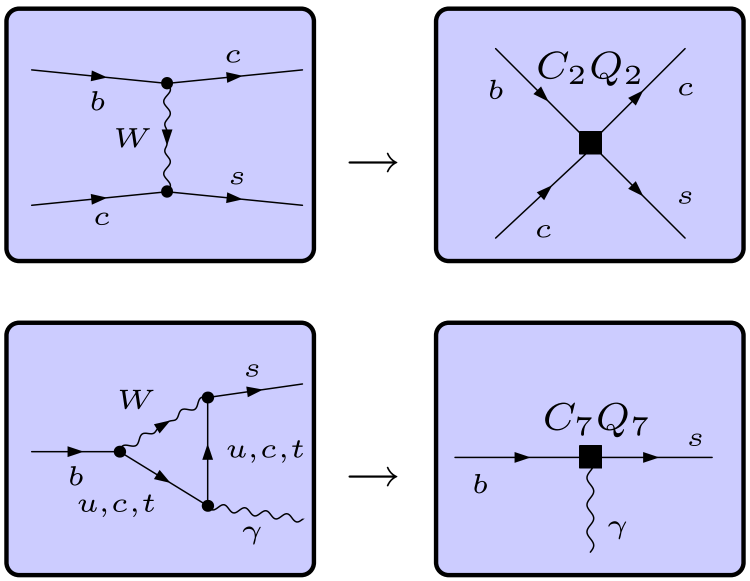
- Matching of $\mathcal{L}_{\rm full}$ and $\mathcal{L}_{\rm eff}$ at μ_0 gives $\delta\mathcal{L}(L)$
- With the help of the Renormalization Group Equation (RGE) we can relate the effective Lagrangian at the high scale to the low scale one

$$\mathcal{L}_{\rm eff} \quad \text{at} \quad \mu_0 \rightarrow \mathcal{L}_{\rm eff} \quad \text{at} \quad \mu$$

- Calculation of the matrix elements (process dependent)

QCD Matching

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum C_i(\mu)\, Q_i$$



- Current-current

$$Q_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)\,,$$

$$Q_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)\,,$$

- Magnetic

$$Q_7 = e/g^2\; m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}\,,$$

$$Q_8 = 1/g\; m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}\,,$$

At NLO: 2-loop matching [Adel, Yao '93; Greub, Hurth '97; Buras et al. '98]

Scale Dependence of the Wilson Coefficients

- The scale dependence of the Wilson coefficients

$$\mu \frac{d}{d\mu} C_i(\mu) = \gamma_{ji} C_j(\mu)$$

  is given by the anomalous dimension matrix

- This RGE can be solved

$$\vec{C}(\mu) = \hat{U}(\mu, \mu_0) \vec{C}(\mu_0)$$

- This gives the low scale Wilson Coefficients

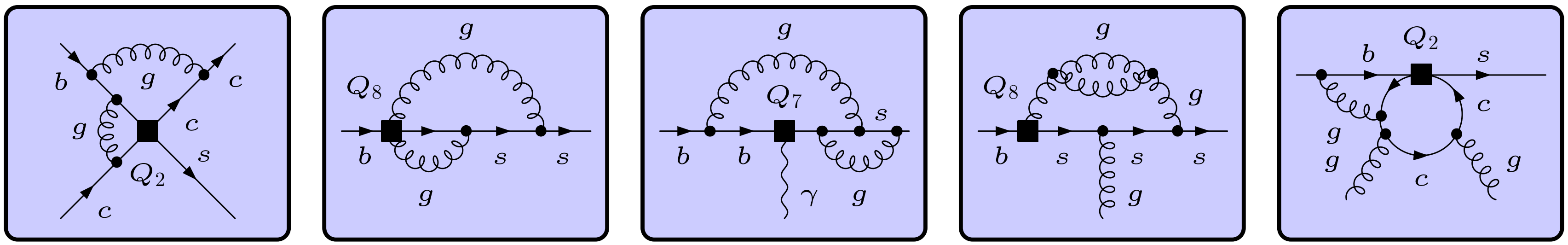
$\lvert C_{1,2}(m_b)\rvert$	$\lvert C_{3-6}(m_b)\rvert$	$C_7(m_b)$	$C_8(m_b)$
$\mathcal{O}(1)$	≤ 0.7	-0.3	-0.15

At NLO we need the 3-loop mixing of Q_2 into Q_7 [Chetyrkin, Misiak, Münz '98; Gambino, Gorbahn, Haisch '03]

Calculation of the ADM

New calculation for the ADM [Gambino, Gorbahn, Haisch]

- Some of the two-loop diagrams



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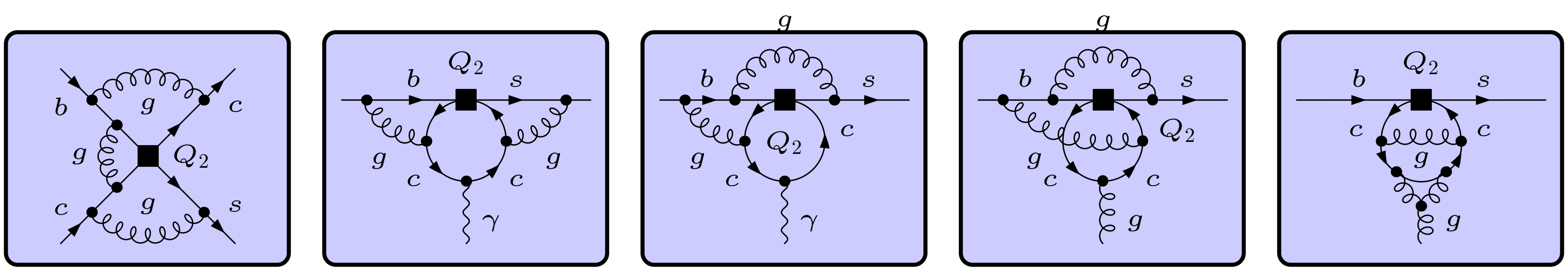


Matrix Elements

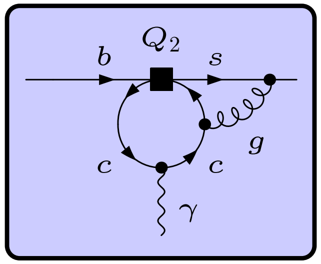
Decay amplitude by on-shell ME

- The 1-loop ME Q_{1-6} give only a constant contribution: Absorb
- The LO result is then given by $C_7(\mu_b)$

At NLO:

- 1-loop ME of $Q_{7,8}$
- 2-loop ME of Q_{1-6} [Greub, Hurth, Wyler '96, Buras et al. '01, Asatrian et al. '04]
- Bremsstrahlung [Ali, Greub '93; Pott '96]

charm mass dependence starts at NLO



First charm dependent Matrix Element is 2 loop:

- Formally, any definition for m_c can be used
- Gambino Misiak pointed out to use m_c in $\overline{\mathrm{MS}}$ at $\mu \sim m_b/2$

Scale Dependence of $\bar{B} \to X_s\gamma$

- At LO the branching ratio can be written

$$\mathrm{BR}[\bar{B} \to X_s\gamma]_{E_\gamma > E_0} =$$

$$\mathrm{BR}[\bar{B} \to X_c e\bar{\nu}]_{\exp} \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \times$$

$$\left[|C_7^{(0)eff}(\mu)|^2 + \mathrm{NP}(E_0) \right]$$

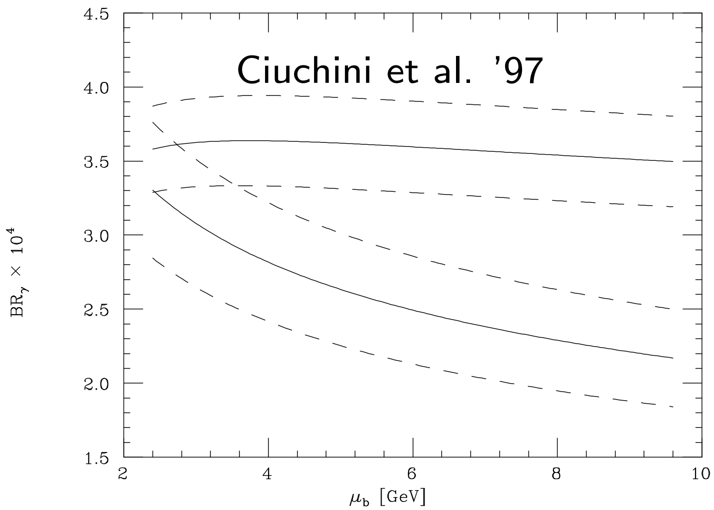
- m_c enters the phase space factor

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]} = 0.581 \pm 0.017$$

- Only other m_c dependence from 2-loop Matrix Elements

- At NLO we get a $\alpha_s \log(\mu_b/m_b)$ term form the matrix elements

- This reduces the scale uncertainty drastically



Beyond NLO QCD

Electroweak corrections

- No $\ln(m_b^2/m_e^2)$ if one uses $\alpha_{\rm em}^{\rm onshell}$ as overall normalisation [Czarnecki, Marciano '98]
- $\alpha_e/\alpha_s \ln(m_W^2/m_b^2)$ negligible [Kagan, Neubert '99; Baranowski Misiak '00]
- Matching reduces $\Gamma[b \to s\gamma]$ by -1.5% for $M_{\rm Higgs} = 115 GeV$ [Gambino, Haisch '00 '01]

Nonperturbative corrections

- $1/m_b^2$ amounts to -3%, $1/m_c^2$ to $+2.5\%$

The dependence on the definition of m_c (formally NNLO)

- if we use m_c in $\overline{\rm MS}$ at $\mu \sim m_b/2$ we get $+10\%$:

$$\mathrm{BR}(\bar{B} \to X_s\gamma)_{\rm th} = (3.70 \pm 0.30) \times 10^{-4}$$

Error Anatomy of $\mathrm{BR}(\bar{B} \to X_s \gamma)$

- Following the analysis of Gambino Misiak '01

$$
\begin{aligned}
\mathrm{BR}(\bar{B} \to X_s\gamma)_{E_\gamma > 1.6\mathrm{GeV}} = 3.61 \times 10^{-4} \times \\
&(1 \pm 0.06_{m_c/m_b} \pm 0.04_{\mathrm{otherNNLO}} \\
&\pm 0.01_{(\mathrm{pert\,C})} \pm 0.02_{\lambda_1} \pm 0.02_{\Delta} \\
&\pm 0.02_{\alpha_s(M_Z)} \pm 0.02_{\mathrm{BR(semilept)_{exp}}} \pm 0.01_{m_t}) \\
&\qquad = (3.61 \pm 0.30) \times 10^{-4}
\end{aligned}
$$

- Total 8% error dominated by charm mass
- This will be improved by going to NNLO

Towards a NNLO prediction of $b \to s\gamma$

- 2-loop matching of Q_{1-6} [Bobeth, Misiak, Urban '00]
- 3-loop matching of $Q_{7,8}$ [Misiak, Steinhauser '04]
- 3-loop mixing of Q_{1-6} [Gorbahn, Haisch in preparation]

To do:

- 4-loop mixing into $Q_{7,8}$ and 3-loop selfmixing [Gorbahn, Haisch, . . .]
- 3-loop matrix elements of $Q_{1,2}$ [Bieri, Greub, Steinhauser '03; Misiak, Steinhauser]
- 2-loop matrix elements of $Q_{7,8}$ [Greub, Hurth, Asatrian]
- Bremsstrahlung

Implications for $\bar{B} \to X_s\gamma$

- The complete NLO prediction of $\bar{B} \to X_s\gamma$ has been done independently by at least two groups
- Important since the LO has large scale uncertainties
- BR_γ is in good agreement with experiment

$$\mathrm{BR}_{\mathrm{th}} = (3.70 \pm 0.30) \times 10^{-4} \sim (3.34 \pm 0.38) \times 10^{-4} = \mathrm{BR}_{\mathrm{exp}}$$

$$\mathrm{BR}_{\mathrm{exp}}^{\mathrm{BELLE}\,'04} = (3.59 \pm 0.32^{+0.30\,+0.11}_{-0.31\,-0.07}) \times 10^{-4}$$

- Experiments are improving: m_c dominant theory error
- NNLO calculation is becoming necessary and has been started recently
- Important to stringently constrain new Physics