Inverted sfermion mass hierarchy and flavour changing neutral currents

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- introduction: the flavour problem in SUSY
- effect of decoupling the sfermions
- constraints on the inverted sfermion mass hierarchy scenario
- conclusions

based on:
- P. Chankowski, K. Kowalska, SL, S. Pokorski (in progress)

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La Thuile, 7 March 2005
The supersymmetric flavour problem

New sources of flavour (and CP) violation in soft terms generally lead to unacceptably large contributions to FCNC processes.

Example: $K^0 - \bar{K}^0$ mixing

$$(\delta_{UL})_{12} \equiv \frac{(\tilde{u}_{12})_{UL}}{m_t^2}$$

$$\Delta m_K^{\text{new}} \sim \frac{\alpha_s^2 m_K f_K^2}{(500 \text{ GeV})^2} \left( \frac{\tilde{(\delta_{UL})}_{12}}{m_t^2} \right)^2 \left( \frac{m_\pi^2}{m_t^2} \right) + \ldots$$

$$\frac{\alpha_s^2 m_K f_K^2}{(500 \text{ GeV})^2} = 7 \times 10^{-7} \text{ MeV} \quad \text{while} \quad \Delta m_K\text{_{exp}} = 3.5 \times 10^{-12} \text{ MeV}$$

Other example: $\mu \rightarrow e\gamma$

$$(\delta_{UL})_{12} \equiv \frac{(\tilde{u}_{12})_{UL}}{m_t^2}$$

$$m_e^2 |_{\text{SHNS}} = R_L m_e^2 R^e_L$$

$$R_L^e H e R^e_R = \left( \frac{m_e}{m_e, m_e} \right)$$

$$\text{BR} (\mu \rightarrow e\gamma) \sim \frac{\alpha^3 |(\tilde{u}_{12})_{UL}|^2}{G_F^2 m_e^4} \tan^2 \beta \sim 10^{-3} \left( \frac{\tilde{(\delta_{UL})}_{12}}{m_t^2} \right)^2 \left( \frac{300 \text{ GeV}}{M_{3/2}} \right)^4 \left( \frac{\text{tan} \beta}{10} \right)$$

Can by far exceed the exp. limit $\text{BR}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$
\[
\rightarrow \text{ for } x \equiv \frac{m_3^2}{m_1^2} = 1 , \quad m_\Sigma = 500 \text{ GeV}
\]

(for fixed \( x \), the limits scale as \( m_\Sigma \))

| \( \Delta m_K \) | \( \sqrt{|\text{Re} (\delta_{12}^d)_{13}^2|}_\text{(RR)} \) | \( \sqrt{|\text{Re} (\delta_{LR}^d)_{12}^2|}_\text{(RL)} \) | \( \sqrt{|\text{Re} (\delta_{LL}^d)_{12} (\delta_{RR}^d)_{12}|} \) |
|---|---|---|
| \( 4.0 \times 10^{-2} \) | \( 4.4 \times 10^{-3} \) | \( 2.8 \times 10^{-3} \) |

| \( \Delta m_{B_d} \) | \( \sqrt{|\text{Re} (\delta_{LL}^d)_{13}^2|}_\text{(RR)} \) | \( \sqrt{|\text{Re} (\delta_{LR}^d)_{13}^2|}_\text{(RL)} \) | \( \sqrt{|\text{Re} (\delta_{LL}^d)_{13} (\delta_{RR}^d)_{13}|} \) |
|---|---|---|
| \( 9.8 \times 10^{-2} \) | \( 3.3 \times 10^{-2} \) | \( 1.8 \times 10^{-2} \) |

| \( \Delta m_D \) | \( \sqrt{|\text{Re} (\delta_{LL}^d)_{12}^2|}_\text{(RR)} \) | \( \sqrt{|\text{Re} (\delta_{LR}^d)_{12}^2|}_\text{(RL)} \) | \( \sqrt{|\text{Re} (\delta_{LL}^d)_{12} (\delta_{RR}^d)_{12}|} \) |
|---|---|---|
| \( 1.0 \times 10^{-1} \) | \( 3.1 \times 10^{-2} \) | \( 1.7 \times 10^{-2} \) |

| \( \mathcal{E}_K \) | \( \sqrt{|\text{Im} (\delta_{LL}^d)_{12}^2|}_\text{(RR)} \) | \( \sqrt{|\text{Im} (\delta_{LR}^d)_{12}^2|}_\text{(RL)} \) | \( \sqrt{|\text{Im} (\delta_{LL}^d)_{12} (\delta_{RR}^d)_{12}|} \) |
|---|---|---|
| \( 6.1 \times 10^{-3} \) | \( 3.7 \times 10^{-4} \) | \( 1.3 \times 10^{-4} \) |

\[
\text{BR} (B \rightarrow \pi K) \quad \left| (\delta_{LL})_{23} \right| \quad \left| (\delta_{LR})_{23} \right| \quad \times \left( \frac{m_\Sigma}{500 \text{ GeV}} \right)^2
\]

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Gabbiani, Gabrielli, Masiero, Silvestrini
(a) $|\delta_{LL}^{d12}|, |\delta_{RR}^{d12}|$

(b) $\sqrt{(\delta_{LL}^{d12})(\delta_{RR}^{d12})}$

(c) $|\delta_{LR}^{d12}|, |\delta_{RL}^{d12}|$

(d) $\sqrt{(\delta_{LR}^{d12})(\delta_{RL}^{d12})}$
singularity and yields a good approximation in the chargino dominance sector\(^2\).

If one relaxes even more the mSUGRA constraints, it becomes legitimate to ask if one can escape the LFV limits on \(\delta_{12}^{LL}\)'s or, conversely, how model independent are, e.g., the more stringent limits on \(\delta_{12}^{\tilde{L}L}\). The only possibility is to play with violations of gaugino mass universality. For instance, by

\(^2\)The SU(2) contribution can be identified with the chargino one, since the latter is always much bigger than the corresponding neutralino contribution.
Possible solutions to the SUSY flavour problem

(i) Degeneracy of sfermion masses: $\delta_{ij} = 0$ (i $\neq$ j)
   either an assumption (mSUGRA) or follows from
   a specific mechanism for SUSY (gauge mediation,
   dilaton dominance in SUGRA mediation...)

(ii) "alignment": sfermion masses approximately
   diagonalized by the same rotations as fermion masses
   $\Rightarrow M^2_{ij} \sim 0$ (i $\neq$ j) $\Rightarrow \delta_{ij} \approx 0$
   (Ni, Seiberg '93)
   can follow from appropriate flavour symmetries

(iii) "decoupling": very heavy sfermions (> 10 TeV)
   suppress SUSY contributions to FCNC
   but lose the benefit of low-energy SUSY
   = protection of the Higgs mass against (quadratically
   divergent) radiative corrections, and reintroduce fine-tuning

   $\Rightarrow$ "effective supersymmetry": sfermions of the first
   two generations, which couple only weakly to
   Higgs bosons, are heavy, while third generation
   sfermions (squarks) are light (< 1 TeV)

   (Cohen, Kaplan, Nelson '96)
assumptions

1) typical hierarchical Yukawa couplings

take for definiteness one of the models of hep-ph/0501071
(charkowski, Kowalska, SL, Fohrenbichl)

based on a spontaneously broken $U(1)$ flavour symmetry

\[
Y_u \sim Y_t \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} Y_d \sim Y_t \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix} Y_e \sim Y_t \begin{pmatrix} \lambda^5 & \lambda^3 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \end{pmatrix}
\]

\[
\lambda = \sin \theta_c = 0.22
\]

$U(1)$ coefficients in each entry chosen to reproduce the observed quark and lepton masses and mixings [consider 100 sets of $U(1)$ coefficients]

2) soft sfermion masses of the form:

\[
\tilde{M}^2 = \begin{pmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{pmatrix} + 6(\lambda^n) \text{ off-diagonal entries (flavour symmetries)}
\]

we are interested in the case $m_3^2 \ll m_1^2, m_2^2$ with $M_{1/2} \leq m_3$ and $A_{ij} \sim m_3 a_{ij} Y_{ij}$ $a_{ij} = U(1)$ [possible origin : D-term SUSY]

→ 2 sources of FCNC

- off-diagonal entries in $\tilde{M}^2$

\[
\tilde{M}_{12}^2 \sim \lambda^{n_{12}} \Rightarrow \delta_{12} \sim \lambda^{n_{12}}
\]

- splittings between diagonal entries of $\tilde{M}^2$

\[
m_1^2 \neq m_2^2 \Rightarrow \delta_{12} \approx 2 \frac{m_1^2 - m_2^2}{m_1^2 + m_2^2} \Rightarrow \delta_{12} \sim R_{12} \text{ unless } m_1^2 \approx m_2^2
\]
Figure 1: Histograms of $R(m_s/m_b)$, $R(m_d/m_b)$, $R(m_c/m_t)$, $R(m_u/m_t)$, $R(V_{us})$ and $R(V_{ub})$, where $R(Q) = Q(M_{GUT})_{\text{predicted}}/Q(M_{GUT})_{\text{evolved}}$; for charge assignment $q_i = u_i = e_i = (3, 2, 0)$, $l_i = d_i = (4, 2, 2)$ and $h_u = h_d = 0$ ($\tan \beta = 15$).
how do the FCNC constraints depend on the sfermion mass scale?

- \( G(\lambda^3) \) off-diagonal entries \([\lambda^3 \times 10^{-2}]\)

  - \( m \leq 1 \text{ TeV} \): effect of off-diagonal entries in the quark sector reduces due to the contribution of the gluino mass to the renormalization of \( \tilde{f} \)

  \[
  16\pi^2 \frac{d R_{ij}}{d m_{\tilde{f}}} = (\text{Yukawa}) - \frac{32}{3} g_s^2 M_3 \Delta_{ij} + (\text{so gauge}) + \ldots
  \]

  enhances diagonal entries by a universal piece

  \[
  M_i^2 (M_Z) \approx M_i^2 (M_{\text{UT}}) + C M_{1/2}^2
  \]

  \( C \approx 6-7 \)

  \[
  \Rightarrow \delta_{12} \sim \frac{m_i^2 \lambda^3}{m_i^2 + C M_{1/2}^2} \ll \lambda^3
  \]

  (see Chengowski, Lebedev, Pokorski 050207)

  A much milder effect in the slepton sector (\( c \leq 0.5 \))

  - \( m \geq 5 \text{ TeV} \): effect of decoupling the sfermion masses - an \( G(\lambda^3) (1,2) \) off-diagonal entry is enough to suppress \( \mu \rightarrow e\gamma \) below exp. limit, but is still problematic for \( \mu \)

- splitting \( m_1 \neq m_2 \) (\( m_1 \in [0.5 m_2, m_2] \))

  - similar effects in the regions \( m \leq 1 \text{ TeV} \) and \( m \geq 5 \text{ TeV} \)

  - again strong constraints from \( \mu \)

    (and for \( m \leq 5 \text{ TeV} \) from \( \mu \rightarrow e\gamma \))

    \Rightarrow some amount of degeneracy needed in the \( \tilde{q} \) sector unless \( m > 50 \text{ TeV} \)

\[ \Delta \text{ since } \delta_{12} \approx \frac{m_i^2 - m_j^2}{m_i^2} R_{ij} \text{, this conclusion strongly depends on the chosen Yukawa couplings} \]

\[ R_L^u \sim R_L^d \sim V_{CKM} \Rightarrow (R_L^d)_{12} \sim \lambda \]

[also \( (R_L^u)_{12} \sim \lambda^2 \), and large phases in both \( R_L^u, R_L^d \)]
\[ M_{1/2} = 400 \text{ GeV} \]
\[ m_1 = m_2 = m_3 \equiv m \]

off-diagonal entries \[ \mathcal{O}(\lambda^3) \approx \mathcal{O}(10^{-2}) \]

\[ M_{1/2} = 400 \text{ GeV}, \ m_1 = m_2 = m_3, \ \mathcal{O}(\epsilon^3), \ A_0 = 0 \]

\[ \Delta m_K \]

\[ \mu \rightarrow e\gamma \]

\[ \epsilon_K \]

\[ \Delta m_0 \]

\[ m \in [100 \text{ GeV}, 50 \text{ TeV}] \]
$M_{1/2} = 400 \text{ GeV}$

$0.5 m_2 \leq m_1 \leq m_2 \quad m_2 = m_3 \equiv m$

no off-diagonal entries

$M_{1/2} = 400 \text{ GeV}, \ 0.5 m_2 < m_1 < m_2, \ m_2 = m_3, \ \text{no off}, \ A_0 = 0$

$m \in [100 \text{ GeV}, \ 50 \text{ TeV}]$
to reduce fine-tuning in the Higgs potential, assume \( m_3 \ll m_{1,2} \) (and \( M_{H^\pm} \leq 500 \text{ GeV} \))

\[
H_2^2 = -\mu^2 + \frac{m_{H^\pm}^2 - m_{H^0}^2 \tan^2 \beta}{\tan^2 \beta - 1} \tan \beta \geq 10 \leq 0
\]

\( m_{H^0}^2 (H_2) \) receives large contributions from third
generation sfermions \((\tilde{y}_t, \tilde{y}_b, \tilde{y}_\tau)\) and from gluinos \((\tilde{g}_2, \tilde{g}_3)\)

\( \Rightarrow m_3, M_{H^\pm} \) should be \( \lesssim 1 \) TeV, while
\( m_{1,2} \) can be heavier

however, several constraints:

(i) \( m_3 \ll m_{1,2} \) \( \Rightarrow \) 2-loop contributions of \( m_1, m_2 \)
to \( \frac{d m_3^2}{d \tan \beta} \) may drive \( m_3^2 < 0 \) (Arkani-Hamed, Hanayama)

schematically:

\[
16 \pi^2 \frac{d m_3^2}{d \tan \beta} = 6 g_3^2 m_3^2 - \sum \frac{C_i m_{H^i}^2}{16 \pi^2} \frac{1}{C_i} \geq 0 \leq 0 \geq 0
\]

\( \Rightarrow \) when \( \frac{m_3}{m_{1/2}} \) \( \downarrow \) and \( M_{H^\pm} \)

\( \Rightarrow \) lower limit on \( \frac{m_3}{m_{1/2}} \) (depends on \( M_{H^\pm} \) and \( m_{1/2} \))

(ii) radiative electroweak symmetry breaking:

driven by stop loops \( \Rightarrow \) may not work due
to the previous effect

(in fact, proper REWSB is a stronger constraint
than \( m_{Z_0} > 150 \text{ GeV} \))

\( \rightarrow \) for \( M_{H^\pm} \leq 500 \text{ GeV} \), \( m_{1/2} > 5 \text{ TeV} \) requires \( \frac{m_3}{m_{1/2}} \geq 0.3 \)
$M_{1/2} = 400$ GeV
$0.3 \ m_2 < m_3 < m_2$  \hspace{1em} m_1 = m_2 \equiv m$
no off-diagonal entries

$m_{1/2}=400$ GeV, $m_1=m_2$, $0.3m_2<m_3<m_2$, no off, $A_0=0$
an explicit example: (dominant) D-term SUSY

the $U(1)$ changes responsible for the hierarchal structure of Yukawa matrices also determine the structure of sfermion mass matrices:

$$\hat{M}_{ij} = \frac{g x \langle D x \rangle X_i D_{ij} + m_F^2 C_{ij} \lambda |X_i - X_j|^2}{\equiv m_0^2}$$

if $m_F << m_0$ then $\begin{cases} X_3 = 0 \\ X_1, X_2 > 0 \end{cases} \Rightarrow m^2_3 - m_F^2 << m_{1,2}^2 - m_F^2$

however, generally $X_1 \neq X_2 \Rightarrow m_1 \neq m_2$

$\therefore$ the $\Sigma^2$ constraint pushes $m_1, m_2$ towards very large values ($\sim 100 \text{ TeV}$)
$M_{1/2} = 400$ GeV

Sfermion mass matrices from D-term SUSY
with $m_F/m_D = 0.3$

$M_{1/2} = 400$ GeV, 320 422, with off, $\varepsilon=0.3$, $A_0=0$
**Conclusions**

\[ FCNC = \text{strong constraint on non-flavour-blind SUSY} \]

- Off-diagonal entries in sfermion mass matrices have to be strongly suppressed in the (1,2) sector.
- Mass splittings on the diagonal are also strongly constrained, but the magnitude of the allowed splittings strongly depends on the Yukawa structure.

decoupling: If relax naturalness arguments, can make all sfermions very heavy - if want to keep fine-tuning at a reasonable level, inverted hierarchy is in principle a good alternative to universality, but \( \frac{m_3}{m_{1/2}} \geq 0.3 \) and generic hierarchical Yukawa couplings are disfavoured.

\( \text{SmK and Ek appear to favour models in which Vckm comes mainly from Yu, while} \) \( \text{Yd provides small rotations on down-type quarks} \).