Thank you for the invitation.

**DARK ENERGY**

**AND**

**DARK MATTER**

[ Three transparency figures enter at pages 71’, 72’ and 74’]
OUTLINE OF DARK ENERGY

● 1. FATE OF DARK ENERGY.
   Future of the universe. Disconnect of geometry and destiny.

● 2. STABILITY ISSUES.
   Can dark energy have any microscopic effect?

● 3. BIGGER RIP WITHOUT D.E.
   DGP modification of gravity.

● SUMMARY.
1. THE FATE OF DARK ENERGY

The concordance of CMB, LSS and SNe1a data lead to the values $\Omega_X \sim 0.7$ and $\Omega_M \sim 0.3$.

One question is: to what extent can precision cosmological data allow us to discriminate between possible future fates of the Universe?

If one assumes that $\Omega_X$ corresponds to a cosmological constant with equation of state given by $w = p/\rho = -1$ then the future evolution of the universe follows from the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} + \frac{\Lambda}{3}$$

in which $a(t)$ is the scale factor normalized at the present time as $a(t_0) = 1$, $\rho(t) = \rho(0)a^{-3}$ is the energy density of the matter component and $\Lambda$ is a constant. Here we assume the universe is flat and neglect radiation.
In such a simple, and still viable, case the behavior of $a(t)$ asymptotically for large $t \to \infty$ is

$$a(t) \sim \exp\left(\sqrt[3]{\frac{\Lambda}{1-t}}\right)$$  \hspace{1cm} (2)

so that dark energy asymptotically dominates and the Universe is blown apart in an infinite time $a(t) \to \infty$ as $t \to \infty$.

Even assuming a constant equation of state $w = p/\rho$ there is a wide spread in the range of allowed $w$ with an upper limit of $w \sim -0.8$ and a lower limit conservatively $w = -2$.

Note that the earliest WMAP analysis in astro-ph/0302207 used a prior that $w \geq -1$. The relaxation of this prior is awaited to find their lower limit on $w$.

Here we consider $W(Z)$ depending on red-shift and will deduce widely-varying future scenarios.
The future fate of the dark energy ranges from a diverging scale factor at a finite future time to a disappearing dark energy with reversion to domination by "ordinary" matter $a \sim t^{2/3}$.

**Constant equation of state**

Keeping only the dark energy term:

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_X a^{-\beta}$$

where $\beta = 3(1+w)$. When $w < -1$, $\beta < 0$ and the solution of Eq.(3) diverges at a finite time $t = t^*$. Integrating

$$\int_{a(t_0)}^{\infty} a^{\beta/2 - 1} = H_0 \sqrt{\Omega_X} \int_{t_0}^{t^*} dt$$

one finds the remaining time $t_r = (t^* - t_0)$ is given by:
the remaining time before the scale factor diverges which is:

\[ t_r = \frac{2}{3H_0\sqrt{\Omega_X(-w - 1)}} = \frac{11 \text{Gyr}}{(-w - 1)} \quad (5) \]

In Eq.(5), putting in \( \Omega_X = 0.7 \) and \( 2/(3H_0) = 9.2 \text{Gyr} \) one finds for \( w = -1.5, -2 \) respectively \( t_r = 22 \text{Gyr}, 11 \text{Gyr} \). In the more extreme case \( w = -2.5 \), one finds \( t_r = 7 \text{Gyr} \).

Note that the Sun will transform into a Red Giant, and swallow the Earth, approximately 5 Gyr from now.

Gravitationally-bound systems could survive longer than \( t_r \) given by Eq.(5) but such systems would be infinitely separated from each other.
Equation of State Varying Linearly with Red-Shift.

As a more general ansatz, we consider the model for the EoS depending linearly on red-shift:

\[ w(Z) = w(0) + CZ\theta(\zeta - Z) + C\zeta\theta(Z - \zeta) \quad (6) \]

where the modification is cut off arbitrarily at some \( Z = \zeta > 0 \). We assume \( C \leq 0 \) and consider the two-dimensional parameter space spanned by the two variables \( w(0) \) and \( C \).

As input data we use the CMB spectrum and CMBFAST. Also we simultaneously make \( \chi^2 \) fits to the SNe1A data.

To set the stage, let us first use only the SNe1A data to constrain the parameters \( w(0) \) and \( C \). The result is shown for \( \zeta = 2 \) in Figure 1 where the allowed region is the region between the two 99\% dashed lines shown. We may de-mark three distinct regions:
(I) $w(0) < (C - 1)$. In this case there is an end of time, at a finite future time.

(II) $(C - 1) \leq w(0) < C$. Here the lifetime of the universe is infinite. The dark energy dominates over matter, as now, at all future times.

(III) $C \leq w(0)$. The lifetime of the universe is again infinite but after a finite time the dark energy will disappear relative to the dark matter and matter-domination will be re-established with $a(t) \sim t^{2/3}$. 
Fig. 1
When we add the constraints imposed by the CMB data, the allowed region is smaller as shown in Figure 2, plotted for $\zeta = 0.5$. Such a small $\zeta$ still allows all three future possibilities (I), (II) and (III). For somewhat larger $\zeta$ only possibilities (I) and (II) are allowed in this particular parametrization.

The case $\zeta = 2$ is exhibited in more detail for different values of $w(0)$ and $C$ in Figures 3 and 4.

Figure 3 shows the variation of the transition red-shift $Z_{trans}$ where deceleration changes to accelerated cosmic expansion defined by $q(Z_{trans}) = 0$.

Figure 4 shows the corresponding fits to the large-Z SNe1A data versus red-shift, also for $\zeta = 2$. 
Fig. 2
Fig. 4

- $w = -1$
- $w = -1 - z$
- $w = -1 - 2z$

$\Delta (m-M)$ (mag)

$z$

0.01 - 1.0
To return to our main point, let us assume that more precise cosmological data will allow an approximate determination of \( w(Z) = f(Z) \) as a function of \( Z \) for positive \( Z > 0 \). Then to illustrate the possible future evolutions write:

\[
w(Z) = f(Z)\theta(Z) + (f(0) + \alpha Z)\theta(-Z) \quad (7)
\]

In this case, the future scenarios (I), (II) and (III) occur respectively for \( \alpha > -f(0) > 0 \), \(-f(0) > \alpha > -f(0) - 1 \) and \( \alpha < -f(0) - 1 \).

Present data are consistent with a simple cosmological constant \( f(Z) = -1 \) in Eq.(7) in which case the end of time scenario occurs for \( \alpha > 1 \), the infinite-time dark energy domination for \( 1 > \alpha > 0 \), and disappearing dark energy for \( \alpha < 0 \).

Since in practice \( F(Z) \) for \( Z \geq 0 \) will never be determined with perfect accuracy the continuation of \( w(Z) \) to future \( Z < 0 \) will be undecidable from observation as will therefore be the ultimate fate of the Universe.
2. Stability Issues for $w < -1$ Dark Energy

Precision cosmological data hint that a dark energy with equation of state $w = P/\rho < -1$ and hence dubious stability is viable. Here we discuss for any $w$ nucleation from $\Lambda > 0$ to $\Lambda = 0$ in a first-order phase transition. The critical radius is argued to be at least of galactic size and the corresponding nucleation rate glacial, thus underwriting the dark energy’s stability and rendering remote any microscopic effect.
Introduction.

The equation of state for the dark energy component in cosmology has been the subject of much recent discussion. Present data are consistent with a constant $w(Z) = -1$ corresponding to a cosmological constant. But the data allow a present value for $w(Z = 0)$ in the range $-1.38 < w(Z = 0) < -0.82$.

If one assumes, more generally, that $w(Z)$ depends on $Z$ then the allowed range for $w(Z = 0)$ is approximately the same.
Interpretation as a limiting velocity

Consider making a Lorentz boost along the 1-direction with velocity $V$ (put $c = 1$). Then the stress-energy tensor which in the dark energy rest frame has the form:

$$T_{\mu\nu} = \Lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & w & 0 & 0 \\ 0 & 0 & w & 0 \\ 0 & 0 & 0 & w \end{pmatrix}$$

is boosted to $T'_{\mu\nu}$ given by
\[ T'_{\mu\nu} = \Lambda \left( \begin{array}{cccc} 1 & V & 0 & 0 \\ V & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \left( \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & w & 0 & 0 \\ 0 & 0 & w & 0 \\ 0 & 0 & 0 & w \end{array} \right) \left( \begin{array}{cccc} 1 & V & 0 & 0 \\ V & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) \]

\[ = \Lambda \left( \begin{array}{cccc} 1 + V^2 w & V(1 + w) & 0 & 0 \\ V(1 + w) & V^2 + w & 0 & 0 \\ 0 & 0 & w & 0 \\ 0 & 0 & 0 & w \end{array} \right) \]

We learn several things by studying this.
First, consider the energy component $T'_{00} = 1 + V^2 w$. Since $V < 1$ we see that for $w > -1$ this is positive $T'_{00} > 0$ and the WEC is respected. For $w = -1$, $T'_{00} \to 0$ as $V \to 1$ and is still never negative. For $w < -1$, however, we see that $T'_{00} < 0$ if $V^2 > -(1/w)$ and this is the first sign that the case $w < -1$ must be studied with great care. Looking at the pressure component $T'_{11}$ we see the special role of the case $w = -1$ because $w = T'_{11}/T'_{00}$ remains Lorentz invariant as expected for a cosmological constant. Similarly the off-diagonal components $T'_{01}$ vanish only in this case.
One alternative is that it is impossible for \( V^2 > -(1/w) \). The highest velocities known are those for the highest-energy cosmic rays which are protons with energy \( \sim 10^{20} \text{eV} \). These have \( \gamma = (1 - V^2)^{-1/2} \sim 10^{11} \) corresponding to \( V \sim 1 - 10^{-22} \). This would imply that:

\[
    w > -1 - 10^{-22}
\]

which is one possible conclusion.
Interpretation as Vacuum Instability.

But let us suppose, that more precise cosmological data reveals a dark matter which has $w$ significantly below $-1$. Then, by boosting to an inertial frame with $V^2 > -(1/w)$, one arrives at $T_{00}' < 0$ and this would be a signal for vacuum instability. If the cosmological background is a Friedmann-Robertson-Walker (FRW) metric the physics is Lorentz invariant and so one should be able to see evidence for the instability already in the preferred frame where $T_{\mu\nu}$ has $T_{00} > 0$. 
This goes back to work in the 1960’s and 1970’s where one compares the unstable vacuum to a superheated liquid. At one atmospheric pressure water can be heated carefully to above $100^0$ C without boiling. The superheated water is metastable and attempts to nucleate bubbles containing steam. However, there is an energy balance for a three-dimensional bubble between the positive surface energy $\sim R^2$ and the negative latent heat energy of the interior $\sim R^3$ which leads to a critical radius below which the bubble shrinks away and above which the bubble expands and precipitates boiling.
For the vacuum the first idea in 1976 was to treat the spacetime vacuum as a four-dimensional material medium just like superheated water. The second idea was to notice that a hyperspherical bubble expanding at the speed of light is the same to all inertial observers. This Lorentz invariance provided the mathematical relationship between the lifetime for unstable vacuum decay and the critical radius of the four-dimensional bubble or instanton. In the rest frame, the energy density is

\[ T_{00} = \Lambda \sim (10^{-3} \text{eV})^4 \sim (\text{mm})^{-4} \]
In order to make an estimate of the dark energy decay lifetime in the absence of a known potential, we can proceed by assuming it is the same Lorentz invariant process of a hyperspherical bubble expanding at the speed of light, the same for all inertial observers.

Let the radius of this hypersphere be $R$, its energy density be $\epsilon$ and its surface tension be $S_1$. Then the relevant instanton action is

$$A = -\frac{1}{2}\pi^2 R^4 \epsilon + 2\pi^2 R^3 S_1$$

where $\epsilon$ and $S_1$ are the volume and surface energy densities, respectively.
The stationary value of this action is

\[ A_m = \frac{27}{2} \pi^2 S_1^4 / \epsilon^3 \]

corresponding to the critical radius

\[ R_m = 3 S_1 / \epsilon \]

We shall assume that the wall thickness is negligible compared to the bubble radius. The number of vacuum nucleations in the past lightcone is estimated as

\[ N = (V_u \Delta^4) \exp(-A_m) \]

where \( V_u \) is the 4-volume of the past and \( \Delta \) is the mass scale relevant to the problem.
This vacuum decay picture led to the proposals of inflation, for solving the horizon, flatness and monopole problems (only the horizon problem was generally known in 1976). None of that work addressed why the true vacuum has zero energy. Now that the observed vacuum has non-zero energy density $+\epsilon \sim (10^{-3} \text{eV})^4$ we may interpret it as a “false vacuum” lying above the “true vacuum” with $\epsilon = 0$.

In order to use the full power of the instanton equations we need to estimate the three mass-dimension parameters $\epsilon^{1/4}, S_1^{1/3}$ and $\Delta$ therein. Let us discuss these three scales in turn.
The easiest of the three to select is $\epsilon$. If we imagine a tunneling through a barrier between a false vacuum with energy density $\epsilon$ to a true vacuum at energy density zero then the energy density inside the bubble will be $\epsilon = \Lambda = (10^{-3}\text{eV})^4$. No other choice is reasonable.

Next we discuss the typical mass scale $\Delta$ involved in the prefactor. The value of $\Delta$ does not matter very much because it appears as a power rather than an exponential so let us put $\Delta = \epsilon^{1/4} = (1\text{mm})^{-1}$ whereupon the prefactor in $N$ is $\sim 10^{116}$. It is easy to check that the conclusions do not depend on the choice.

The third and last scale to discuss is the surface tension, $S_1$. Here we appeal to stimulated decay and its absence to understand why $N << 1$. 
Spontaneous dark energy decay brings us to the question of whether such decay can be initiated in an environment existing within our Universe. The question is analogous to one of electroweak phase transition in high energy particle collision. This was first raised in 1976 and revisited for cosmic-ray collisions. That was in the context of the standard-model Higgs vacuum and the conclusion is that high-energy colliders are safe at all present and planned foreseeable energies because much more severe conditions have already occurred (without disaster) in cosmic-ray collisions within our galaxy. More recently, this issue has been addressed in connection with fears that the Relativistic Heavy Ion Collider (RHIC) might initiate a disastrous transition but according to careful analysis there was no such danger.
The dark energy density is some 58 orders of magnitude smaller \([(10^{-3} eV)^4\)] compared to \((300 GeV)^4\)] than for the electroweak case and so the nucleation scales are completely different. One is here led away from microscopic towards astronomical size scales.
We see that the critical radius cannot be microscopic. Think first of a macroscopic scale e.g. 1 meter and consider a magnetic field practically-attainable in bulk on Earth such as 10 Tesla. Its energy density is given by

$$\rho_{mag} = \frac{1}{\mu_0} B^2$$

Using the value $\mu_0 = 4\pi \times 10^{-7} N A^{-2}$ and $1T = 6.2 \times 10^{12} (MeV.s.m^{-2})$ leads to an energy density $\rho_{mag} = 2.5 \times 10^{17} eV/(mm)^3$, over 20 orders above the value of Eq.() for the interior of nucleation. Magnetic fields in bulk exist in galaxies with strength $\sim 1\mu G$ and the rescaling by $B^2$ then would give $\rho_{mag} \sim (2.5 \times 10^{-5} eV)(mm)^{-3}$, slightly below the dark energy value.
Assuming the dark energy can exchange energy with magnetic energy density the observed absence of stimulated decay would then imply a critical radius of at least galactic size, say, $\sim 10^{kpc}$. Using Eq() then gives for the surface tension $S_1 > 10^{23}(mm)^{-3}$ and number of nucleations in Eq.( ) $N < exp(-10^{92})$. The spontaneous decay is thus glacial. Note that the dark energy has appeared only recently in cosmological time and has never interacted with background radiation of comparable energy density. Also, this nucleation argument does not require $w < -1$. 
Discussion

As a first remark, since the critical radius $R_m$ for nucleation is astronomical, it appears that the instability cannot be triggered by any microscopic process. While it may be comforting to know that the dark energy is not such a doomsday phenomenon, it also implies at the same time the dreadful conclusion that dark energy may have no microscopic effect. If any such microscopic effect in a terrestrial experiment could be found, it would be crucial in investigating the dark energy phenomenon. We note that the present arguments are less model-dependent than those given elsewhere in the literature.
One may speculate how such stability arguments may evolve. One may expect most conservatively that the value $w = -1$ will eventually be established empirically in which case both quintessence and the “phantom menace” will be irrelevant. In that case, indeed for any $w$, we may still hope that dark energy will provide the first connection between string theory and the real world as in e.g. the BFM stringy dark energy. Even if precise data do establish $w < -1$, as in the “phantom menace” scenario, the dark energy stability issue is still under control.
3. BIGGER RIP WITHOUT DARK ENERGY

The conclusion about the make-up of our Universe depends on assuming that general relativity (GR) is applicable at the largest cosmological scales. Although there is good evidence for GR at Solar-System scales there is no independent evidence for GR at scales comparable to the radius of the visible Universe. The expansion rate of the Universe, including the present accelerating rate of cosmic expansion can be parameterized in the right-hand side of the Friedmann equation by including a dark energy density term with some assumed time dependence on the scale parameter $a(t)$: $\rho_{DE} \sim a^\beta$ with $\beta = -3(1 + w)$. This is a rather restricted function if we assume that the equation of state $w$ is time-independent. But as soon as we admit that it may depend on time $w(t)$ then the function on the right hand side of the Friedmann equation becomes completely arbitrary, just as does its designation as a dark energy density.
Present data are fully consistent with constant $w = -1$ corresponding to a cosmological constant which we may accommodate on the left hand side of the Friedmann equation describing the expansion rate of the Universe. But cases with $w \neq -1$, including $w < -1$, are still permitted by observations. In this case, there is a choice between cooking up a “dark energy” density with a particular time dependence $\rho_{DE} \sim a^\beta(t)$ on the right-hand side of the Friedmann equation or changing the left-hand-side by changing the relationship between the geometry and the matter density, i.e. by changing GR. Even if we do the latter, from the viewpoint of the Friedmann equation, we can always find a time-dependent term on the right-hand-side which is equivalent and which we may call ”dark energy”. Eventually, this distinction may come down to observational tests of whether a particular change in the geometry predicted by GR can be detected, other than by the expansion rate of the Universe.
The case constant $w < -1$ has the interesting outcome for the future of the Universe that it will end in a finite time at a “Big Rip” before which all structure disintegrates.
Here we study an amalgam of the modification of GR due to Dvali, Gabadadze and Porrati (DGP) and the idea of a Big Rip, in fact here a Bigger Rip. This will be based on admittedly *ad hoc* ansatz for terms in modified Friedmann equations but the results are sufficiently interesting to examine and such modifications may be constrained by observational data.
Set-up

The DGP gravity arises from considering the four-dimensional gravity which arises from five-dimensional general relativity confined to a brane with three space dimensions. The underlying action is:

\[ S = M(t)^3 \int d^5 X \sqrt{G} \mathcal{R}^{(5)} + M_{\text{Planck}}^2 \int d^4 x \sqrt{g} R \]

where \( \mathcal{R}^{(5)} \) and \( R \) are the scalar curvature in 5- and 4-dimensional spacetime respectively, and \( G \) and \( g \) are the determinant of the 5- and 4-dimensional spacetime metric.
This leads to an interesting modification of GR which embodies a time-dependent length scale \( L(t) = \frac{M_{\text{Planck}}^2}{M(t)^3} \). For cosmology it is natural to identify, within a coefficient of order one, \( L(t) \) with the Hubble length \( L(t) = H(t)^{-1} \) at any cosmological time \( t \). Actually we are slightly generalizing the original DGP approach to include time dependence of the length scale \( L(t) \).
Taking the four dimensional coordinates to be labeled by \(i, k = 0, 1, 2, 3\) leads to the following modification of Einstein’s equation:

\[
\left( R^{ik} - \frac{1}{2} R g^{ik} \right) + \frac{2\sqrt{G}}{L(t)\sqrt{g}} \left[ \left( \mathcal{R}^{(5)ik} - \frac{1}{2} G^{ik} \mathcal{R}^{(5)} \right) \right] = 0
\]  

(9)

where \([\ldots]\) means:

\[
\int dx \left[ (f(x)) \right] \equiv f'(0)
\]  

(10)
It is interesting to generalize the Schwarzschild solution to this modification of GR. One finds that the modification of the Newton potential at short distances is given by:

\[
V(r) = -\frac{Gm}{r} - \frac{4\sqrt{Gm}\sqrt{r}}{L(t)}
\]  
(11)

\[
= -\frac{r_g}{2r} - \frac{2\sqrt{2}\sqrt{r_g}r}{L(t)}
\]  
(12)

where \(r_g = 2Gm\) is the Schwarzschild radius.
The fractional change in the Newtonian gravitational potential at cosmological time $t$ at orbital distance $r$ from an object with Schwarzschild radius $r_g$ is therefore

$$\left| \frac{\Delta V}{V} \right| = \sqrt{\frac{8r^3}{L(t)^2r_g}}$$

(13)
In the Bigger Rip scenario we will describe the characteristic length $L(t)$ will decrease with time according to

$$L(t) = L(t_0)^{-1}T(t)^p$$  \hspace{1cm} (14)

where the power satisfies $p \geq 0$ ($p < 0$ implies that $L(t)$ would increase) and where

$$T(t) = \frac{(t_{rip} - t)}{(t_{rip} - t_0)}$$  \hspace{1cm} (15)

in which $t_{rip}$ is the time of the Rip.
A bound system will become unbound at a time $t_U$ when the correction to the Newtonian potential becomes large. We make adopt the value of $t_U$ defined from

$$\sqrt[4]{\frac{8r^3}{L(t_U)^2r_g}} = 1$$

(16)

We can rewrite Eq.(16) as:

$$(t_{rip} - t_U) = \frac{1}{\gamma} \left( \frac{8l_0^3}{L_0^2r_g} \right)^{1/2p}$$

(17)

where $\gamma = (t_{rip} - t_0)^{-1}$.
We shall define another later time $t_{\text{caus}}$ as the time after which the two objects of a bound system become causally disconnected from $t_{\text{caus}}$ until $t_{\text{rip}}$. This is defined by the equation:

$$(t_{\text{rip}} - t_{\text{caus}}) = \frac{l_0}{c} \left( \frac{a(t_{\text{caus}})}{a(t_U)} \right) \tag{18}$$
As an example taking $p = 1$ with the values $L_0 = H_0^{-1} = (14 \text{Gy})^{-1} = 1.3 \times 10^{28} \text{cm}$ and $\gamma = (20 \text{Gy})^{-1}$ we arrive at the entries in the following Table:

<table>
<thead>
<tr>
<th></th>
<th>$l_0(cm)$</th>
<th>$r_g(cm)$</th>
<th>$t_{rip} - t_U$</th>
<th>$t_{rip} - t_{cs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gal</td>
<td>$5 \times 10^{22}$</td>
<td>$3 \times 10^{16}$</td>
<td>100My</td>
<td>4My</td>
</tr>
<tr>
<td>S-E i</td>
<td>$1 \times 10^{13}$</td>
<td>$3 \times 10^{5}$</td>
<td>5mos</td>
<td>2days</td>
</tr>
<tr>
<td>E-M</td>
<td>$3 \times 10^{10}$</td>
<td>0.866</td>
<td>2weeks</td>
<td>1hr</td>
</tr>
</tbody>
</table>
Note that the values we find for \((t_{ri} - t_U)\) are consistent with those found in Kamionkowski et al. The corresponding dark energy would have equation of state \(w = -1 - \frac{2}{3} \gamma L_0 = -1.466\) which is now outside of the range allowed by observations if we assume a constant equation of state although it is allowed in the present model with its time-dependence.
As another example, with more normal present $w$, we can increase the time to the Rip to $\gamma = (50 G y)^{-1}$ in which case $w(t_0) = -1.19$ and the Table is modified to:

<table>
<thead>
<tr>
<th>$l_0(cm)$</th>
<th>$r_g(cm)$</th>
<th>$t_{rip} - t_U$</th>
<th>$t_{rip} - t_{cs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gal</td>
<td>$5 \times 10^{22}$</td>
<td>$3 \times 10^{16}$</td>
<td>250My</td>
</tr>
<tr>
<td>S-E</td>
<td>$1 \times 10^{13}$</td>
<td>$3 \times 10^{5}$</td>
<td>5mos</td>
</tr>
<tr>
<td>E-M</td>
<td>$3 \times 10^{10}$</td>
<td>0.866</td>
<td>1mo</td>
</tr>
</tbody>
</table>

so with the more lengthy wait until the Big Rip the disintegration of structure and causal disconnection occur correspondingly earlier before the eventual Rip.
The Bigger Rip

The modified Friedmann equation for DGP gravity is

$$H^2 - \frac{H}{L(t)} = 0 \quad (19)$$

so that we arrive at:

$$\frac{\dot{a}}{a} = H(t) = H(t_0) \frac{1}{T_p} \quad (20)$$

In Eqs.(19,20) we can neglect, for the future evolution, the term $(\rho_M + \rho_\gamma)/(3M_{Planck}^2)$ on the right-hand-side of the modified Friedmann equation.
Defining \( \gamma = -dT/dt = (t_{ri} - t_0)^{-1} \) gives:

\[
\ln a(t) = - \int_1^{T(t)} \frac{dT}{\gamma L(t_0) T^p} \tag{21}
\]

and hence, for \( p = 1 \), which is similar to dark energy with a constant \( w < -1 \) equation of state:

\[
a(t) = T^{-\frac{1}{\gamma L(t_0)}} \tag{22}
\]

while for the Bigger Rip case \( p > 1 \) one finds

\[
a(t) = a(t_0) \exp \left[ \left( \frac{1}{T^{p-1}} - 1 \right) \frac{1}{(p - 1)\gamma L(t_0)} \right] \tag{23}
\]

Here we see that the scale factor diverges more singularly in \( T \) for \( p > 1 \), hence the designation of Bigger Rip. In particular we study the values \( p = 2, 3, \cdots \) as alternative to the “dark energy” case \( p = 1 \).
Inverting Eq.(23) gives:

\[ T = [1 + (p - 1)\gamma L(t_0) \ln a(t)]^{-\frac{1}{(p-1)}} \]  \hspace{1cm} (24)

In this case there is strictly no dark energy, certainly not with a constant equation of state, but we can mimic it with a fictitious energy density \( \rho_L \) by noticing that \( H^2 \sim T^{-2p} \) and writing

\[ \rho_L \sim [1 + (p - 1)\gamma L(t_0) \ln a(t)]^{\frac{2p}{(p-1)}} \]  \hspace{1cm} (25)

If we use Eqs.(23) and (25) in conservation of energy

\[ \frac{d}{dt}(\rho_L a^3) = -p \frac{d}{dt}(a^3) = -w_L(t)\rho_L \frac{d}{dt}(a^3) \]

we find a time-dependent \( w_L(t) \) for the “fictitious” dark energy as follows.
\[ w_L(t) = -1 + \frac{2dL(t)}{3} \frac{dt}{dt} \]  
\[ = -1 - \frac{2}{3} \frac{p\gamma L(t_0)}{1 + (p - 1)\gamma L(t_0) \ln a(t)} \] (28)

so the effective \( w_L(t) \) has the limiting values 
\[ w_L(t_0) = -1 - \frac{2}{3} p(\gamma L(t_0)) \] and \( w_L(t_{rip}) = -1 \).

We may check consistency with the space-space components of Einstein’s equations which are

\[ \frac{\ddot{a}}{a} = -\frac{1}{2} a^2 - 4\pi G p \] (29)

which leads to

\[ \dot{H} = -4\pi G (\rho_L + p_L) = -4\pi \rho_L (1 + w_L) \] (30)

so that with \( H = L^{-1} \) and \( \rho_L = 3H/(8\pi GL) \)
we find a \( w_L(t) \) consistent with Eq.(28).
Keeping the value \( L_0 = H_0^{-1} = (14\text{Gy})^{-1} = 1.3 \times 10^{28} \text{ cm} \) and putting \( \gamma = (20\text{Gy})^{-1} \) and \( p = 2 \) we arrive at the entries in the following Table:

<table>
<thead>
<tr>
<th></th>
<th>( l_0(\text{cm}) )</th>
<th>( r_g(\text{cm}) )</th>
<th>( t_{rip} - t_U )</th>
<th>( t_{cs} - t_U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>gal</td>
<td>( 5 \times 10^{22} )</td>
<td>( 3 \times 10^{16} )</td>
<td>( 2.37\text{Gy} )</td>
<td>( 1.14\text{Gy} )</td>
</tr>
<tr>
<td>S-E</td>
<td>( 1 \times 10^{13} )</td>
<td>( 3 \times 10^{5} )</td>
<td>( 9 \times 10^{4}\text{y} )</td>
<td>( 7\text{y.} )</td>
</tr>
<tr>
<td>E-M</td>
<td>( 3 \times 10^{10} )</td>
<td>0.866</td>
<td>( 2 \times 10^{4}\text{y} )</td>
<td>( 6\text{mos.} )</td>
</tr>
</tbody>
</table>

Note that for the \( p = 2 \) case we have tabulated the difference \( (t_{caus} - t_U) \) rather than \( (t_{rip} - t_{caus}) \) because in this case the expansion is so rapid.
Observational Constraints

Next we turn to observational constraints on the parameters $L_0$ and $\gamma$ for $p = 2$.

In the previous section, we discussed the future universe in the model. In this section, we discuss constraints on model parameters from SNeIa data. To discuss the constraint, we have to include other component such as cold dark matter (CDM) and baryon. Including all components, we can write the Friedmann equation as

$$H^2 + \frac{k}{a^2} = \left(\frac{\rho_m}{3M_{\text{Planck}}^2} + \frac{1}{4L^2} + \frac{1}{2L}\right)^2$$  \hspace{1cm} (31)

If we define the density parameter $\Omega_m \equiv \rho_m/\rho_{\text{crit}} = \rho_{m0}(1 + z)^3$, we can rewrite Eq. (31) as

$$H^2 = H_0^2 \left[\Omega_k(1 + z)^2 + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_m(1 + z)^3}\right)^2\right]$$  \hspace{1cm} (32)
where $\Omega_k$ and $\Omega_L$ are defined as

$$\Omega_k \equiv \frac{-k}{H_0^2}, \quad \Omega_L \equiv \frac{1}{4L^2H_0^2}. \tag{33}$$

Thus at the present time, we have the relation among the density parameters,

$$\Omega_k + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_m}\right)^2 = 1. \tag{34}$$

To obtain a constraint from the SNeIa observations, we assume that red-shift dependence of $\Omega_L$ is as in Eq. (25) and that $L$ is dependent on time as Eq. (14); thus we can say that we consider a new component $\rho_L$ which is defined as Eq. (25).
Now we discuss the constraint on this model from SNeIa data using recent results. In Fig.1 we show contours of 95 and 99 % C.L. in $\Omega_L$-$\Omega_m$ plane for $\gamma L_0 = 0$ (which is the constant $L$ case), 0.5 and 1 with $p = 2$. In the figure, we also plot the line for the flat universe. Notice that the line is different from the standard case because we have the modified Friedmann equation in this model. To obtain the constraint, we marginalize the Hubble parameter dependence by minimizing $\chi^2$ for the fit.
Figure 1: Constraint from SNeIa observation in $\Omega_L-\Omega_m$ plane for $\gamma L_0 = 0$ (bottom), $\gamma L_0 = 0.5$ (middle) and $\gamma L_0 = 2$ (top). Contours are for 95 % (dotted line) and 99 % (dashed line) C.L. constraints respectively. The solid line indicates parameters which give a flat universe.
In Fig. 2 we show the constraint on $\Omega_m - \gamma L_0$ plane assuming the flat universe ($\Omega_k = 0$). If we take the value $\Omega_m = 0.3$, we can find an upper limit of $\gamma L_0 < 0.7$. This implies that the time remaining from now until the Rip is constrained to be generically at least somewhat longer than the current age. If we make $L_0$ larger than the length corresponding to the age of the Universe then the upper bound on $\gamma$ diminishes and hence the time until the Rip increases.
Figure 2: Constraint from SNeIa observation in $\Omega_r$-$\Omega_m$ plane. Contours are for 95% (dotted line) and 99% (dashed line) C.L. constraints respectively. In this figure, we assume a flat universe.
We note that because the effective equation of state \( w(t) \) is varying with time its present value \( w(t_0) \) can be more negative than allowed by constraints derived from assuming constant \( w \). Our constraint on \( \gamma L_0 < 0.7 \) permits 
\[
 w(t_0) = -1 - \frac{2}{3}p\gamma L_0
\]
to be as negative as 
\[
 w(t_0) = -1.9 \quad \text{for} \quad p = 2.
\]
Assuming constant \( w \), on the other hand, gives \( w > -1.2 \).
Discussion

No amount of observational data can, by itself, tell us the fate of dark energy if we allow for an arbitrarily varying equation of state.

Three possibilities are:

• there may be a Big Rip,

• dark energy may dominate but with an infinite lifetime

• the dark energy may eventually disappear leaving a matter-dominated Universe.

Given that observational data are insufficient, only a convincing theory of the past could inform of the future of the Universe, and no such theory is at hand.
The Big Rip was the most exotic of the fates and there seemed tied to a phantom $w < -1$ dark energy. However, here we have studied a Bigger Rip, in which the scale factor is even more divergent at a future finite time than for the Big Rip, which is achieved by modifying gravity and omitting dark energy. In the model, as with the phantom case, structures become unbound and subsequently their components become causally disconnected before the Universe is torn apart in the Rip.
If we allow an arbitrarily-varying equation of state any new term on the left-hand-side of the Friedmann equation can apparently be taken to the right-hand-side and reinterpreted as a dark energy. But this will, in general, lead to a relationship between pressure and energy which is not truly simple, but very complicated as in Eq.(28) above. We use the term “dark energy” when the time dependence of such a term governing the expansion rate of the Universe is sufficiently simple, e.g. it could be required that \( w(t) \) is either constant or polynomial in \( t \). This is not the case for the present model and hence justifies the title chosen for the study “without dark energy”.
SUMMARY OF DARK ENERGY

1. The future of dark energy cannot be predicted from precision observation of the past.

2. DE cannot have effect on microscopic level at scales sub-galactic.

3. Modification of gravity can eliminate need for dark energy and can lead to a more singular future than the Big Rip.
OUTLINE OF DARK MATTER

• 1. WIMPS and their detection

• 2. HEAT positron excess

• 3. γ-Rays from Galactic Bulge

• 4. DAMA Annual Modulation

• Summary of Dark Matter

This makes generous use of a review by P. Gondolo. hep-ph/0501134.
Non-Baryonic Cold Dark Matter.

Existence is supported by disparate cosmological measurements.

Values of energy and matter densities at the present time, determined by: the temperature fluctuations in the CMB data; distance-luminosity for supernovae type 1A; distribution of galaxies on large scales (LSS); abundance of light elements (BBN).

In terms of the critical density $\Omega$ for the various components is found to be as follows (taking $h^2 = 0.5, h = 0.707$).
- Relativistic particles, radiation \textit{e.g.} the CMB photons. Only \( \Omega_\gamma = 5.934 \pm 0.008 \times 10^{-5} \).

- \( \Omega_\Lambda = 0.72 \pm 0.08 \) in a smoothly distributed dark energy.

- \( \Omega_M = 0.27 \pm 0.016 \) in non-relativistic particles (Matter) of which

\[ \Omega_b = 0.0448 \pm 0.0018 \] in baryons (protons and neutrons)

\[ \Omega_{HDM} < 0.0152 (95\% CL) \] in non-baryonic hot dark matter.

\[ \Omega_{CDM} = 0.223 \pm 0.016 \] in non-baryonic cold dark matter.

The excess of total matter density (0.27) over baryonic mass density (0.0224) constitutes the evidence for non-baryonic dark matter.
No known elementary particle can account for the non-baryonic dark matter.

One obvious candidate, the neutrinos, are so light they constitute hot dark matter and contribute to the $\Omega_{HDM} < 0.0152$.

Many hypothetical particles have been proposed for the CDM. Some come from extensions of the standard model, most notably the axion and the lightest supersymmetric particle.

Other possibilities include Wimpzillas, solitons, self-interacting dark matter, Kaluza-Klein dark matter, etc.

The class of non-baryonic dark matter candidate of greatest interest are the Weakly Interacting Massive Particles (WIMPs). Therefore I shall focus on their detection and the claims to have discovered them.
1. WIMPs and their detection

WIMPs are appealing because of the simple mechanism by which they can achieve the appropriate present cosmic density. In the early universe they were in thermal and chemical equilibrium with the rest of matter and radiation. With the expansion of the Universe, their reactions (including annihilation) slowed down and decoupled from the rest of the world leaving a constant number of WIMPs expanding with the Universe.

The correct present density is obtained for WIMPs with couplings of order the weak interactions and masses in the 1 GeV - 1 TeV range. The neutralino is the most popular example.
Detection can be direct or indirect.  

**Direct** signals are from collisions with nuclei in a detector. A very sensitive low-background detector (bolometer) records the amount of energy deposited by WIMPs and (in the future) the direction of motion of the struck nucleus.

**Indirect** signals come from WIMP reactions in planets, stars or galaxies. The most common reaction is WIMP annihilation with anti-WIMP. Out of this annihilation come $\nu$, $e^+$, $\bar{p}$ and high-energy $\gamma$. Such annihilations occur at a detectable rate where the anti-WIMPs are concentrated *e.g.* in the center of the Sun, the center of the Earth and in galactic centers including the Milky Way. Neutrino telescopes, gamma-ray telescopes and cosmic-ray detectors can be used in these indirect searches.

Now we look at three claims for seeing WIMPs.
2. HEAT positron detection

Two separate balloon flights with different detectors have seen more cosmic ray positrons above $\sim 7$ GeV than predicted in models for cosmic ray propagation in the galaxy. Wimp annihilation can be invoked to explain this excess.

As seen in the Figure (next transparency) the extra positrons can be fitted by assuming neutralino annihilation. The best fits requires WIMP mass of 238 GeV.

The positron spectrum lacks any discriminat- ing feature which clearly singles out WIMP annihilation.
3. $\gamma$-Rays from Galactic Bulge.

Gamma rays from WIMP annihilation offer a characteristic signature in the spectrum: a gamma-ray line. Each photon will carry an energy equal to the WIMP mass, 10GeV to 100 TeV. No competing process is known that could produce such a line.

No line has been detected yet. The estimates for GLAST (launch scheduled 2006) are encouraging (see next transparency).

Another suggestion has been that the 511 keV gamma excess from the galactic bulge arises from positrons associated with unexpectedly light WIMP annihilation. The necessary WIMP mass is in the region between 1 MeV and 100 MeV.
4. DAMA Annular Modulation.

Because the Earth’s motion changes the relative speed of the Earth and WIMPs the WIMP detection rate varies and repeats itself every year. The maximum occurs in June for the canonical halo model with Maxwellian velocity distribution but may occur in December for the cold infall model of Sikivie.

The DAMA group has claimed to have detected such annual modulation in their NaI data. No alternative explanation of the DAMA data has been forthcoming.

No other direct detection of such a WIMP signal has been made by any other group but there are differences between the targets used as well as the nuclear spin thereof. So comparison between experiments requires some theoretical assumptions.
Nevertheless, it does appear that CDMS data *completely* (left Figure) or *almost (?)* (Right Figure) excludes the DAMA claim (see next transparency).

Future detectors will measure the direction of motion of the recoil nucleus and enable a more clearcut WIMP signature.
SUMMARY OF DARK MATTER

How to be Sure of WIMP Detection?

We require features that can be due to WIMPs and nothing else.

- (i) Gamma-ray annihilation from WIMP annihilation should show a gamma-line in correspondence with the WIMP mass.

- (ii) Annual modulation should show the correct periodicity both in rate and, in future, directional dependence.

Compatible indirect (i) and direct (ii) detection could provide compelling evidence for WIMPs.

Better would be production in a collider consistent with cosmological detection!

Thank you for your attention.