B MESONS AND FORM FACTORS

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THE CKM MATRIX ELEMENT $V_{cb}$

$B$ Physics probes the flavor structure of $SM$:

Exclusive decays of $B \rightarrow$ determination of $|V_{cb}|$

$$\bar{B}^0 (b\bar{d}) \rightarrow D^+(c\bar{d}) e^- \bar{\nu}_e$$

Bound-state effects parametrized by form factors

$$\frac{d\Gamma(\bar{B} \rightarrow D^{(*)} l\bar{\nu})}{dw} = \mathcal{K}|V_{cb}|^2 \left\{ \begin{array}{l} (w^2 - 1)^{1/2} |\mathcal{F}^*(w)|^2 \\ (w^2 - 1)^{3/2} |\mathcal{F}(w)|^2 \end{array} \right.$$ 

$$w \equiv \mathbf{v} \cdot \mathbf{v}' = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

Measurements of $|V_{cb}| \mathcal{F}^*(w) \rightarrow$ extrapolation at zero recoil $w = 1$

$$\mathcal{F}^*(1) = \eta_A [1 + \delta_{1/m_Q^2}] = 0.91 \pm 0.04$$

$$\left\{ \begin{array}{l} \mathcal{F}^*(1)|V_{cb}| = (38.2 \pm 0.9) \times 10^{-3} \\ \rho^2 = 1.56 \pm 0.13 \end{array} \right. \quad [\chi^2/d.o.f. = 1.83]$$

$$\Rightarrow |V_{cb}|_{excl} = (42.0 \pm 1.1_{exp} \pm 1.9_{th}) \times 10^{-3}$$
Inclusive determination $B \rightarrow X_c l \bar{\nu}$:

$$|V_{cb}|_{incl} = (41.0 \pm 0.5_{exp} \pm 0.9_{th}) \times 10^{-3}$$

- Exclusive and inclusive determinations: very different hadronic problem
  
  $\Rightarrow$ convergence?

- Large dispersion of data in the $(F(1)|V_{cb}|, \rho^2)$ plane
  
  $\Rightarrow$ strong correlation
THE HEAVY QUARK SYMMETRY (HQS)

Two length scales within heavy-light quarkonium $Q\bar{q}$:

- hadronic length scale: $R_{\text{had}} \sim 1/\Lambda_{\text{QCD}}$
- Compton wave length of the heavy quark: $\lambda_Q \sim 1/m_Q$

$$m_Q \gg \Lambda_{\text{QCD}} \Rightarrow \lambda_Q \ll R_{\text{had}}$$

The heavy quark and the light cloud (light quark and gluons) interacts via soft gluons ($q^2 \sim \Lambda_{\text{QCD}}^2$)

$\Rightarrow$ When $m_Q \rightarrow \infty$, the heavy quark $Q$ is a static color source.

Configuration of the light cloud does not depend on:

- $m_Q$ (flavor symmetry) $\Rightarrow SU(2N)$
- $s_Q$ (spin symmetry)

- For $j$ fixed: degenerate doublets $J = j \pm s_Q$:

  $l = 0$:
  $$j^P = \frac{1^-}{2} = \begin{pmatrix} D &= 0^{-}_{1/2} \\ D^* &= 1^{-}_{1/2} \end{pmatrix}$$

  $l = 1$:
  $$j^P = \frac{1^+}{2} = \begin{pmatrix} D_0^* &= 0^+_{1/2} \\ D_1^* &= 1^+_{1/2} \end{pmatrix} \quad j^P = \frac{3^+}{2} = \begin{pmatrix} D_1 &= 1^+_{3/2} \\ D_2^* &= 2^+_{3/2} \end{pmatrix}$$
Form factors → universal functions: IW functions
(large distance physics of the light cloud)

\[ \bar{B} \rightarrow D^{(*)} l \nu \ (l = 0) : \]

6 form factors → elastic Isgur-Wise function \( \xi(w) \).

\[ \bar{B} \rightarrow D^{**} l \nu \ (l = 1) : \]

14 form factors → 2 functions \( \tau_{1/2}(w) \) and \( \tau_{3/2}(w) \).

HQS: model-independent normalisation \( \xi(1) = 1 \)

when \( m_Q \rightarrow \infty \): \( F(1) = F^*(1) = \xi(1) = 1 \)

⇒ model-independent derivation of \( |V_{cb}| \)

HQS is broken by corrections of order \( O(\alpha_s) \) and \( O(\Lambda_{QCD}/m_Q) \):

\[
F(1) = \xi(1) + c_V(\alpha_s) + (\ldots) \frac{\Lambda_{QCD}}{m_Q} + (\ldots) \left( \frac{\Lambda_{QCD}}{m_Q} \right)^2 + \ldots
\]

\[
F^*(1) = \xi(1) + c_A(\alpha_s) + 0 \frac{\Lambda_{QCD}}{m_Q} + (\ldots) \left( \frac{\Lambda_{QCD}}{m_Q} \right)^2 + \ldots
\]
DERIVATION OF THE GENERIC SUM RULE

Systematic method to obtain Sum Rules


$T-$product of 2 heavy-heavy currents:

$$T_{fi}(q^2) = i \int d^4 x e^{-iq\cdot x} < B_f(v_f)|T\{J_f(0)J_i^+(x)\}|B_i(v_i) >$$

$$J_{i,f}(x) = \bar{c}(x)\Gamma_{i,f}b(x)$$

- **Completeness** in the 2 chronological orders and OPE.

$\Rightarrow$ Generic Sum Rule:

$$\sum_D < B_f(v_f)|J_f(0)|D(v') > < D(v')|J_i(0)|B_i(v_i) >$$

$$= < B_f(v_f)|\bar{b}(0)\Gamma_f P_+(v')\Gamma_i b(0)|B_i(v_i) >$$

$$P_+(v') = \frac{1+y'}{2}$$

- **The Trace Formalism** (Bjorken and Falk, 1992):

  - matrix elements:

    $$< D(v')|\bar{h}_v^{(c)}\Gamma h_v^{(b)}|B(v) > = \xi(w)Tr[\bar{D}(v')\Gamma B(v)]$$

  - spin wave functions $J_{f}^{\pi}$ (matrices $4 \times 4$):

    $$0_{1/2}^{-} : B(v) = P_+(-\gamma_5)$$

    $$1_{1/2}^{-} : B(v) = P_+c^\mu \gamma_\mu$$
Heavy quark limit and Trace formalism \((m_Q \to \infty)\):

\[
\sum_{D=P,V} \sum_n \xi^{(n)}(w_i) \xi^{(n)}(w_f) \text{Tr}[\overline{\mathcal{B}}_f(v_f) \Gamma_f \mathcal{D}^{(n)}(v')] \text{Tr}[\mathcal{D}^{(n)}(v') \Gamma_i \mathcal{B}_i(v_i)] + \text{other excited states}
\]

\[
= -2 \xi(w_{if}) \text{Tr}[\overline{\mathcal{B}}_f(v_f) \Gamma_f P_+(v') \Gamma_i \mathcal{B}_i(v_i)]
\]

\(w_i = v_i \cdot v'\), \(w_f = v_f \cdot v'\), \(w_{if} = v_i \cdot v_f\)

- **non-forward amplitude**: \(v_i \neq v_f\)

\[\Rightarrow \xi(w_{if}) \neq 1\]

- \((w_i, w_f, w_{if})\): **compact** parameter space

\[
w_i \geq 1 \quad w_f \geq 1
\]

\[
w_i w_f - \sqrt{w_i^2 - 1} \sqrt{w_f^2 - 1} \leq w_{if} \leq w_i w_f + \sqrt{w_i^2 - 1} \sqrt{w_f^2 - 1}
\]

\[\Rightarrow \text{relevant boundary conditions: rigorous results.}\]
CONSTRAINTS ON THE DERIVATIVES OF $\xi(w)$

In the infinite quark mass limit of the generic Sum Rule:

$$L_{Hadrons}(w_i, w_f, w_{if}) = R_{OPE}(w_i, w_f, w_{if})$$

By introducing all the intermediate states $J_j^P$ with:

- **vector and axial** currents $\Gamma = \not{p}, \not{p}\gamma_5$

- **pseudoscalar** initial and final states $B = P_+(-\gamma_5)$

**Generalized Bjorken and Uraltsev Sum Rules:**

- $\xi^{(l)}(1) = \frac{1}{4}(-1)^l \prod \{4 \sum_{2l+1} |\tau_{l+1/2}^{(l)}(n)(1)|^2 + \sum_n |\tau_{l-1/2}^{(l-1)}(n)(1)|^2 + \sum_n |\tau_{l+1/2}^{(l)}(n)(1)|^2\}$

$\Rightarrow l = 1$, Bjorken Sum Rule:

$$\rho^2 = \frac{1}{4} + \sum_n |\tau_{1/2}^{(n)}(1)|^2 + 2\sum_n |\tau_{3/2}^{(n)}(1)|^2$$

$\Rightarrow$ bound $\rho^2 \geq \frac{1}{4}$

$\Rightarrow \xi(w)$ is an alternate series in powers of $(w - 1)$

- $\frac{1}{2l+1} \sum_n |\tau_{l+1/2}^{(l)}(n)(1)|^2 - \frac{1}{4} \sum_n |\tau_{l-1/2}^{(l)}(n)(1)|^2 = \frac{1}{4} \sum_n |\tau_{l-1/2}^{(l-1)}(n)(1)|^2$

$\Rightarrow l = 1$, Uraltsev Sum Rule:

$$\sum_n |\tau_{3/2}^{(n)}(1)|^2 - \sum_n |\tau_{1/2}^{(n)}(1)|^2 = \frac{1}{4}$$

$\Rightarrow$ much stronger bound $\rho^2 \geq \frac{3}{4}$
Bounds on the derivatives of $\xi(w)$ at zero recoil $w = 1$:

- $l = 1$: slope $\rho^2 \geq \frac{3}{4}$
- $l = 2$: curvature $\sigma^2 \geq \frac{5}{4} \rho^2 \geq \frac{15}{16}$
- for any $l$:

$$(-1)^l \xi^{(l)}(1) \geq \frac{2l+1}{4} (-1)^{l-1} \xi^{(l-1)}(1)$$

$$\geq \frac{(2l+1)!!}{2^{2l}}$$

Improved bound on the curvature $\sigma^2$ of $\xi(w)$:

$$\frac{4}{3} \rho^2 + (\rho^2)^2 - \frac{5}{3} \sigma^2 + \sum_{n \neq 0} |\xi^{(n)}(1)|^2 = 0$$

$$\Rightarrow \sigma^2 \geq \frac{1}{5} [4 \rho^2 + 3 (\rho^2)^2]$$

$$\Rightarrow \sigma^2 \geq \frac{15}{16} \text{ when } \rho^2 \geq \frac{3}{4}$$

Actually, the second term $\frac{3}{5} (\rho^2)^2$ is the leading term in the non-relativistic limit when the light quark mass is large.
SUM RULES FOR SUBLEADING FORM FACTORS

Sum Rule in the heavy quark limit $m_Q \rightarrow \infty$:

$$\sum_D < B_f(v_f)|J_f(0)|D(v')> < D(v')|J_i(0)|B_i(v_i) >$$

$$= < B_f(v_f)|\bar{b}(0)\Gamma_f P_+(v')\Gamma_i b(0)|B_i(v_i) >$$

Non-pertubative corrections $O(1/m_Q)$ of the matrix elements:

$$< M'(v')|\bar{Q}'\Gamma Q|M(v) >$$

$\Rightarrow$ Corrections $\bullet$ to the current (Falk and Neubert, 1992)

$\bullet$ to the wave function
Non-pertubative corrections \(O(1/m_Q)\):

\[
L_0 + \frac{1}{2m_b} L_b + \frac{1}{2m_c} L_c = R_0 + \frac{1}{2m_b} R_b + \frac{1}{2m_c} R_c
\]

\[\Rightarrow\] Correction terms: subleading \(IW\) functions

- Corrections \(O(1/m_b)\) between ground states \((l = 0)\):
  - 6 functions \(L_i(w)\) \((i = 1, \ldots, 6)\) (Falk et Neubert, 1992)

\[
< D(\frac{1}{2}^-)(v')|\bar{c} \Gamma b|B(v)> = -\xi(w)Tr \left\{ \overline{D}(v')\Gamma B(v) \right\}
- \frac{1}{2m_b} Tr\left\{ \overline{D}(v')\Gamma \left[ P_+(v)L^B_+(w) + P_-(v)L^B_-(w) \right] \right\}
\]

\[
0^-_{1/2} : \begin{cases}
L_+ = -(\gamma_5)L_1(w) \\
L_- = -(\gamma_5)L_4(w)
\end{cases}
1^-_{1/2} : \begin{cases}
L_+ = \epsilon_v L_2(w) + (\epsilon_v \cdot v')L_3(w) \\
L_- = \epsilon_v L_5(w) + (\epsilon_v \cdot v')L_6(w)
\end{cases}
\]

- Corrections \(O(1/m_b)\) between excited states \((l=1)\):
  - Corrections to the current: 7 functions \(\zeta_i\) et \(\tau_i\)
  - Magnetic corrections to the Lagrangian: 5 functions \(\chi_i\) et \(\eta_i\)
  - Kinetic corrections to the Lagrangian: 2 functions \(\chi_{ke}\) et \(\eta_{ke}\)
Generic Sum Rule at $O(1/m_b)$:

\[ G_0 + E_0 = R_0 \quad , \quad O(1/m_Q)^0 \]
\[ G_b + E_b = R_b \quad , \quad O(1/m_b) \]

$G_0$ and $G_b$ : ground state $\frac{1}{2}^-$

$E_0$ and $E_b$ : excited states $\frac{1}{2}^+$ and $\frac{3}{2}^+$

$R_0$ and $R_b$ : OPE

⇒ New Sum Rules for subleading quantities.

• $B$ meson Sum Rule ($\Delta E_j = E_{j^+} - E_{1/2^-}$ with $j = \frac{1}{2}, \frac{3}{2}$):

\[ L_A(w) = -6 \sum_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w) \]

• $B^*$ meson Sum Rule:

\[ -L_5(w) + (w + 1)L_6(w) = 2 \sum_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w) \]
\[ -4(w + 1) \sum_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w) \]

$(w_i, w_f, w_{if}) = (w, 1, w) \Rightarrow only \ l = 0 \ , \ l = 1 \ contribute$
\( L_i(w) \) \((i = 4, 5, 6)\) not independent (equations of motion):

\[
L_4(w) = -\overline{\Lambda} \xi(w) + 2\xi_3(w)
\]

\[
L_5(w) = -\overline{\Lambda} \xi(w)
\]

\[
L_6(w) = -\frac{2}{w+1}[\overline{\Lambda} \xi(w) + \xi_3(w)]
\]

\( \overline{\Lambda} = m_B - m_b \): energy of the light cloud in the heavy meson

(fundamental parameter of HQET)

\( \overline{\Lambda} \xi(w) \) and \( \xi_3(w) \) parametrize the current corrections

\[
\overline{\Lambda} = \frac{2}{\xi(w)} \left[ \sum_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w) + (w + 1) \sum_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w) \right]
\]

\[
\xi_3(w) = (w + 1) \sum_n \Delta E_{3/2}^{(n)} \tau_{3/2}^{(n)}(1) \tau_{3/2}^{(n)}(w) - 2 \sum_n \Delta E_{1/2}^{(n)} \tau_{1/2}^{(n)}(1) \tau_{1/2}^{(n)}(w)
\]

At zero recoil \( w = 1 \):

- Voloshin sum rule (1992):

\[
\frac{\overline{\Lambda}}{2} = \sum_n \Delta E_{1/2}^{(n)} |\tau_{1/2}^{(n)}(1)|^2 + 2 \sum_n \Delta E_{3/2}^{(n)} |\tau_{3/2}^{(n)}(1)|^2
\]

- Orsay sum rule (Le Yaouanc et al., 2000):

\[
\xi_3(1) = 2 \sum_n \Delta E_{3/2}^{(n)} |\tau_{3/2}^{(n)}(1)|^2 - 2 \sum_n \Delta E_{1/2}^{(n)} |\tau_{1/2}^{(n)}(1)|^2
\]
Phenomenological applications

- The function (Grinstein et Ligeti, 2001):
  \[ \eta(w) \equiv \frac{\xi_3(w)}{\xi(w)} = \eta(1) + \eta'(1)(w - 1) + \frac{1}{2} \eta''(1)(w - 1)^2 \ldots \]

- The subleading Isgur-Wise function \( L_4 \):
  \[ L_4(w) = -6 \sum_n \Delta E_{1/2}^{(n)} T_{1/2}^{(n)}(1) T_{1/2}^{(n)}(w) = L_4(1) + L'_4(1)(w - 1) + \ldots \]
  \( \Rightarrow \) These functions appear at \( O(1/m_Q) \) in the decay rates

Class of quark models \( à la \) Bakamjian-Thomas:

\[ \xi(w) = \left( \frac{2}{w + 1} \right)^{2\rho^2} \]

\[ \rho^2 = 1.02 \]

\[ \tau^{(0)}_{3/2}(w) = \tau^{(0)}_{3/2}(1) \left( \frac{2}{w + 1} \right)^2 \sigma^2_{3/2} \]

\[ \tau^{(0)}_{1/2}(w) = \tau^{(0)}_{1/2}(1) \left( \frac{2}{w + 1} \right)^2 \sigma^2_{1/2} \]

\[ \begin{cases} 
\tau^{(0)}_{3/2}(1) = 0.54 \\
\sigma^2_{3/2} = 1.50 
\end{cases} \]

\[ \begin{cases} 
\tau^{(0)}_{1/2}(1) = 0.22 \\
\sigma^2_{1/2} = 0.83 
\end{cases} \]

From our \( SR \), we obtain:

\[ \tilde{\Lambda} = 0.513 \pm 0.015 \]

\[ L_4(1) = -0.123 \quad L'_4(1) = 0.102 \]

\[ \eta(1) = 0.376 \quad \eta'(1) = -0.003; \quad \eta''(1) = 0.0003 \]
CONCLUSIONS

• Heavy quark limit Sum Rules provide us bounds on the derivatives of $\xi(w)$

  $\Rightarrow$ these bounds must be taken into account to obtain $|V_{cb}|$.

• Heavy quark limit Sum Rules have been extended to subleading quantities using the OPE and the non-forward amplitude (when $v_i \neq v_f$).

• Some subleading form factors (of current type) can be expressed in terms of leading measurable quantities.

• Consistency of our rigorous results with input of $IW$ functions of $n = 0$ states drawn from Quark Models à la Bakamjian-Thomas with $QCD$ Sum Rule results.

• Continuation of the program for the other subleading form factors $L_1$, $L_2$ and $L_3$. 