DIVERGENCIES AND SYMMETRIES IN HIGGS-GAUGE UNIFICATION THEORIES

(see also hep-ph/0410226)

Outline:
1. Introduction: motivations for Higgs-gauge unification theories
2. Gauge theories on orbifolds
3. Symmetries @ fixed points and localized terms
4. The residual $O_f$ symmetry
5. Conclusions and outlook

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ELECTROWEAK INTERACTIONS & UNIFIED THEORIES
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A possible motivation:

**Little Hierarchy Problem (LHP)**

Barbieri & Strumia 00  Giudice 03

- **Standard Model (SM):** effective theory with cutoff $\Lambda_{SM}$

\[ \delta m_H = \frac{3G_F}{4\sqrt{2}\pi^2} \left( 2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2 \right) \Lambda_{SM}^2 = \left( \frac{200\text{GeV}}{0.7\text{TeV}} \right)^2 \]

No fine-tuning $\rightarrow$ $\Lambda_{SM} \lesssim 1\text{ TeV}$

- **New physics** $\leftrightarrow$ non-renormalizable (dimension six) operators $O$

\[ L = L_{SM} \pm \frac{1}{\Lambda_{LH}^2} O \]

Precision tests $\rightarrow$ $\Lambda_{LH} \gtrsim 5-10\text{ TeV}$

One order of magnitude of discrepancy: LHP
A possible solution:

**Supersymmetry (SUSY)**

- SUSY → no quadratic divergences → (grand) HP solved:
  SUSY SM (MSSM) can be extended up to $M_{Pl}$

  \[ \Lambda_{SM} \sim M_{SUSY} \]

- If R-parity is conserved → SUSY virtual loops are suppressed →

  \[ \Lambda_{LH} \sim 4\pi\Lambda_{SM} \quad \rightarrow \quad \text{LHP solved} \]

However:
- SUSY not yet been observed → fine-tuning
- SUSY breaking sector not well defined
- ...

Worthwhile looking for alternative solutions
An alternative solution:

**Higgs-gauge unification**

Consider a gauge theory in a D-dimensional space-time

\[
A_M^A = \left\{ A_{\mu}^A, A_i^A \right\}
\]

4D Lorentz scalars \( \rightarrow \) Higgs fields!

\( \Lambda^{\text{SM}} \sim 1/R \sim \text{TeV} \)

D-dimensional theory UV completion

4D Lorentz vector

they can acquire mass through the Hosotani mechanism

\( \Lambda_D \geq 10 \text{ TeV} \)

Higgs mass in the bulk is protected by higher-dimensional gauge invariance

finite corrections \( \sim (1/R)^2 \) allowed

\( \Lambda^2 A_i^A A_j^B \)

\( \Lambda_{\text{SM}} \sim 1/R \sim \text{TeV} \)

\( \Lambda_D \geq 10 \text{ TeV} \)

→ LHP solved
Gauge theory in D dimensions

Spacetime: $M^D$
coord.: $x^M = (x^\mu, y^i)$

$$L_D = -\frac{1}{4} F_{MN} F^{MN} + i\bar{\psi} \Gamma^D D_M \psi$$

Invariant under gauge group $G$ ($SO(1,D-1)$)

Compactification on the orbifold $M^4 \times T^d / G_{orb}$

- $T \cdot y = y + u \quad u \in \Lambda^d$

Torus: $y \equiv y + u$

- $k \in G_{orb} \quad k \cdot y = R_k y + u \quad R_k \in SO(d)$

Orbifold: $y \equiv R_k y + u$

Fixed points: invariants under $G_{orb}$

$$k \cdot y_f = y_f$$

5D: $S^1 / Z_2$

$\cdots \quad y \quad y + 2\pi R \quad \cdots$

$\downarrow$

$y = y + 2\pi R$

Circle $S^1$

$\pi R \quad 0 \quad -\pi R$

$\downarrow$

$y = -y$

Orbifold $S^1 / Z_2$

bulk $\pi R$

fixed points
Action of $G_{\text{orb}}$ on the fields

$$k \cdot \phi_R(y) = \lambda^k_R \otimes P^k_\sigma \phi_R(k^{-1} \cdot y)$$

acts on Lorentz indices

acts on gauge and flavour indices

acts on Lorentz indices

unconstrained

fixed by requiring invariance of lagrangian

it can be used to break symmetries

$S^1$: $\phi(x^\mu, y) = \sum_{n=-\infty}^{+\infty} e^{i R y} \phi_n(x^\mu)$ → 4D fields with mass $m_n = \frac{n}{R}$

$S^1/Z_2$: \[ \begin{align*}
\phi(y) &= +\phi(-y) & \phi^+(x^\mu, y) &= \sum_{n=0}^{+\infty} \cos\left(\frac{n}{R} y\right) \phi^+_n(x^\mu) \\
\phi(y) &= -\phi(-y) & \phi^-(x^\mu, y) &= \sum_{n=1}^{+\infty} \sin\left(\frac{n}{R} y\right) \phi^-_n(x^\mu)
\end{align*} \]

• zero mode: only for $\phi^+_0(x^\mu)$

• @ $y_f$ $\phi^-(x^\mu, k\pi R) = 0$
Gauge symmetry breaking @ $y_f$

- Why looking @ $y_f$?
  → lagrangian terms localized @ the fixed points can be radiatively generated
    (if compatibles with symmetries)

- $G \equiv \{ T^A \} \xrightarrow{G_{\text{orb}}} H_f \equiv \{ T^a_f \} @ y_f$

\[
\begin{bmatrix}
T^a_f, \chi^k_R
\end{bmatrix} = 0
\]

Non-zero fields @ $y_f$:
- $A^a_{\mu} A^{\dot{a}}_i$ (for some $i$ & $\dot{a}$) (with zero modes)
- some derivatives of non-invariant fields
  (without zero modes)

\[\downarrow\]
Residual global symmetry $K$

Gorsdorff, Irges & Quirós 02

[Diagram showing the transformation and fixed points]

Georgi, Grant & Hailu 00
Effective 4D lagrangian

\[ L_{4}^{\text{eff}} = \int d^{d}y \left[ L_{D} + \sum_{f} L_{f} \delta^{d}(y - y_{f}) \right] \]

\( L_{f} \to \) most general 4D lagrangian compatible with symmetries @ \( y_{f} \)

The symmetries @ \( y_{f} \) are: \( G_{\text{orb}} \), \( \text{SO}(3,1) \), \( H_{f} \), \( K \)

Forbidden terms:

If \( A_{i}^{\hat{a}} \) is \( G_{\text{orb}} \)-invariant \( \Rightarrow \) a “shift” symmetry forbids a direct mass term:

\[ \Lambda^{2} A_{i}^{\hat{a}} A_{j}^{\hat{b}} \]

Allowed terms:

\[ F_{\mu \nu}^{a} F^{a \mu \nu} \to \text{localized kinetic term for } A_{\mu}^{a} \]

\[ F_{\mu \nu}^{a} \tilde{F}^{a \mu \nu} \to \text{localized anomaly} \]

\( \to \) If \( A_{i}^{\hat{a}} \) and \( F_{ij}^{a} \) are orbifold invariant

\[ F_{ij}^{a} F^{aij} \to \text{localized quartic coupling for } A_{i}^{\hat{a}} \quad (D \geq 6) \]

\[ F_{i \mu}^{\hat{a}} F^{\hat{a} i \mu} \to \text{localized kinetic term for } A_{i}^{\hat{a}} \]

All these are dimension FOUR operators \( \to \) renormalize logarithmically
... another (worse) allowed term...

If $H_f = U(1)^a \times ...$ and $A_i^a$ and $F_{ij}^a$ are orbifold invariant

$$F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a - g f^{abc} A_i^b A_j^c$$

is invariant under $G_{\text{orb}} \to \text{SO(3,1)}$  $H_f \to K$

- tadpole for $\partial_i A_j^a$
- mass term for $A_i^b A_j^c$

This is a dimension TWO operator $\to$ quadratic divergencies

$D \geq 6$ it seems it always exists

- $D=6$ (QFT)  
  Gersdorff, Irges & Quirós 02  
  Csaki, Grojean & Murayama 02  
  Scrutti, Serone, Silvestrini & Wulzer 03 (SSSW03)
- $D=10$ (strings)  
  Groot-Nibbelink et al. 03

* How can we avoid this?

1. global cancellation of tadpoles  
2. ...
But...

another symmetry must be considered


- d-dimensional smooth manifold:
  at each point can be defined a TANGENT SPACE $\rightarrow$ SO(d)

- when orbifolding:
  as $G \xrightarrow{G_{\text{orb}}} H_f$ such that $\left[ \lambda^k_R, H_f \right] = 0$

  so $SO(d) \xrightarrow{G_{\text{orb}}} O_f$ such that $\left[ P^k_\sigma, O_f \right] = 0$

The symmetries @ $y_f$ are: $G_{\text{orb}} \ SO(3,1) \ H_f \ K \ O_f$

* Can this $O_f$ forbid the tadpole?
The tadpole $F_{ij}$ and the symmetry $O_f$

- If $O_f = SO(2) \times \ldots$ then the Levi-Civita tensor $\varepsilon^{ij}$ exists
  
  \[ \varepsilon^{ij} F_{ij}^\alpha \] is $O_f$ invariant  \rightarrow \text{TADPOLES ARE ALLOWED} 

- If $O_f = SO(p_1) \times SO(p_2) \times \ldots \ (p_i > 2)$ then the Levi-Civita tensor is $\varepsilon^{i_1i_2\ldots i_p}$
  
  \[ \varepsilon^{i_1i_2\ldots i_p} B_{i_1i_2\ldots i_p} \]  \rightarrow \text{NO TADPOLES} 

Sufficient condition for the absence of localized tadpoles

\[ O_f = \prod SO(p_i) \quad p_i > 2 \]  

$O_f$ is orbifold-dependent: we studied the $T^d/Z_N$ case
Orbifolds $T^d/Z_N$ \hspace{1cm} (d even)

- $O_f$ depends on $R_{N_f}$:
  
  on $T^d/Z_N \rightarrow R_{N_f} \sim \text{diag}(r_1...r_i...r_{d/2})$ with $r_i$ rotation in the $i$-plane

- If $N_f>2 \Rightarrow O_f = \prod_{i=1}^{d/2} SO(2) \supseteq SO(d)$ which acts on $(y_{2i-1}, y_{2i})$

  $\Rightarrow$ in every subspace $(y_{2i-1}, y_{2i})$ $\varepsilon^{IJ}$ exists

  $\Rightarrow \sum_{i=1}^{d/2} \sum_{I,J=2i-1}^{2i} \varepsilon^{IJ} F^\alpha_{IJ} \delta^{d/2} (y - y_f)$

- If $N_f=2 \Rightarrow R_{N_f} = -1 \Rightarrow [-1, SO(d)] = 0$

  $\Rightarrow O_f = SO(d)$ the Levi-Civita tensor is $\varepsilon^{i_1i_2...i_d}$

  $\rightarrow$ only invariants constructed with $d$-forms are allowed

  $\rightarrow$ TADPOLES ONLY FOR $d=2$ (D=6) valid also for odd D

* $T^d/Z_2 \rightarrow$ explicitely checked @ 1- and 2-loop for any D \hspace{0.5cm} BQ'04
Conclusions

In Higgs-gauge unification theories \((\text{Higgs} = A_i)\)

- bulk gauge symmetry \(G\) prevents the Higgs from acquiring a quadratically divergent mass in the bulk
- “shift” symmetry \(K\) forbids a direct mass @ \(y_f\)

If \(H_f = U(1)^a \times \ldots \quad F_{ij}^\alpha\) can be radiatively generated @ \(y_f\) giving rise to a quadratically divergent mass for the Higgs

\(F_{ij}^\alpha\) can be generated \(\leftrightarrow\) it is \(O_f\)-invariant

\(O_f \subseteq SO(d)\) such that \(\left[ O_f, P^k_\sigma \right] = 0\)

If \(O_f = SO(p_1) \times SO(p_2) \times \ldots \quad (p_i>2)\) \(\rightarrow\) NO TADPOLES

If \(O_f = SO(2) \times \ldots\) \(\rightarrow\) TADPOLES

- \(T^d/Z_N\) (d even, \(N>2\): if \(N_f>2\) \(\rightarrow\) \(O_f = SO(2) \times \ldots \times SO(2)\) \(\rightarrow\) TADPOLES
- \(T^d/Z_2\) (any d): \(O_f = SO(d)\) \(\rightarrow\) TADPOLES ONLY FOR \(d=2\) (D=6)
Outlook

The absence of tadpoles is a necessary but not sufficient condition for a realistic theory of EWSB without SUSY

Other issues:

• REALISTIC HIGGS MASS
  
  \( D > 6 \) (\( D = 5 \) no quartic coupling, \( D = 6 \) tadpoles)
  
  \( \mathbb{T}_d / \mathbb{Z}_2 \rightarrow \text{d Higgs fields} \Rightarrow \text{non-minimal models} \)
  
  \( \rightarrow \) we have to obtain only one SM Higgs
    even if this is achieved
  
  \( \rightarrow \) Higgs mass must be in agreement with LEP bounds

• FLAVOUR PROBLEM
  
  - matter fermions in the bulk coupled to a background
    which localizes them at different locations

  Burdman & Nomura 02

  - matter fermions localized and mixed with extra heavy
    bulk fermions

  Csaki, Grojean & Murayama 02