The hydrogen atom for quantum gravity

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This talk is about a discovery that should have been made decades ago, but it was only made very recently:

In an unusual basis, the scattering matrix $S$ describing the evolution of a black hole can be written in an almost closed form. Unless we choose proper boundary conditions, firewalls arise, violating the basic principles of GR. Every point on the horizon has to be identified with its antipode. The quantum black hole has no interior! Unlike the classical black hole in its usual description.
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Using an algebra derived some 30 years ago, we had all knowledge needed to derive these facts.
All needed to be done, was to handle the black hole the way the hydrogen atom was handled when Schrödinger discovered his equation. Matter in the form of spherical harmonics has to be inserted into this algebra to make exactly clear what happens.

These very simple arguments lead to surprising new physics!
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Hartle-Hawking vacuum:

\[
|HH\rangle = C \sum_{E,n} e^{-\frac{1}{2} \beta E} |E,n\rangle_I |E,n\rangle_{II}
\]

\(I = \text{outside}\)
\(II = \text{inside}\) [?]
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**Superstring theory**

finds something that looks like a black hole, and it is described in terms of pure quantum states!

"microstates".

Claim: that thing is not well understood . . .

How are these microstates related to vacuum fluctuations?
But then:

**Difficulties regarding entanglement and no-cloning:**

Almheiri, Marolf, Polchinski, Sully (AMPS):  *If we start in a pure state, and consequently Hawking radiation is in a pure state, then that pure state must be entangled also with earlier radiation.* Therefore it can’t be in the state originally considered by Hawking.

They want to put the black hole in a *pure state, living in region I only*. Such a state differs from the HH vacuum as seen by local observer. This difference produces a curtain of infinitely energetic particles along the future event horizon: a firewall . . .

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*And yes, the price is: there will be new physics*

String theory is *far from fool-proof*
Let us begin by getting the maximum out of *standard Einstein General Relativity* and *standard Quantum Mechanics*

Then, one has to discover 3 *important things*:

(i) Particles going into a black hole, interact *gravitationally* with particles going out. If you want to describe these as pure quantum states, you cannot ignore that (only if they are in mixed states, you may) because this grav. force is *strong* and because it *diverges*; all microstates used are ⊥ to the HH state

(ii) This force generates an algebra that is *linear* in the coordinates & momenta of the in- and out-particles, and therefore, you can superimpose solutions! → Make an expansion in *spherical harmonics*.

(iii) We have always been wrong in thinking that the mirror particles of the Hawking particles, going into the BH, will get lost. The mirror particles reappear at the other side!
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If we ignore any **one** of these **3** points, we fail to understand what happens
The gravitational backreaction:
calculate the grav. field due to a fast moving particle,
simply Lorentz boost the field of a particle at rest:

\[
\delta u^- (\tilde{x}) = -4G p^- (\tilde{x}') \log |\tilde{x} - \tilde{x}'| .
\]

(Rindler space simplification)
An extra particle $\delta(\text{in})$ with momentum $\delta p^-$ going in interacts gravitationally with the out-going particles, causing a shift $\delta u^-$:

\[
\delta u^-(\Omega) \approx -4G \delta p^-(\Omega') \log |\Omega - \Omega'| , \quad \Omega \equiv (\theta, \varphi) .
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In 1986, we argued as follows:

1. Consider ONE pure quantum state of a black hole, defined by ALL of its initial, in-going particles. Call this state \( |BH(1)\rangle_{in} \)

2. Assume that this quantum state evolves into a pure final state of out-going particles, \( |BH(1)\rangle_{out} \).
   Of course, \( |BH(1)\rangle_{out} \) consists of a superposition of a very large number of elementary particle states. But, it is pure.

3. Now add a single particle entering the horizon with momentum \( p_{in}^- \) at \( \Omega = (\theta, \varphi) \), so that we modify the in-state: \( |BH(2)\rangle_{in} = |BH(1) + p_{in}^-\rangle_{in} \)

4. All particles in \( |BH(1)\rangle_{out} \) are dragged by an amount \( \delta u_{out}^- \), given by our drag formula:
   \[
   |BH(2)\rangle_{out} = e^{-i \int d^2\Omega P^+(\Omega) \delta u_{out}^- (\Omega)} |BH(1)\rangle_{out}
   \]

5. Repeat this by adding or removing in-particles, as many as you want. This way, we get the \( |BH(3)\rangle_{out} \) that correspond to any conceivable in-state. Call the new in-states \( |BH(1), p_{in}^- (\theta, \varphi)\rangle_{in} \). Then
   \[
   |BH(3)\rangle_{out} = e^{-i \int d^2\Omega d^2\Omega' P^+_{out} (\Omega) f(\Omega, \Omega') p_{in}^- (\Omega') } |BH(1)\rangle_{out}
   \]

6. Thus, notice that this gives a unitary (?) S-matrix, in terms of the states \( |p^- (\Omega)\rangle_{in} \) and \( |p^+ (\Omega)\rangle_{out} \).
   * You must allow all values for \( u^\pm (\Omega) \) and \( p^\pm (\Omega) \)!!
The sum of all momenta going in, drags the positions of all out-going things.

This leads to a linear algebra relating the momenta of all in-going particles combined, to the average positions of all out-going ones.

These obey simple commutation relations,

$$\left[ \frac{1}{N} \sum_i u_i(\Omega), \sum_j p_j(\Omega') \right] = i\delta^2(\Omega, \Omega'), \quad \Omega \equiv (\theta, \varphi)$$

Integrate all particles that cross the horizon.
This leads to a local algebra for the total momentum densities and the average position operators, as distributions on the horizon:

\[ u_{\text{out}}^{-} (Ω) = \int d^2 Ω' \ f(Ω, Ω') \ p_{\text{in}}^{-}(Ω') , \]

This equation allows us to transform in-going particles into out-going particles

\[ (ΔΩ - 1) f(Ω, Ω') = -8πG \ δ^2(Ω, Ω') \]

\[
\begin{align*}
(ΔΩ - 1)u_{\text{out}}^{-}(Ω) &= -8πG \ p_{\text{in}}^{-}(Ω) ; \tag{1} \\
[u_{\text{out}}^{-}(Ω), p_{\text{out}}^{+}(Ω')] &= [u_{\text{in}}^{+}(Ω), p_{\text{in}}^{-}(Ω')] = i \ δ^2(Ω, Ω') . \tag{2} \\
[u_{\text{in}}^{+}(Ω), u_{\text{out}}^{-}(Ω')] &= i f(Ω, Ω') ; \tag{3} \\
(ΔΩ - 1)u_{\text{in}}^{+}(Ω) &= +8πG p_{\text{out}}^{+}(Ω) . \tag{4}
\end{align*}
\]

This algebra is extremely simple, but also tricky. How to interpret the sign switch in ↔ out? (it is correct, in our notation)
All states, both in the initial and the final black hole, are a representation of this algebra.

So much for 1986. We concluded that the relation between in- and out- is now not much more than a Fourier transformation: $u^\pm \leftrightarrow p^\mp$, with $p = -i \frac{\partial}{\partial u}$

Is that all?
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Is that all? NO!

There is a complication.

Let’s calculate the representation:

Do the partial wave expansion
Partial waves on the spherical black hole:

\[
\left( \frac{u}{p} \right)(\theta, \varphi) = \sum_{\ell, m} \left( \frac{u_{\ell,m}}{p_{\ell,m}} \right) Y_{\ell,m}(\theta, \varphi)
\]

\[
[u_{\ell,m}^\pm, p_{\ell',m'}^\mp] = i \delta_{\ell \ell'} \delta_{mm'}
\]

\[
(1 - \Delta_\Omega)f(\Omega) = 8\pi G \delta^2(\Omega) \rightarrow (\ell^2 + \ell + 1)f_{\ell,m} = 8\pi G .
\]

\[
u_{\ell,m}^\pm = \mp \frac{8\pi G/R^2}{\ell^2 + \ell + 1} p_{\ell,m}^\pm
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Just as in the hydrogen atom, the different \((\ell, m)\) modes decouple.

This is not a "classical approximation" - it happens because the operators at different \((\ell, m)\) all commute.
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The out-going wave is the Fourier transform of the in-going wave.

Indeed, if \(u_{\text{in}}\) approaches horizon at \(u = 0\) as: \(u_{\text{in}} \rightarrow \lambda u_{\text{in}}\), then \(u_{\text{out}}\) will be driven out: \(u_{\text{out}} \rightarrow \lambda^{-1}u_{\text{out}}\)
Partial wave expansion in black hole: at given $\ell, m$, the out-state is the Fourier transform of the in-state.

However, $\text{Radius } r \geq 0$ and $r < 0$ both occur!
The Penrose diagram

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Regions III and IV are quantum clones of regions I and II, but they cannot be seen by the outside observers. The states in I and II together must span all of Hilbert space.
In the 1920s, partial, spherical wave expansions showed how the quantum dynamics of the hydrogen atom can be understood.

Now, we see that partial, spherical wave expansions solve the quantum dynamics of a black hole.

The underlying mathematics is almost the same, but the physics is different:

In an atom, we expand the wave function; in the black hole we expand the local momentum density.
The connection between regions I and II is called the Einstein-Rosen bridge.

The ER bridge seems to connect two black holes.

To restore unitarity for a single black hole, we have only one option: region II describes the points on the horizon that are antipodal to the points in region I.

Sanchez-Whiting 1986/87
This gives us the Schrödinger equation (generates dilatations):

\[ H = \{u^+ p^-\} = -\{u^- p^+\} \]

with boundary conditions as given in the algebra.
\[
\begin{pmatrix}
\psi_{\text{out}}^+ \\
\psi_{\text{out}}^-
\end{pmatrix}
= \frac{1}{\sqrt{2\pi}} \Gamma\left(\frac{1}{2} - i\kappa\right) e^{i\phi_\ell(\kappa)} \begin{pmatrix}
e^{-\frac{1}{2}\pi\kappa} & ie^{\frac{1}{2}\pi\kappa} \\
ie^{\frac{1}{2}\pi\kappa} & e^{-\frac{1}{2}\pi\kappa}
\end{pmatrix}
\begin{pmatrix}
\psi_{\text{in}}^+ \\
\psi_{\text{in}}^-
\end{pmatrix}
\]

To be read as follows: $\kappa$ is the energy of a partial wave, as defined by the outside observer.

At each $\ell$, $m$, we have a wave function $\psi^\text{in}_\sigma(\kappa)$, and $\psi^\text{out}_\sigma(\kappa)$ related by this unitary transformation matrix.

The two signs, $\sigma = \pm 1$ mix.

This means that our particles fill region $I$ as well as region $II$ in the Penrose diagram.

This BH universe has two asymptotic regions.

$I$ and $II$ talk to each other! What does this mean?
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you would re-emerge at the other side, \(CPT\) inverted!
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\[
\frac{2GM}{CPT}
\]

you would re-emerge at the other side, *CPT* inverted!
Firewalls

Picture emerging:

the “old” picture has firewalls of Hawking particles hanging on the future horizon, and imploding matter on the past horizon.

These are now regarded as quantum clones of particles far away from the black hole.

So they are redundant, and have to be discarded.

This does away with the firewalls, and allows us to use the Penrose diagram for eternal black holes.

But it does require an unconventional attitude towards black holes, or:
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NEW PHYSICS
\( \ell \) must be odd!

\[
\begin{align*}
\ell \pm (\theta, \varphi) &= -\ell \pm (\pi - \theta, \varphi + \pi) \\
p \pm (\theta, \varphi) &= -p \pm (\theta, \varphi)
\end{align*}
\]

only obeyed at odd \( \ell \):

\[
Y_{\ell m}(\pi - \theta, \varphi + \pi) = (-1)^\ell Y_{\ell m}(\theta, \varphi)
\]
What are the implications for the nature of space-time?
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but more work must be done . . . we will have to be much more precise in our analysis!