Lepton flavour violating processes with full renormalisation group running*

Yasutaka Takanishi
ICTP, Trieste, Italy

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Overview:

- **MSSM + right-handed Majorana neutrinos**
- **Assumptions**
- **Numerical method**
- **Lepton flavour violating processes**
- **Results**
- **Conclusion**

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*S. T. Petcov, T. Shindou, Y. T., to appear*
MSSM + right-handed Majorana Neutrinos

The superpotential with three right-handed Majorana neutrinos:

\[
\mathcal{W} = \sum_{i,j=1,2,3} (Y_e)_{ij} \epsilon_{\alpha\beta} H_d^{\alpha} E^c_i L_j^{\beta} + (Y_d)_{ij} \epsilon_{\alpha\beta} H_d^{\alpha} D^c_i Q_j^{\beta} \\
+ (Y_u)_{ij} \epsilon_{\alpha\beta} H_u^{\alpha} U^c_i Q_j^{\beta} + \mu \epsilon_{\alpha\beta} H_1^{\alpha} H_2^{\beta} \\
+ (Y_\nu)_{ij} \epsilon_{\alpha\beta} H_u^{\alpha} N^c_i L_j^{\beta} + \frac{1}{2} (M_R)_{ij} N^c_i N^c_j,
\]

where

- \(L_i\): the chiral multiplet of a \(SU(2)_L\) doublet lepton
- \(E^c_i\): a \(SU(2)_L\) singlet charged lepton
- \(N^c_i\) a right-handed Majorana neutrino
- \(H_d\) and \(H_u\) two Higgs doublets with opposite hypercharge
- \(Q, U\) and \(D\) chiral multiplets of quarks of a \(SU(2)_L\) doublet and two singlets with different \(U(1)_Y\) charges.
- \(\epsilon_{\alpha\beta}\) is an anti-symmetric tensor with \(\epsilon_{12} = 1\).

Using see-saw mechanism (ignoring left-handed Majorana term) effective neutrino mass matrix becomes:

\[
M_{\text{eff}} \approx M_\nu^T M_R^{-1} M_\nu.
\]
Our Assumptions

- **Gauge unification at GUT scale** \((=2.3222 \times 10^{16} \text{ GeV})\)
  \[ g_1 = g_2 = g_3 = 0.71472 \]
- **All Yukawa coupling constants are real \(\Rightarrow\) no CP violation**
- **The right-handed Majorana Yukawa coupling constants are one at GUT scale:** degenerate \(M_R\): \(M_{R1} = M_{R2} = M_{R3}\).
- **All fermion masses and mixing angles are positive defined quantities.**
- **The first- and third-generation mixing angle in the neutrino sector:** \(\tan \theta_{13} = 0.1\).
- **soft SUSY breaking**
- **\(m\text{SUGRA}\) at GUT scale**
  - **Gauginos:** \(M_1 = M_2 = M_3\)
  - **Scalers:** \((m^2_L)_{ij} etc. = \delta_{ij}m^2_0\) and \(m^2_{Hu} = m^2_{Hd} = m^2_0\)
- **Low energy at \(M_Z\)**

<table>
<thead>
<tr>
<th>Experimental values</th>
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<tbody>
<tr>
<td>(m_u = 3.00 \text{ MeV})</td>
<td>(m_c = 1.20 \text{ GeV})</td>
<td>(m_t = 166.71 \text{ GeV})</td>
</tr>
<tr>
<td>(m_d = 6.75 \text{ MeV})</td>
<td>(m_s = 117.50 \text{ MeV})</td>
<td>(m_b = 4.25 \text{ GeV})</td>
</tr>
<tr>
<td>(m_e = 0.511 \text{ MeV})</td>
<td>(m_\mu = 105.64 \text{ MeV})</td>
<td>(m_\tau = 1.7770 \text{ GeV})</td>
</tr>
<tr>
<td>(V_{us} = 0.22)</td>
<td>(V_{cb} = 0.041)</td>
<td>(V_{ub} = 0.0035)</td>
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<tr>
<td>(\Delta m^2_{\odot} = 7.32 \times 10^{-5} \text{ eV}^2)</td>
<td>(\Delta m^2_{\text{atm}} = 2.6 \times 10^{-3} \text{ eV}^2)</td>
<td>(\tan^2 \theta_{\odot} = 0.41)</td>
</tr>
<tr>
<td>(\tan^2 \theta_{\odot} = 0.41)</td>
<td>(\tan^2 \theta_{\text{atm}} = 1.0)</td>
<td>(\tan^2 \theta_{\text{chooz}} = 0.01)</td>
</tr>
</tbody>
</table>

N.B. hierarchical left-handed neutrino solution with large mixing angles.

- **diagonal** \(Y_d\) and \(Y_e\) at GUT scale
Method of numerical computation

To calculate LFV processes we need all Yukawa coupling constants under our assumptions:

- \(3 + 3\) in Up-type quark sector (masses and CKM mixing angles)
- \(3\) in Down-type quark sector (masses)
- \(3\) in Charged lepton sector (masses)
- \(9\) in Dirac Yukawa coupling
- \(1\) in Majorana sector (universal \(M_R\) scale) as input parameter

For numerical computations:

- use one-loop MSSM renormalisation group from GUT scale to \(M_R\) (including \(M_R\))
- stop \(Y_\nu, M_R\) at \(M_R\)
- use \(M_{\text{eff}}\) running until \(M_{\text{susy-breaking}}\)

\[
16\pi^2 \frac{d}{dt} M_{\text{eff}} = \left\{-\frac{6}{5} g_1^2 - 6 g_2^2 + 6 \text{ Tr } (Y_u^\dagger Y_u) \right\} M_{\text{eff}}
+ (Y_e^\dagger Y_e)^T M_{\text{eff}} + M_{\text{eff}} (Y_e^\dagger Y_e).
\]

\(M_{\text{susy-breaking}} = \sqrt{m_{i_1} m_{i_2}}\).

- use non-susy RGE (usual ones) + \(M_{\text{eff}}\) until \(M_Z = 91.188\) GeV
We do these processes unless “best possible fit” becomes less than $10^{-2}$:

$$\text{b.p.f.} \equiv \sum \left[ \ln \left( \frac{\langle m \rangle}{m_{\text{exp}}} \right) \right]^2. \quad (1)$$

Here \( \langle m \rangle \) are the fitted masses and mixing angles, \emph{i.e.}, Yukawa coupling constants which we will use for the numerical calculation for LFV can fit the 17 low energy fermion quantities within an average of \( \exp\left(\sqrt{10^{-2}/17}\right) \approx 1.025 \).

\text{N.B.: Yukawa’s are \emph{only} one of a lot of possibilities!}
Lepton Flavour Violation processes

- (a) slepton-neutralino interaction
- (b) sneutrino-chargino interaction

An approximation for the additional contribution to the off-diagonal element of the slepton mass matrix:

\[
(\Delta m_L^2)_{ji} \approx -\frac{\ln(M_{GUT}/M_R)}{16\pi^2} \left(6m_0^2 + 2A_0^2\right) \left(Y_\nu^\dagger Y_\nu\right)_{ji} \tan^2 \beta
\]

Using this approximation we can naively estimate

\[
Br(l_j \rightarrow l_i\gamma) \approx \frac{\alpha^3}{G_F^2} \frac{\left(m_L^2\right)_{ji}^2}{m_S^8}
\]

But we do not really know what is really \(m_s\)? We have to be careful because such high powers! Later you will see that we can fit this parameter using full RGE calculation and log approx. of slepton mass matrix off-diagonal elements.
Branching ratios $\mu \rightarrow e\gamma$

![Graphs showing branching ratios $\mu \rightarrow e\gamma$](image_url)
Exact RG evolution effects from $\mu$ and $A_0$

Fixed $M_R = 10^{11}$ GeV and $m_0 = 200$ GeV with $\tan \beta = 10$ for $A_0 = 0$, $A_0 = \pm 1$ TeV and $A_0 = \pm 2$ TeV. Log app. does not depend on sign of $A_0$!
Fixed $M_R = 10^{12}$ GeV with $\tan \beta = 3$, $A_0 = 0$ for sign $\mu$ is positive/negative.
The Coannihilation region
The effective SUSY mass $m_S$

$$m_S^8 \simeq 0.5 \, m_0^2 \, M_{1/2}^2 \left( m_0^2 + 0.6 \, M_{1/2}^2 \right)^2$$
Conclusion

We have calculated lepton flavour violating processes using full RGEs including non-renormalisable RGE for the $M_{\text{eff}}$ under many assumptions (too much restrictive!)

- log approx. give order magnitude wise nice results, but in some (still) allowed region parameter space – small $m_0$ – this approx. is not so nice.
- using $m_8$ with log approx. we can estimate BR very easily $\Rightarrow$ you can save your time!

\[ \text{Br}(l_j \rightarrow l_i \gamma) \approx \frac{\alpha^3}{G_F} \left[ \frac{(m_{L}^2)_{ji}}{m_S^8} \right]^2, \]

with

\[ (\Delta m_{L}^2)_{ji} \approx \frac{-\ln(M_{\text{GUT}}/M_R)}{16\pi^2} \left( 6m_0^2 + 2A_0^2 \right) \left( Y_{\nu}^\dagger Y_{\nu} \right)_{ji} \tan^2 \beta \]

\[ m_S^8 \simeq 0.5 \ m_0^2 \ M_{1/2}^2 \left( m_0^2 + 0.6 \ M_{1/2}^2 \right)^2 \]