Effects of Right-handed Neutrino Supermultiplets on Electric Dipole Moments and other low energy observables

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Y. F., Phys. Rev. D69 (073009) 2004

Y. F., JHEP 02 (2005) 025
Plan of the Talk

Introduction

– Lepton Flavor Violation (LFV) in the MSSM;
– Electric Dipole Moments (EDMs) in the MSSM;
– MSSM with universal SUSY breaking parameters
– Neutrino anomaly \textit{seesaw mechanism}:

right-handed neutrino supermultiplet, neutrino $B$-term, $Y_{\nu}$;
Effects of the neutrino $B$-term on LFV
(Y. F., PRD69 (2004) 073009, hep-ph/0310055);
Effects of the neutrino $B$-term on the EDMs
(Y. F., PRD69 (2004) 073009, hep-ph/0310055);
Effects of the neutrino $B$-term on the Higgs mass parameters
(Y. F., JHEP 02 (2005) 025)

Conclusions
MSSM

\[ W = \mathcal{Y}_{l}^{ij} H_d \tilde{l}_{Ri}^c \tilde{L}_j + \mu H_u H_d \]

Without loss of generality: real diagonal \( \mathcal{Y}_{l}^{ij} \)

General form of soft supersymmetry breaking masses

\[ V_{soft} = (m_{L}^2)_{\alpha\beta} \tilde{L}_{\alpha} \tilde{L}_{\beta} + (m_{E}^2)_{\alpha\beta} \tilde{l}_{R\alpha} \tilde{l}_{R\beta} + A_{l}^{\alpha\beta} H_d \tilde{l}_{R\alpha} \tilde{L}_{\beta} \]

\( A_l \) and \( Y_l \) create mass terms: \( (m_{l_{RiL}}^2)_{\alpha\beta} \tilde{l}_{R\alpha} \tilde{l}_{L\beta} \),

where

\[ (m_{l_{RiL}}^2)_{\alpha\beta} = (A_l)_{\alpha\beta} \langle H_d \rangle - \mu \langle H_u \rangle \langle Y_l \rangle \delta_{\alpha,\beta} \]

with \( \frac{\langle H_u \rangle}{\langle H_d \rangle} = \tan \beta \)

Off-diagonal elements are LFV and can induce LFV rare decays.
J. Casas and Ibarra, hep-ph/0109161:

\[ \text{Br}(l_\alpha \rightarrow l_\beta + \gamma) \sim \frac{\alpha^3}{G_F} \frac{|(m_L^2)_{\alpha\beta}|^2}{m_{\text{susy}}^8} \tan^2 \beta, \]

and \[ \text{Br}(l_\alpha \rightarrow l_\beta + \gamma) \sim \frac{\alpha^3}{G_F^2} \frac{|(A_l)_{\alpha\beta}|^2}{m_{\text{susy}}^4}. \]

**Particle Data Group Booklet**

\[ \text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \quad \text{Br}(\tau \rightarrow e\gamma) < 2.7 \times 10^{-6} \]

and


\[ \text{Br}(\tau \rightarrow \mu\gamma) < 3.1 \times 10^{-7} \]
These bounds can be translated into bounds on the off-diagonal elements.

In particular, for $m_{\text{susy}} \simeq 200$ GeV,

\[
\frac{m^2_{\tilde{L}} e}{m^2_{\text{susy}}} < \text{few} \times 10^{-4} \quad \frac{m^2_{\tilde{R}} e}{m^2_{\text{susy}}} < \text{few} \times 10^{-6} \]

\[
\frac{m^2_{\tilde{L}} \tau}{m^2_{\text{susy}}} < 0.1 \quad \frac{m^2_{\tilde{R}} \tau}{m^2_{\text{susy}}} < \text{few} \times 10^{-2} \quad \frac{m^2_{\tilde{R}} \tau}{m^2_{\text{susy}}} < \text{few} \times 10^{-2} \quad \frac{m^2_{\tilde{L}} \tau}{m^2_{\text{susy}}} < 10^{-2} \]

Universal MSSM

At the GUT scale, \( m_L^2 = m_E^2 = m_0^2 \) and \( A_{ij}^{(l)} = a_0 Y_{ij}^{(l)} \)

Sources of CP-violation within MSSM with Universal Couplings \( \mu \quad a_0 \)

EDM of elementary particles \( \leftrightarrow \) CP-violation

\[
d_e < 1.5 \times 10^{-27} \text{ e cm} \quad d_\mu < 7 \times 10^{-19} \text{ e cm}
\]
\[
d_\tau < 5 \times 10^{-16} \text{ e cm} \quad d_n < 6 \times 10^{-26} \text{ e cm}
\]
For $m_{\text{susy}} \sim 200$ GeV, $d_e < 1.5 \times 10^{-27}$ e cm

[hep-ph/0211283]

\[ \text{Im} \mu = 0 \Rightarrow \text{Im} \frac{a_0}{m_{\text{susy}}} \lesssim 0.1 \]

\[ \text{Im}a_0 = 0 \Rightarrow \sin \phi_\mu \tan \beta \frac{\tan \beta}{10} \lesssim 10^{-3} \]

In general, there can be a cancellation between the contributions of $\mu$ and $a_0$ to $d_e$, allowing phases of order of 1.
M. V. Romalis et al., PRL 86 (2001) 2505:

\[ d_{Hg} < 2.1 \times 10^{-28} \text{ e cm} \quad \text{90 \%} \]

Combining the bounds on \( d_e \) and \( d_{Hg} \), one finds that such a cancelation is not possible and \( \text{Im}(a_0) \) and \( \phi_\mu \) are indeed very small.


In the following we will assume that \( \mu \) and \( a_0 \) are both real.
Neutrino anomalies: seesaw mechanism

$N_1, N_2$ and $N_3$ with $M_1, M_2, M_3 \gg m_{\text{susy}}$

$W = Y^{ij}_l H_d L^c_{Ri} L_j - Y^{ij}_u H_u N_i L_j + \mu H_u H_d +
\frac{1}{2} M^{ij} N_i N_j,$

$Y^{ij}_l = \text{diag}(Y_e, Y_\mu, Y_\tau) \quad M^{ij} = \text{diag}(M_1, M_2, M_3).$

At the GUT scale

$V_{\text{soft}} = m^2_\nu \tilde{N}^\dagger_i \tilde{N}_i + (B M_{ij} \tilde{N}_i \tilde{N}_j/2 + \text{h.c.})$

$- A^{ij}_\nu \tilde{N}_i H_u \tilde{L}_j,$

where $A^{ij}_\nu = a_0 Y^{ij}_\nu$
Neutrino anomalies: $Y_\nu$ has large non-diagonal elements
Non-diagonal elements of $Y_\nu \Rightarrow$ LFV left-handed slepton masses

\[
(m_L^2)_{\alpha\beta} = - \sum_k \frac{Y_{\nu}^k Y_{\nu}^k}{16\pi^2} [3m_0^2 + a_0^2] \log \frac{M_{\text{GUT}}^2}{M_k^2}.
\]

As we will see, if $B$-term is large there will be an additional effect.
Adding three right-handed neutrinos

**New sources of CP-violation**

\[ B_\nu \quad Y_\nu \]

**Effect of the phase of** $B_\nu$ **on EDMs:**

**Effect of the phases of** $Y_\nu$ **on EDMs**
J. R. Ellis, J. Hisano, S. Lola and M. Raidal, Nucl. Phys. B 621, 208 (2002);
J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Lett. B 528, 86 (2002);
J. R. Ellis, J. Hisano, M. Raidal and Y. Shimizu, Phys. Rev. D 66, 115013 (2002);
J. R. Ellis and M. Raidal, Nucl. Phys. B 643, 229 (2002);
Neutrino $B$-term: $\frac{1}{2}B_\nu \tilde{N} M \tilde{N}$

Right-handed neutrinos are electroweak singlets so $B_\nu$ can be much larger than $m_{EW}$. In principle it can be as large as $M_i$. 
$B_\nu$ gives a correction to the neutrino mass equal to

$$-\frac{g^2}{32\pi^2\cos^2\theta_W} \frac{2B_\nu Y_T \langle H_u \rangle^2}{m_\nu} \sum_j f(y_j) |Z_j Z|^2, \quad f(y_j) = \frac{\sqrt{y_j}[y_j-1-\log(y_j)]}{(1-y_j)^2},$$

where $y_j \equiv m_\nu^2/m_{\tilde{\chi}_j^0}^2$ and $Z_j Z \equiv Z_j 2 \cos \theta_W - Z_j 1 \sin \theta_W$ is the neutralino mixing matrix element that projects out the $\tilde{Z}$ eigenstate from the $j$th neutralino.


$$Y_T \langle H_u \rangle^2 \frac{Y_\nu}{M} \sim m_\nu \rightarrow B_\nu/m_{susy} < 10^3$$

What if $Y_T \langle H_u \rangle^2 \frac{Y_\nu}{M} \ll m_\nu$?

$B$-term gives radiative correction to $m_{H_u}^2$ and $B_H$

⇒ Stronger bound on $B_\nu$

Y. Farzan, JHEP 02 (2005) 025
What is the theoretical prediction for $B_\nu$?

\[ \int d^4 \theta (1 + B_\nu \theta^2)(1 + B_\nu^* \bar{\theta}^2)N^\dagger N + \int d^2 \theta MNN \]

$\Rightarrow B_\nu M_i \tilde{N}_i \tilde{N}_i$ with $B_\nu \sim A_\nu \sim m_{\text{susy}}$

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One way to have large $B$-term while other supersymmetric parameters are below TEV:
Let us introduce a field $X$ from the hidden sector and assign it a lepton number equal to 2.

\[ \int d^2 \theta XNN \]

Large $\langle \tilde{X} \rangle \Rightarrow$ Large Majorana Mass term

Large $\langle F_X \rangle \Rightarrow$ Large $B$-term
\[-i \Delta m_{H_u}^2 = 2 \sum_k \int \frac{M_k^2 \text{Re}[B_{\nu} \sum_i (Y_{\nu})_{ki} (A_{\nu}^*)_{ki}] d^4k}{k^2(k^2-M_k^2)^2} = -i 2 \sum_k \text{Re} \left[ B_{\nu} \text{Tr}(Y_{\nu} A_{\nu}^*) \right].\]

\[-i \Delta b_H = - \sum_k \int \frac{M_k^2 \text{Tr}[(Y_{\nu})_{ki} (Y_{\nu}^*)_{ki}] \mu d^4k}{k^2(k^2-M_k^2)^2} = \frac{i B_{\nu} \mu \text{Tr}[Y_{\nu} Y_{\nu}^*]}{(4\pi)^2}.\]
Dimensional analysis ⇒ No correction to cubic or quartic terms

\[ V = (|\mu|^2 + m^2_{H_u} + \Delta m^2_{H_u})|H^0_u|^2 + (|\mu|^2 + m^2_{H_d})|H^0_d|^2 + [(b_H + \Delta b_H)H^0_u H^0_d + H.C.] + \frac{g^2 + g'^2}{8}(|H^0_u|^2 - |H^0_d|^2)^2. \]

Requiring \( m_Z^2 = (g^2 + g'^2)(\langle H_u \rangle^2 + \langle H_d \rangle^2)/2 \) and \( \partial V/\partial H^0_u = \partial V/\partial H^0_d = 0 \),

we find

\[ |\mu|^2 + m^2_{H_d} = |b_H + \Delta b_H| \tan \beta - (m_Z^2/2) \cos 2\beta \]

and

\[ |\mu|^2 + m^2_{H_u} + \Delta m^2_{H_u} = |b_H + \Delta b_H| \cot \beta + (m_Z^2/2) \cos 2\beta \]

where \( \tan \beta = \langle H_u \rangle / \langle H_d \rangle \).

Assuming \( |\mu|^2 \sim m^2_{H_u} \sim m^2_{\text{susy}} \), we find

\[ |b_H - B_{\nu} \mu \frac{\text{Tr}[Y_{\nu} Y_{\nu}^\dagger]}{16\pi^2}| \sim m^2_{\text{susy}} / \tan \beta \Rightarrow B_{\nu} \frac{\text{Tr}[Y_{\nu} Y_{\nu}^\dagger]}{16\pi^2} < m_{\text{susy}} \]
The one-loop effect is suppressed by $B_{\nu m_{\text{susy}}}/M^2$. However the neutrino $B$-term can affect the LFV rare decays as well as EDMs through corrections to $A_{\ell}$ and $\Delta m_{ij}^2$. 
Diagrams contributing to the slepton masses.

\[ m^2_{(2)\alpha\beta} = -2 \sum_k Y_{\nu\alpha}^k Y_{\nu\beta}^k \Re[a_0 B_{\nu}] \frac{\text{Re}[a_0 B_{\nu}]}{(4\pi)^2} \]
Diagram contributing to $A_\ell$.

$$-iA_{ij}^i = -ia_0 Y_l^{jj} \delta_{ij} - \frac{i}{(4\pi)^2} Y_l^{jj} (Y_{\nu k}^{kj})^* Y_{\nu}^{ki} B_{\nu}$$
As it is shown in hep-ph/0502022 by E. J. Chun, A. Masiero, A. Rossi and S. K. Vempati, an imaginary $B_\nu$ induces an imaginary correction to $A_u$ through a correction to $\langle F^\dagger_{H_u} H_u \rangle$:

$$\delta A_u = \frac{1}{16\pi^2} Y_u \text{Tr}[Y^\dagger_\nu B_\nu Y_\nu].$$
So, \( d_l \propto \text{Im}[\langle A_l \rangle_{ii}] \)
Combining the results

\[ \text{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}, \ \text{Br}(\tau \to e\gamma) < 2.7 \times 10^{-6}, \ \text{Br}(\tau \to \mu\gamma) < 3.1 \times 10^{-7} \]
give
\[ \text{Re}(a_0 B^*_\nu) (Y^{\dagger}_\nu Y_\nu)_{\mu e} < \text{few} \times 10^{-4} \]
and
\[ \text{Re}(a_0 B^*_\nu) (Y^{\dagger}_\nu Y_\nu)_{\tau e} < 0.1 \]
\[ \text{Re}(a_0 B^*_\nu) (Y^{\dagger}_\nu Y_\nu)_{\tau \mu} < \text{few} \times 10^{-2}. \]

However, these bound are only on the off-diagonal.
On the other hand, the dependence of \( b_H \) on \( B_\nu \) is through
\[ \sum_{k l} |(Y_\nu)_{k l}|^2 \]
which is larger than maximum \( |(Y_\nu)_{k l}|^2 \).
Thus, bounds from LFV rare decay and the condition of electroweak symmetry
breaking are complementary.

\[ d_e < 1.4 \times 10^{-27} \ \text{e cm implies Im}(B_\nu) \sum_i |(Y_\nu)_{ie}|^2/(16\pi^2) < 0.1m_{\text{susy}} \]
which is again complementary to the bound we found
What do we expect, if $B$-term is the only source of CP-violation?

$$d_\tau/(m_\tau \sum_k |Y_{\nu}^{k\tau}|^2) = d_\mu/(m_\mu \sum_k |Y_{\nu}^{k\mu}|^2) =$$
$$d_e/(m_e \sum_k |Y_{\nu}^{k\ell}|^2)$$

$$d_e \sim 10^{-27} \text{ e cm} \rightarrow d_\mu \sim 10^{-25} \text{ e cm} \text{ which can be tested}$$

in the proposed experiments

(Storage ring of nuFactory)

J. Aysto et. al., hep-ph/0109217
Now let us relax the assumption that $\mu$ and $a_0$ are real.
Then we will have 4 independent complex parameters to fit the EDM data:

Phases of $\mu$ and $A_e = a_0 Y_e + Y_e (Y_\nu B_\nu Y_\nu)_{ee}/(16\pi^2)$ give corrections to $d_e$.

Phases of $\mu$, $A_d = Y_d a_0 = A_s (m_d/m_s)$ and $A_u = a_0 Y_u + Y_u \text{Tr}\{Y_\nu^\dagger B_\nu Y_\nu\}/(16\pi^2)$ give corrections to $d_n$ and $d_{Hg}$.

This opens the possibility of $\phi_\mu \sim O(1)$.

Y. F., work in progress
Conclusions

The condition of electroweak symmetry breaking \( \rightarrow |b_H - B_\nu \mu \frac{\text{Tr}[Y^\dagger Y]}{16\pi^2}| \sim \frac{m^2_{\text{susy}} \tan \beta}{16\pi^2} \)

An upper bound on \( |B_\nu| \) one or two orders of magnitude stronger than the previous one.

Unlike the bound from radiative correction to \( m_\nu \), this bound does not \( M \).

Even within this bound, the effect of \( B_\nu \) on both LFV left-hand slepton masses \((m^2_L)_{\alpha\beta} \ (\alpha \neq \beta)\) and EDMs can be dominant.

The effect of an imaginary \( B_\nu \) on the EDM of charged lepton, \( l_i \)
\[
d_i \sim 10^{-27} \frac{\text{Im}(B_\nu)}{m_L} Y^\dagger Y \left( \frac{200 \text{ GeV}}{m_L} \right)^2 \frac{m_{l_i}}{m_e} \text{ e cm}
\]

Radiative LFV correction induced due to \( B_\nu \)
\[
m^2_L = -2\frac{\text{Re}(B_\nu a_0)}{16\pi^2} Y^\dagger Y
\]
There are two ways to suppress LFV rare decays:

(I) The off-diagonal elements are much smaller than the diagonal elements

At mass insertion approximation:

\[ \text{Br}(l_\alpha \rightarrow l_\beta + \gamma) \sim \frac{\alpha^3}{G_F^2} \frac{|(m^2_L)_{\alpha\beta}|^2}{m^8_{\text{susy}}} \tan^2 \beta \]

(II) The correction to \(m^2_L\) due to \(B_\nu\) is much larger than \(m_{\text{susy}}\).

At this range \(\text{Br}(l_\alpha \rightarrow l_\beta + \gamma) \sim \frac{\alpha^3}{G_F^2 m^4_L} \tan^2 \beta\)

We cannot derive any conclusive bound on \(B_\nu\) only from LFV rare decay.
What about a correction proportional to $|B_\nu|^2$?

Surprisingly, the two diagrams cancel each other.
What about higher loop orders?

The “eyeglasses-diagram.”

This diagram is equal to $|B_\nu|^2 \left[ \text{Tr}\{Y_\nu^\dagger Y_\nu\}/16\pi^2 \right]^2$.

Diagrams with a different topology have a different dependence on $Y_\nu$ and cannot cancel the effect of the “eyeglasses-diagram.” This demonstrates that the cancellation at one-loop level is completely accidental.