Weak interaction phenomenology from lattice QCD

Laurent Lellouch

lellouch@cpt.univ-mrs.fr

CPT, Marseille

Objectives of lattice QCD

- Establish that QCD is theory of strong interactions also in NP domain
- Fix the fundamental parameters of QCD
- Determine the NP QCD corrections to weak processes involving quarks
- Understand QCD at finite-T and/or density
- Make predictions (postdictions) for exotic hadrons
- Understand mechanism(s) of confinement and χSB

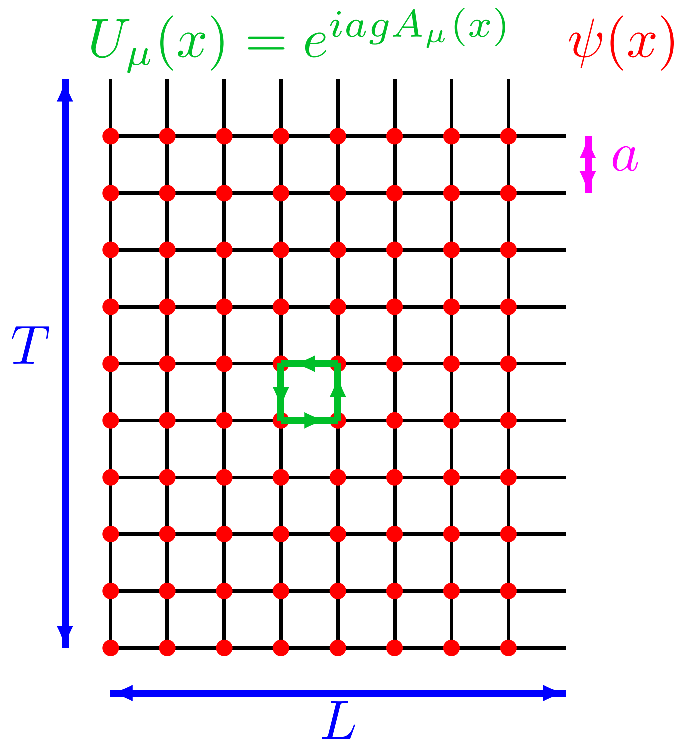
In particle phenomenology $\longrightarrow$ contribute to and learn from experiment: B-factories (constraining UT, rare decays, ...), Tevatron Run II (ΔM_{B_s}, $\Delta\Gamma_{B_s}$, b-hadron lifetimes, ...), CLEO-c, BES-III (leptonic and semileptonic D decays, masses of quarkonia, hybrids, glueballs, ...), LHC ...

What is lattice QCD?

Confinement $\Rightarrow$ need non-perturbative (NP) QCD tool to relate experiment (hadrons) to underlying theory of quarks and gluons

Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (and IR) cutoffs and a well defined path integral
- finite # of dof's $+$ euclidean spacetime
  $\Rightarrow$ numerical evaluation of path integral using stochastic methods



$\Rightarrow$ hadronic observables obtained *directly* from QCD lagrangian $\longrightarrow$ not a model

$\Rightarrow$ errors can be made arbitrarily small w/ stats $\rightarrow \infty$, $a \rightarrow 0$ and $V \rightarrow \infty$

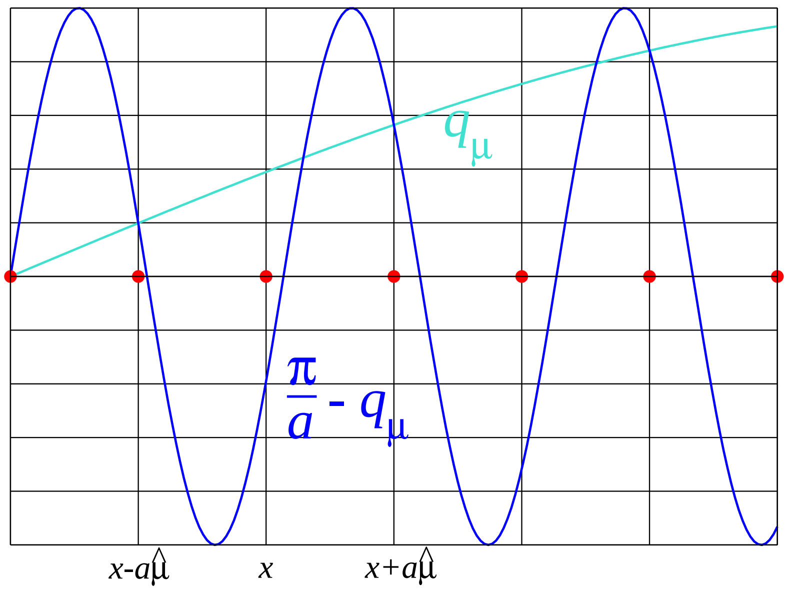
In practice, approximations are required ...

Fermion doubling

$$S = a^4 \sum_x \bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x)$$

Naive discretization

$$\begin{aligned} \partial_\mu \psi(x) &\rightarrow \tfrac{1}{2}\left[\nabla^*_\mu \psi(x) - \nabla_\mu \psi(x)\right] \\ &\equiv \tfrac{1}{2a}\left[\psi(x+a\hat{\mu}) - \psi(x) + \psi(x) - \psi(x-a\hat{\mu})\right] \end{aligned}$$



$\frac{\pi}{a} - q_\mu$ mode gives same contrib. to action as a q_μ mode w/ opposite chirality

$\Rightarrow$ naive discretization $\longrightarrow 2^d$ flavors

Solutions to doubling problem

Wilson fermions: add $-\frac{a}{2}\bar{\psi}(x)\nabla^*_\mu\nabla_\mu\psi(x)$ to lagrangian
$\Rightarrow$ 15 doublers decouple, but break axial sym. @ $O(a)$
$\Rightarrow$ N_f fields: $SU(N_f)_V$ at $a \neq 0$ and $SU(N_f)_L \times SU(N_f)_R$ when $a \to 0$, w/ fine tuning

Rather cheap but loss of axial sym. is a problem

Staggered fermions: keep 1 Dirac component of $\psi(x)$ at each x
$\Rightarrow$ 16/4 flavors or *tastes*, but break flavor sym. @ $O(a^2)$
$\Rightarrow$ N_f fields: $U(N_f)_e \times U(N_f)_o$ at $a \neq 0$ and $SU(4N_f)_L \times SU(4N_f)_R$ when $a \to 0$ (?), w/out fine tuning

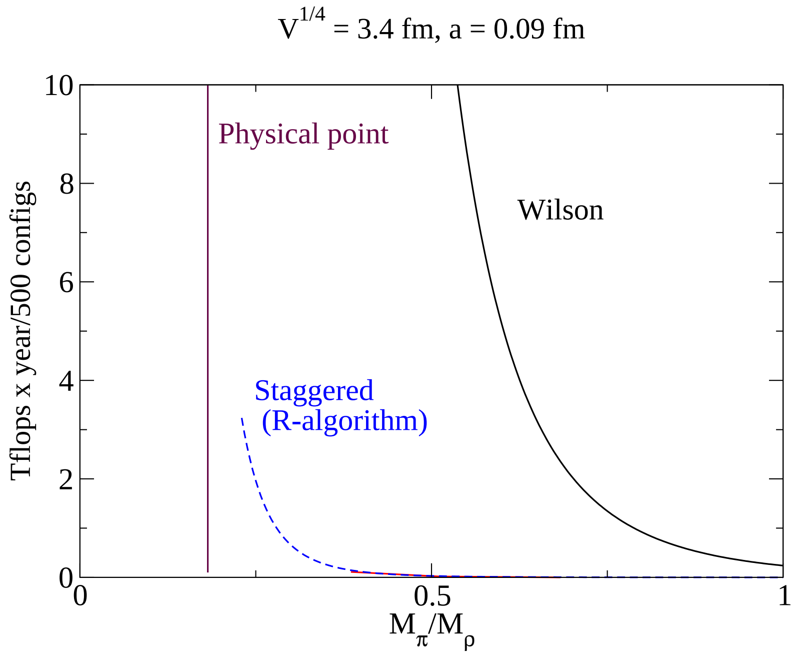
Cheap but have to deal with additional tastes

Ginsparg-Wilson fermions (domain wall/overlap): implement modified chirality
$\Rightarrow$ 15 doublers decouple as w/ Wilson fermions
$\Rightarrow$ N_f fields: full $SU(N_f)_L \times SU(N_f)_R$ at $a \neq 0$ and no fine tuning

By far the cleanest theoretically, but very expensive $\sim O(100)\times$ Wilson

Lattice luminosity: cost wall

Numerically very demanding: # of d.o.f. $\sim \mathcal{O}(10^9)$



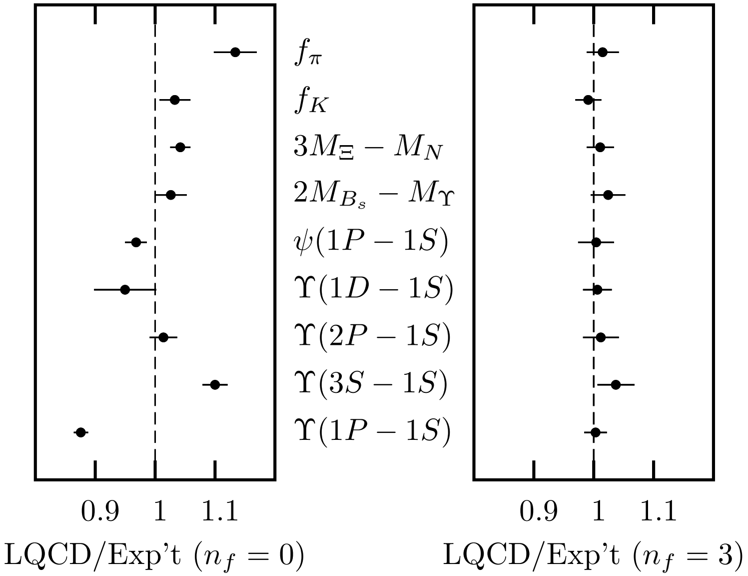
- cost $\sim A\left(\frac{M_\pi}{M_\rho}\right)^{-6} V^{5/4} a^{-7}$
  (Ukawa '02, Gottlieb '02)
- Both formulations have a cost wall
- Wall appears for much lighter quarks w/ Staggered, but algorithm not exact

$\longrightarrow$ MILC has gone for the gusto: $N_f = 2+1$ simulations with $m_{u,d} \gtrsim 0.1 m_s$!

$\Rightarrow$ masses small enough to match onto χPT (at least for mesons)

Are we there?

Certainly looks like it!



(Davies et al '04)

Devil's advocate! $\longrightarrow$ potential problems:

- $[\det(D+m)]^{1/4}$ to get rid of 4 spurious tastes $\Rightarrow$ possible non-localities
- algorithm used not exact
- at current a, significant lattice artefacts
- insensitivity to topology?
- renormalizablity of Staggered fermions not shown to all orders in PT

However, "best" we can do *now* $\longrightarrow$ should be pursued, *but approach should be put on firmer ground*

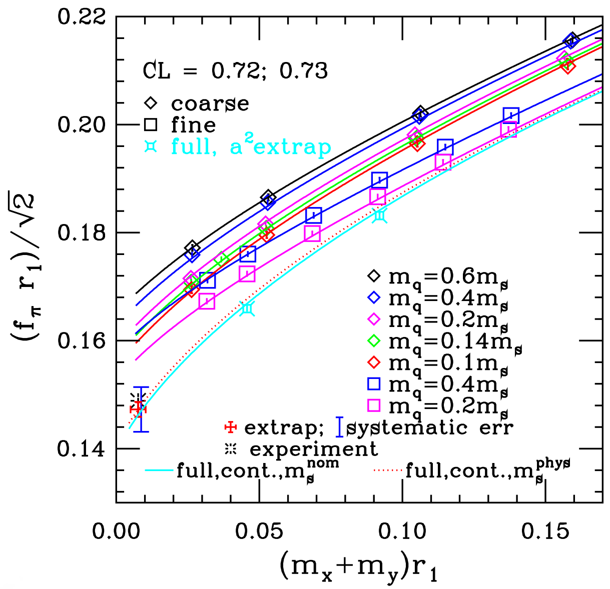
$|V_{us}|$ *from the lattice*

Marciano '04: obtain $|V_{us}|$ from

$$\frac{\Gamma(K\to\mu\bar{\nu}(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}(\gamma))} = 0.9930(35)\times\frac{|V_{us}|^2}{|V_{ud}|^2}\times\frac{f_K^2 M_K\left(1-\frac{m_\mu^2}{M_K^2}\right)^2}{f_\pi^2 M_\pi\left(1-\frac{m_\mu^2}{M_\pi^2}\right)^2} = 1.334(4)$$

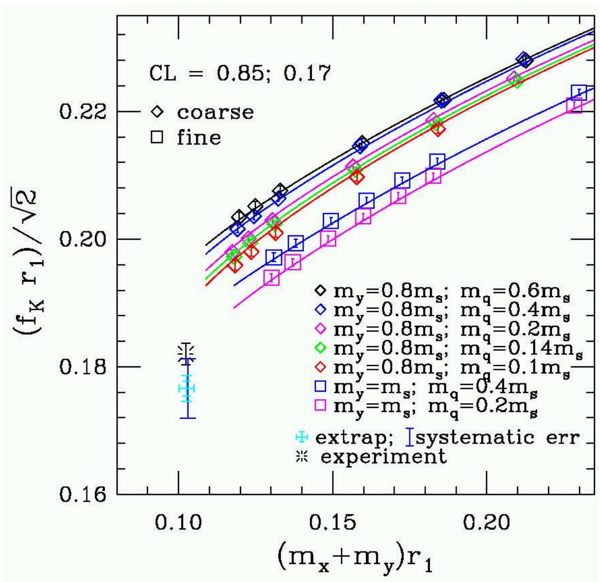
w/ f_K/f_π from $N_f = 2+1$, staggered fermion calculation (MILC '03-'04)

Fit M_π^2 and f_π computed at $a \simeq 0.12$ and $0.09\,\mathrm{fm}$, as fn of valence and sea q masses, to partially quenched, staggered χPT (pqSχPT) at NLO $++$: 46 parameters of which 27 account for lattice artefacts



A): f_π, L_i, $m_{u,d}^{phys}$ from "data" w/ $m \lesssim 0.8 m_s^{phys}$ (194 points)

B): f_K, m_s^{phys} from "data" w/ $m \lesssim m_s^{phys}$ (324 points)



In addition to potential pbs w/ simulation itself:

- Very complex fitting procedure $\rightarrow$ stability?
- Chiral and continuum extrap's significant compared to final errors: 12% for f_π and 10% for f_K
- Validity of χPT expansion in presence of taste partners with masses up to $\sim 750\,\mathrm{MeV}$ (set A) or even $\sim 850\,\mathrm{MeV}$ (set B)?

Preliminary results:

$$\begin{aligned} f_\pi &= 129.3 \pm 1.1 \pm 3.5\,\mathrm{MeV} \\ f_K &= 155.0 \pm 1.8 \pm 3.7\,\mathrm{MeV} \end{aligned} \qquad \frac{f_K}{f_\pi} = 1.201 \pm 0.008 \pm 0.015$$

Dominant systematic on $f_{\pi,K}$: 2.2% scale uncertainty $\leftrightarrow O(10\%)$ + quenching error, minimum systematic in a quenched calculation!

Using $|V_{ud}|$ from super-allowed $0^+ \rightarrow 0^+$ nuclear beta decays (Marciano '04):

$$|V_{us}| = 0.2236(1)(3)(4)(30) \qquad \text{and} \qquad |V_{ud}|^2 + |V_{us}|^2 = 0.9987(17)$$

$\longrightarrow$ dissipates 2 standard deviation departure from 3 generation unitarity (with $|V_{us}|$ from PDG '02)

$|V_{us}|$ *from the lattice: "spin-offs"*

Obtain, at m_η (preliminary)

$$2L_8 - L_5 = -0.1(1)^{+1}_{-3} \times 10^{-3} \quad L_5 = 1.9(3)^{+6}_{-2} \times 10^{-3}$$
$$2L_6 - L_4 = 0.5(2)^{+1}_{-3} \times 10^{-3} \quad L_4 = 0.3(3)^{+7}_{-2} \times 10^{-3}$$

cf. $L_5 = 2.3(3) \times 10^{-3}$ and $L_6 \approx L_4 \approx 0$ (Gasser, Leutwyler '85)

$2L_8 - L_5$ outside $[-3.4, -1.8] \times 10^{-3}$ which allows $m_u = 0$ (Kaplan, Manohar '86; Cohen et al '99), in accord with the results of Nelson et al '03.

Also get

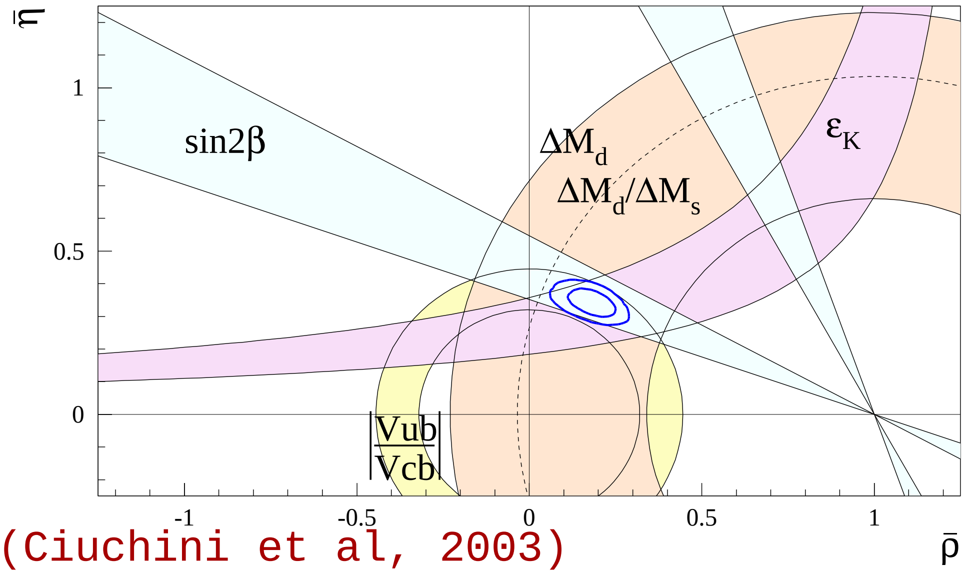
$$m_s^{\overline{\text{MS}}}(2\,\text{GeV}) = 70(15)\,\text{MeV} \qquad \hat{m}^{\overline{\text{MS}}}(2\,\text{GeV}) = 2.7(6)\,\text{MeV}$$

compatible with the preliminary $N_f = 3$ results of CP-PACS/JLQCD '03, but well below sum-rule results and dangerously close to the rigorous lower bounds of Lellouch et al '97, Bhattacharya et al '98 (recent review, Gupta '03).

Lattice QCD for the unitarity triangle

"Standard UT fit is now entirely in the hands of Lattice QCD (up to, perhaps, $|V_{ub}|$)

Martin Beneke @ Lattice 2001



(Ciuchini et al, 2003)

- Only $\sin 2\beta$ is free from hadronic uncertainties
- Only $|V_{ub}|/|V_{cb}|$ (tree-level decays) is insensitive to physics beyond the SM
- $\Rightarrow$ $|V_{ub}|/|V_{cb}|$ (and γ) fix the summit and other constraints probe of new phys.

$$\Delta M_d = C_B M_{B_d} f_{B_d}^2 \hat{B}_{B_d} A^2 \lambda^6 [(1-\bar\rho)^2 + \bar\eta^2]$$

$$\frac{\Delta M_d}{\Delta M_s} = \frac{M_{B_d}}{M_{B_s}} \xi^{-2} \lambda^2 [(1-\bar\rho^2) + \bar\eta^2]$$

$$|\epsilon_K| = C_K \hat{B}_K A^2 \lambda^6 \bar\eta [A^2 \lambda^4 (1-\bar\rho) S_{tt} + S_{tc}]$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \lambda/(1-\lambda^2/2)\sqrt{\bar\rho^2 + \bar\eta^2}$$

Lattice QCD $\rightarrow f_{B_d}\sqrt{\hat{B}_{B_d}},\ \xi,\ \hat{B}_K$

$|V_{cb}|$ can be obtained from $B \rightarrow D^*(D)\ell\nu$

$|V_{ub}|$ from $B \rightarrow \pi(\rho)\ell\nu$

Lattice QCD $\rightarrow$ form factors

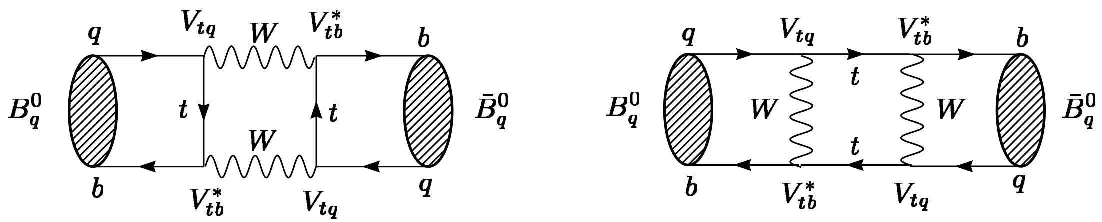
All are gold-plated quantities on the lattice

B^0_q and $\bar{B}^0_q$ ($q = d, s$) are not eigenstates of the weak hamiltonian $\longrightarrow$ they can oscillate into one another:

$$\mathcal{P}_{B^0_q \to \bar{B}^0_q}(t) = \frac{1}{2\tau_q}\, e^{-t/\tau_q}\, [1 - \cos(\Delta M_q\, t)]$$

with ΔM_q mass difference of eigenstates of full hamiltonian

In Standard Model :



$$\Delta M_q \simeq \frac{G_F^2}{8\pi^2}\, M_W^2\, |V_{tq} V^*_{tb}|^2\, \eta_B S_0(x_t) C_B(\mu)\, \frac{|\langle \bar{B}_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B_q \rangle|}{2 M_{B_d}}$$

$$C_B(\mu) \langle \bar{B}^0_q | (\bar{b}q)_{V-A} (\bar{b}q)_{V-A}(\mu) | B^0_q \rangle = \frac{8}{3} M^2_{B_q} f^2_{B_q} \hat{B}_{B_q}$$

$B^0_{(d,s)} - \bar{B}^0_{(d,s)}$ ***mixing: lattice issues (1)***

Consider f_{B_q} and B_{B_q} separately, because systematics very different

1) Discretization errors: on presently accessible lattices $1/m_b < a < 1/m_c$

$\Rightarrow m_b$ cannot be simulated directly

$\Rightarrow$ must rely on effective field theories (heavy-light case): relativistic quarks, HQET (static approx.), "NRQCD", FNAL

Approaches are complementary: should agree within systematic errors

2) Chiral extapolation: all calculations to date have $m_{u,d} \gtrsim m_s^{phys}/2 \Rightarrow$ must extrapolate to $m_{u,d}^{phys}$

Guide: heavy meson χPT (HMχPT)

Pb: coefficient of χral log in HMχPT expression f_B is large

$\Rightarrow$ large chiral log corrections ($O(20\%)$) previously ignored (Yamada '02; Kronfeld et al '02)

Analysis relies on trusting NLO HMχPT up to $M_\pi \sim 1\,\mathrm{GeV}$! Corrections closer to 10% (LL '02, Becirevic et al '03, Cillero et al '03, Aoki et al '03)

$B^0_{(d,s)}-\bar{B}^0_{(d,s)}$ ***mixing: lattice issues (2)***

Issue *not* critical for B_B since coefficient of χral log is small

3) Quenching: Have $N_f = 2$ results w/ $m_{u,d} \gtrsim m_s^{phys}/2$, awaiting $N_f = 2+1$ results.

Preliminary result for f_{B_s} on $N_f = 2+1$ MILC configurations `(Wingate et al '03)`

$$f_{B_s} = 260(7)(28)\,\mathrm{MeV}$$

$B^0_{(d,s)}-\bar{B}^0_{(d,s)}$ **mixing: summary**

Keep summary of ICHEP 2002 `(LL '02)` (extrapolation to $N_f = 2+1$)

$$f_B = 203(27)^{+0}_{-20}\ \mathrm{MeV} \qquad f_{B_s} = 238(31)\ \mathrm{MeV} \qquad \frac{f_{B_s}}{f_B} = 1.18(4)^{+12}_{-0}$$

$$\hat{B}_B^{NLO} = 1.34(12) \qquad \hat{B}_{B_s}^{NLO} = 1.34(12) \qquad \frac{\hat{B}_{B_s}^{NLO}}{\hat{B}_B^{NLO}} = 1.00(3)$$

$$f_B\sqrt{\hat{B}_B^{NLO}} = 235(33)^{+0}_{-24}\ \mathrm{MeV} \qquad f_{B_s}\sqrt{\hat{B}_{B_s}^{NLO}} = 276(38)\ \mathrm{MeV}$$

$$\xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}} = 1.18(4)^{+12}_{-0}$$

where flat asymmetric error is due to uncertainty in chiral extrapolation (can be symmetrized in UT fits)

$|V_{cb}|$ **from** $B \to D^* \ell \nu$

$|V_{cb}|$ plays important rôle in constraining UT $\to$ must be determined precisely

- Can extracted from differential rate (Neubert '91)

$$\frac{d\Gamma}{d\omega} \sim |V_{cb}|^2 |\mathcal{F}_{D^*}(w)|^2$$

  extrapolated to zero recoil, i.e. $w = v_B \cdot v_{D^*} = 1$

- HQET and Luke's theorem predict: $\mathcal{F}_{D^*}(1) = 1 + \mathcal{O}(1/m_{c,b}^2)$, but precise measurement of $|V_{cb}|$ requires reliable determination of $\mathcal{F}_{D^*}(1) - 1$

- Through clever use of double ratios of matrix elements for $D^{(*)}, B^{(*)} \to D^{(*)}, B^{(*)}$ Kronfeld et al (2001) reconstruct, in a quenched calculation at 3 values of the lattice spacing

$$\mathcal{F}_{D^*}(1) = 0.913^{+24+17}_{-17-30}$$

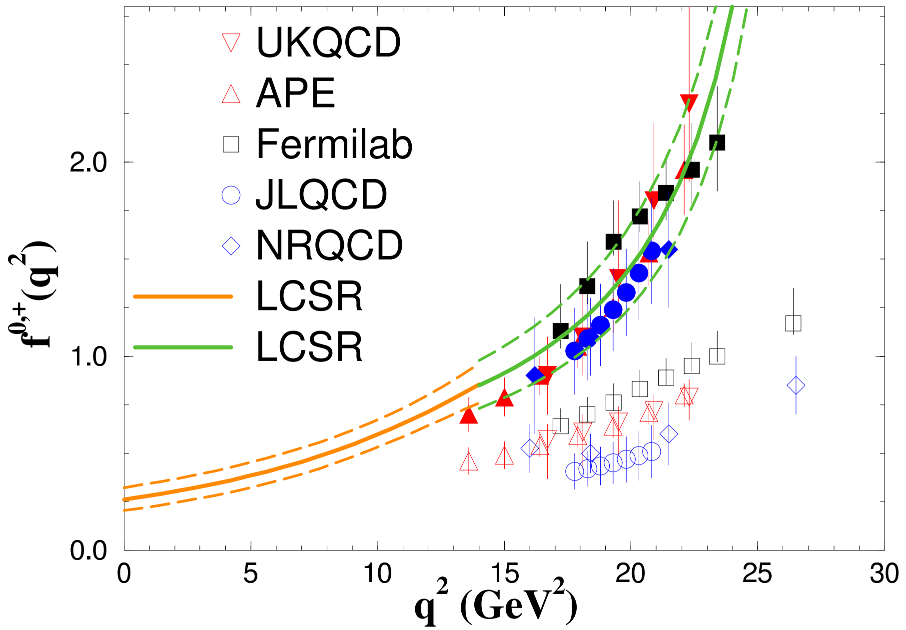
This important calculation, which requires excellent control of statistical and systematic errors, should be performed by other groups and also unquenched

$|V_{ub}|$ **from** $B \to \pi\ell\nu$

Enables measurement of $|V_{ub}|$ (no normalization by HQS here)

$$\langle \pi(k)|\bar{u}\gamma_\mu b|\bar{B}(p)\rangle \longrightarrow f_+(q^2),\ f_0(q^2)$$

Quenched calculations by four groups using relativistic, FNAL and NRQCD quarks:



(Onogi, CKM '03)

- good consistency on $f^+(q^2)$ which determines rate when $m_\ell \to 0$
- error is of $\mathcal{O}(20\%)$
- Fit of lattice results to BK parametrization (Becirevic et al '00) which incorporates most of known constraints on the form factors $\longrightarrow$ extrapolation consistent with LCSR (Khodjamirian et al '00)

Need now an unquenched calculation $\longrightarrow$ Shigemitsu et al '03

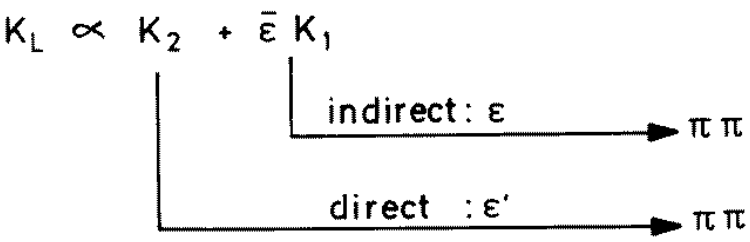
Non-leptonic weak decays of kaons

$K \to \pi\pi$ ***decays: phenomenology***

$$T[K^0 \to \pi^+\pi^-] = \sqrt{\tfrac{2}{3}} A_0 e^{i\delta_0} + \sqrt{\tfrac{1}{3}} A_2 e^{i\delta_2} \qquad T[K^+ \to \pi^+\pi^0] = \sqrt{\tfrac{3}{2}} A_2 e^{i\delta_2}$$

$$T[K^0 \to \pi^0\pi^0] = \sqrt{\tfrac{2}{3}} A_0 e^{i\delta_0} - 2\sqrt{\tfrac{1}{3}} A_2 e^{i\delta_2}$$

CP violation implies $A_I^* \neq A_I$



$$\epsilon \equiv \frac{T[K_L \to (\pi\pi)_{I=0}]}{T[K_S \to (\pi\pi)_{I=0}]} \simeq \frac{1}{\sqrt{2}} e^{i\pi/4} \frac{\mathrm{Im} M_{12}}{\Delta M_K}$$

$$\epsilon' \simeq \frac{1}{\sqrt{2}} e^{i\pi/4}\, \mathrm{Im}\left(\frac{A_2}{A_0}\right)$$

Experimentally:

$|A_0/A_2| \simeq 22.2$ ($\Delta I = 1/2$ rule)

$|\epsilon| = (2.282 \pm 0.017) \cdot 10^{-3}$

$\mathrm{Re}(\epsilon'/\epsilon) = (16.6 \pm 1.6) \cdot 10^{-4}$

K^0-$\bar{K}^0$ mixing in the SM: B_K

$$2M_K M_{12}^* = \langle \bar{K}^0 | H_{\rm eff}^{\Delta S=2} | K^0 \rangle \sim C_K(\mu) \langle \bar{K}^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A}(\mu) | K^0 \rangle$$

and

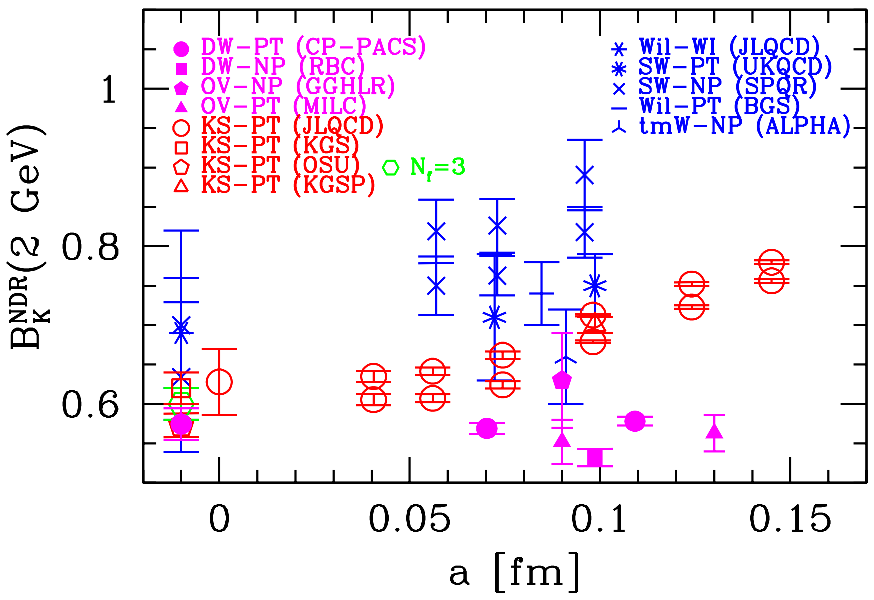
$$C_K(\mu) \langle \bar{K}^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A}(\mu) | K^0 \rangle = \frac{8}{3} M_K^2 f_K^2 \hat{B}_K$$

Recent developments (quenched):

- 2 determinations of B_K in formulations with *exact* chiral-flavor symmetry:
  - Neuberger fermions, $L \simeq 1.5\,{\rm fm}$, $a = 0.09\,{\rm fm}$ and *non-perturbative* renormalization (Garron et al '04)
  - HYP overlaps fermions, $L \simeq 1.3\,{\rm fm}$, $a = 0.09,\ 0.013\,{\rm fm}$ and *perturbative* renormalization (DeGrand '04)

  $\longrightarrow$ validate use of GW fermions for weak matrix element calc's
- Twisted-mass Wilson fermions: break flavor and parity but no operator mixing; $L \simeq 1.5\,{\rm fm}$, $a = 0.09\,{\rm fm}$ and *non-perturbative* renormalization and running (ALPHA '03, prelim.)

K^0-$\bar{K}^0$ *mixing: summary (1)*



- Only one preliminary unquenched result (OSU '97)
- Results consistent in continuum limit
- Some GW results are slightly lower
- Reference result: quenched staggered JLQCD '98 calculation (weak point: perturbative renormalization)

- Quenching: $\delta B_K \sim 15\%$ (OSU $N_f = 3$ and QχPT)
- $m_d = m_s = m_s^{phys}/2 \rightarrow m_d \neq m_s$: $\delta B_K \sim 5\%$ (χPT)

(Sharpe '92, '96)

Summary number

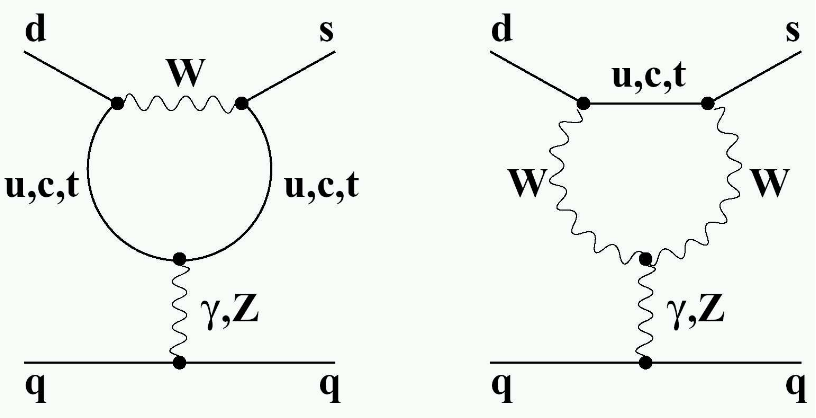
$$B_K^{NDR}(2\,\mathrm{GeV}) = 0.628(42)(99) \longrightarrow \hat{B}_K^{NLO} = 0.86(6)(14)$$

with $\hat{B}_K^{NLO}$ two-loop RGI B-parameter

- Same result as in `LL, Lattice 2000`
- Clarify situation regarding lower GW results
- Error dominated by quenching $\Rightarrow$ need unquenched studies to maintain impact of ϵ in UT fits

ϵ': electroweak penguins

Dominant $\Delta I = 3/2$ contribution to ϵ' determined by EW penguins (Flynn et al '87)



$$Q_{7,8}^{3/2} = \frac{1}{2}[(\bar{s}d)_{V-A}(\bar{u}u)_{V+A}$$
$$+(\bar{s}u)_{V-A}(\bar{u}d)_{V+A} - (\bar{s}d)_{V-A}(\bar{d}d)_{V+A}]$$

Q_7 color diagonal and Q_8 color mixed

In limit $m_u = m_d$, no eye contractions $\Rightarrow$ much simplified lattice calculation

Compute $\langle\pi^+|Q_{7,8}^{3/2}|K^+\rangle$ and obtain physical amplitude at LO in χPT:

$$\langle(\pi\pi)_{I=2}|Q_{7,8}|K^0\rangle = -\frac{1}{F_0}\langle\pi^+|Q_{7,8}^{3/2}|K^+\rangle$$

Electroweak penguins: results in the chiral limit

Summary of results, in the χral limit, in the $\mathrm{NDR}-\overline{\mathrm{MS}}$ scheme at $2\,\mathrm{GeV}$, in units of GeV^3.

All lattice results quenched

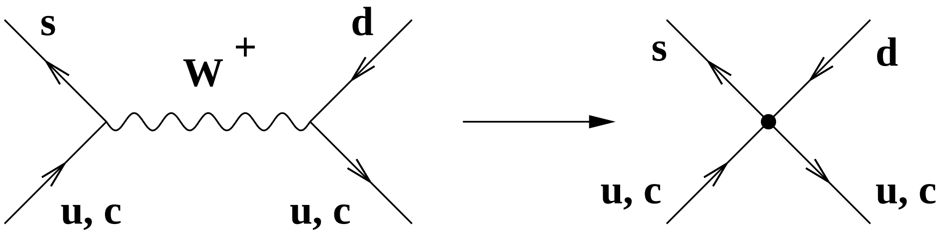
Group	Action	$\langle(\pi\pi)_{I=2}\|Q_7\|K\rangle$	$\langle(\pi\pi)_{I=2}\|Q_8\|K\rangle$
Bijnens et al ’01		0.24(3)	1.2(7)
Cirigliano et al ’02		0.22(5)	1.5(3)
Knecht et al ’01		0.11(3)	3.5(1.1)
Narison ’00		0.17(5)	1.4(3)
SPQR (prelim)	Wilson	0.14(1)(?)	0.8(1)(?)
RBC ’02	Domain-Wall	0.27(3)(?)	1.1(2)(?)
CP-PACS ’01	Domain-Wall	0.24(3)(?)	1.0(2)(?)
Berruto et al (prelim)	Neuberger	0.35(7)(?)	1.4(3)(?)

Careful estimate of systematics required for lattice results (e.g. large uncertainty from choice of a^{-1})

Need unquenched results and control on final state interactions

$\Delta I = 1/2$ ***rule with an active charm (1)***

OPE (CP-conserving $\Delta S = 1$ transitions)



$$+\text{rad.corr.} \longrightarrow \mathcal{H}^{\Delta S=1} = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us}^* \sum_{i=\pm} C_i(\mu, M_W)\, \tilde{\mathcal{O}}_i(\mu)$$

$$\mathcal{O}_\pm = [(\bar{s}d)_{V-A}(\bar{u}u)_{V-A} \pm (\bar{s}u)_{V-A}(\bar{u}d)_{V-A}] - [u \to c]$$

- $\mathcal{O}_-$ is pure $I = 1/2$
- $\mathcal{O}_+$ has both $I = 1/2, 3/2$

Short distance enhancement?

$$|C_-(M_W)/C_+(M_W)| = 1 + \mathcal{O}(\alpha_s(M_W)) \quad \longrightarrow \quad |C_-(2\text{GeV})/C_+(2\text{GeV})| \sim 2$$

(Gaillard et al '74; Altarelli et al '74; Shifman et al '75-'77)

$\Delta I = 1/2$ *rule with an active charm (2)*

Most of enhancement must come from long distance QCD effects in

$$\frac{\langle(\pi\pi)_{I=0}|C_+\mathcal{O}_+ + C_-\mathcal{O}_-|K^0\rangle}{\langle(\pi\pi)_{I=2}|C_+\mathcal{O}_+|K^0\rangle}$$

Some evidence from quenched lattice studies of $K \to \pi$ w/ integrated charm (CP-PACS '01; RBC '01; Pekurovsky et al '01): require subtraction of $1/a^2$ divergences; have intrinsic quenched ambiguity (Golterman et al '01); accurate only to $O(1/m_c^0)$

LQCD w/ χral symmetry $\Rightarrow$ renormalized, continuum operators are given by:

$$\hat{\mathcal{O}}_\pm(\mu) = Z_\pm(\mu a, g_0)\tilde{\mathcal{O}}_\pm(g_0) + \mathcal{O}(a^2)$$

$$\tilde{\mathcal{O}}_\pm = O_\pm + (m_c^2 - m_u^2)C_\pm^m O_m \quad \text{and} \quad \mathcal{O}_m = (m_s + m_d)\bar{s}d - (m_s - m_d)\bar{s}\gamma_5 d$$

$\Rightarrow$ no power divergent subtractions w/ GW fermions, even when simpler $K \to \pi$ transitions are studied (e.g. Capitani et al '01)

$\Rightarrow$ GW fermions provide a unique opportunity to elucidate $\Delta I = 1/2$ puzzle

Conclusion

- Large range of quantities of central importance to particle physics is being computed w/ lattice QCD simulations, many of which could not be presented here
- Dominant systematics are **(partial)-quenching** and **chiral extrapolations** (also discretization errors for HQ) $\rightarrow$ beginning to be addressed head on
- $\Rightarrow$ reliable simulations w/ $N_f = 2+1$ and chiral $m_{u,d}$ will immediately lead to large reduction in systematic error (e.g. $\delta^{quenched}_{syst} f_{\pi,K} \sim 10-15\% \longrightarrow \delta^{N_f=2+1}_{syst} f_{\pi,K} \sim 3\%$!)
- Breakthrough of GW fermions, with their full chiral-flavor symmetry at $a \neq 0 \longrightarrow$ new possibilities for the calculation of weak matrix elements, in particular those associated with the $\Delta I = 1/2$ rule and ϵ'(cf DWF calculations by CP-PACS and RBC '01; analytical work by Capitani et al '00, '01)
- Not mentioned: potentially large reduction in uncertainties obtained by combining ratios of b-quark to equivalent charm quark matrix elements computed on the lattice with charm measurements from e.g. CLEO-c
  $\longrightarrow$ see e.g. Ryan '01 for a review of some charm results

Conclusion (2)

Also not discussed:

- The finite-volume solution to the longstanding problem of simulating non-leptonic decays such as $K \to \pi\pi$ directly on the lattice (LL et al '01)
- Despite much hard work, difficult to improve on $0.03 < \frac{\Delta\Gamma_s}{\Gamma_s} < 0.15$, at $\sim 3\sigma$, due to large cancellation in leading contribution and large uncertainties in $1/m_b$ correction (e.g. CKM report '03)
- Much work on b-hadron lifetimes (e.g. review by Tarantino, EPS '03)
- GW fermions allow investigation of a numerically unexplored regime of QCD in which the correlation length of pion fields $\gg L$(ϵ regime of Gasser et al '87)
- In the quenched approximation $\to$ the calculation of low-energy constants (LECs) of the strong chiral lagrangian (Hernández et al '99; DeGrand '01; Hasenfratz '01; Damgaard et al '02; Giusti et al '04)
- It is possible to generalize this approach to extract the LECs of the weak chiral Lagrangian by studying the weak interactions in the ϵ regime (Damgaard et al '02; Giusti et al '02; Hernández et al '02)

Conclusion (3)

- Quenched calculations of the hadronic vacuum polarization relevant for $(g-2)_\mu$, but limited information in dominant kinematical region (Blum '03, Göckeler et al '03)
- Very interesting developments in lattice SUSY: theories with SUSY at finite a have been constructed by orbifolding matrix models $\Rightarrow a \to 0$ SUSY theories in $d = 1, 2, 3$ w/out fine tuning (Kaplan et al '02-'03; see also Catterall et al '02-'03)
- ...
- More generally, the range of approaches and the quantities studied on the lattice is constantly expanding $\longrightarrow$ should be many exciting results in years to come