

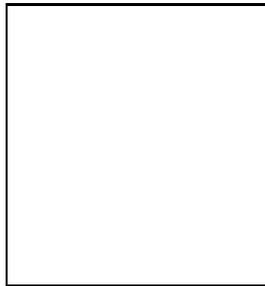
CONSTRAINTS ON CLUSTER PROPERTIES USING X-RAY & OPTICAL DATA

J.M DIEGO^{1,2}, E. MARTINEZ-CONZALEZ,¹, J. L. SANZ,¹, L. CAYON,¹, J. SILK,³

¹*Instituto de Física de Cantabria, Consejo Superior de Investigaciones Científicas-Universidad de Cantabria, Santander, Spain*

²*Departamento de Física Moderna, Universidad de Cantabria, Avda. Los Castros s/n, 39005 Santander, Spain*

³*Department of Physics, Nuclear & Astrophysics Laboratory, Keble Road, Oxford OX1 3RH, UK*



In this work we show that combining different cluster data sets is a powerful tool to constrain both, the cosmology and cluster properties. We assume a model with 9 parameters and fit them to 5 cluster data sets. From that fit, we conclude that only low density universes are compatible with the previous data and we also get some interesting conclusions on the rest of parameters.

1 Introduction

Our main purpose when we started this work was to perform realistic simulations of the Sunyaev-Zel'dovich effect (SZE) in order to check different subtraction techniques applied to this effect. By realistic we understand that our simulations must take into account first, the cluster population; that is, how is the cluster distribution in the mass-redshift space. Second, we must be able to describe cluster properties such as the scalings temperature-mass ($T - M$) and X-ray luminosity-mass ($L_x - M$). These descriptions of the cluster population and cluster scalings must be realistic in the sense that both descriptions must be in agreement with recent cluster data.

The idea of this work is to find such a realistic description of the clusters (population + intrinsic properties) by comparing our model with recent X-ray and optical data and fitting the free parameters of the model to the data in order to search for the best fitting values.

Several authors have used different cluster data sets in an attempt to constrain the cosmology. The usual procedure is, starting from the Press-Schechter (PS) mass function, fitting the experimental mass function or by using a given (that is fixed or non-free parameter) $T - M$ or $L_x - M$ relation construct some of the other cluster functions (temperature, X-ray luminosity or flux

functions) and then compare with the corresponding data sets. This can be a dangerous process for two reasons. First, when considering just one data set one ensures that his best fitting model is compatible with that data set but it can be inconsistent with others. We have observed this behaviour in many cases. For instance some of the models inside the 68% confidence level region in the $\Omega - \sigma_8$ space in Bahcall & Fan (1998) are inconsistent with, for example, the luminosity function of Ebeling et al. (1997). This point suggests that a consistent analysis should take into account the information coming from different experiments to avoid these incompatibilities. The second problem comes when some authors fix the $T - M$ or $L_x - M$ relations. The scatter in this correlations is large enough to introduce important uncertainties in the final result (Voit & Donahue (1998)). When these authors fix some of these relations, they probably are introducing a systematic bias in their conclusions due to the fact that they suppose that the virial relation, for instance, is a good enough approximation everywhere. We want to avoid all those uncertainties by fitting different data sets simultaneously without doing any assumption about the cosmology and the $T - M$ and $L_x - M$ relations.

2 Data

In this section we present the five different data sets we have used in our fit. The first data set is the cluster mass function given in Bahcall & Cen. (1993). To take into account evolutionary effects of the cluster mass function we use the evolution of the mass function given by Bahcall & Fan 1998. With the $T - M$ and $L_x - M$ relations we can build the cluster temperature, X-ray luminosity and flux functions. For the temperature function we use the one given by Henry & Arnaud (1991). For the X-ray luminosity function, that of Ebeling et al. 1997 and finally for the X-ray cluster flux function, the one determined by Rosati et al. 1998 for low-flux clusters, and the one obtained by De Grandi et al. (1999) for high-flux clusters. With all these data curves and with our model we are now able to fit the model and say something about its free parameters. The results will be robust and consistent in the sense that we did not make any assumption about the cosmology (we did not fix any cosmological parameter) nor about the intrinsic cluster properties ($T - M$ and $L_x - M$).

3 The model

Our model consists of two parts. First the description of the cluster population and second the intrinsic cluster properties. For the cluster population we assume the standard Press-Schechter (PS) formalism (Press & Schechter 1974). This formalism depends on three parameters: the density of the universe Ω , the amplitude of the power spectrum in units of σ_8 and finally the shape parameter of the power spectrum Γ . In this work we have only considered low density models with $\Lambda = 0$. In a subsequent paper we will also include in our analysis flat Λ CDM models.

The PS approach is supported by N-body numerical simulations which do show a good agreement with the PS parametrization (Lacey & Cole 1994, White et al. 1993, Efstathiou et al. 1988, etc).

With the PS formalism we know the comoving number density of clusters with $M \in [M, M + dM]$ and at a given redshift. This will allow us to distribute the clusters in the $M - z$ space for a given solid angle and cosmology (Ω, σ_8, Γ). What we need now is the second part of the model, the intrinsic cluster properties. Basically what we need is the $T - M$ relation for which the virial relation is usually assumed (Navarro et al. 1995, Bryan & Norman 1998). One problem is that it is not clear to what extend the virial relation is true for high or even intermediate redshift. In

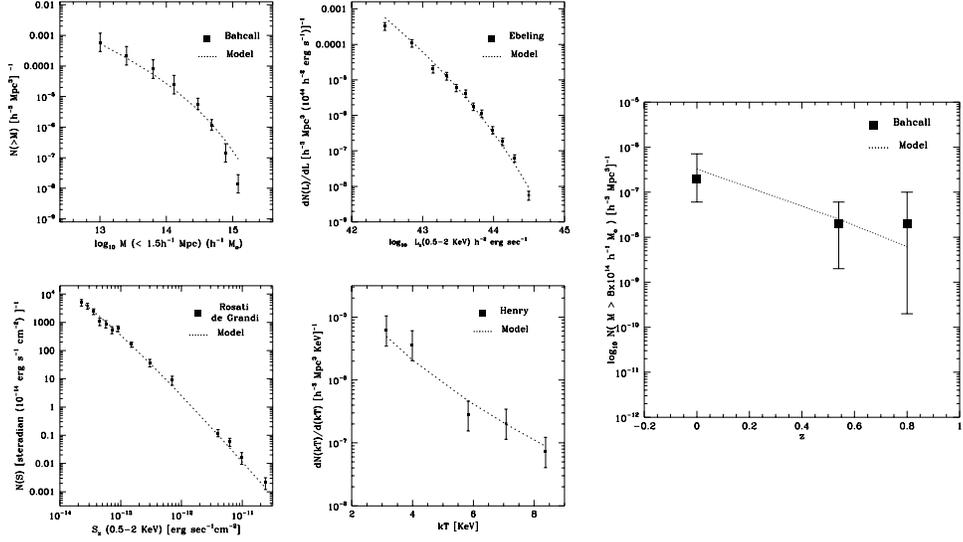


Figure 1: Expected curves compared with the data for the best CDM ($\lambda = 0$) model.

this work we have decided to consider this scaling as a free parameter relation:

$$T_{gas} = T_0 M_{15}^\alpha (1+z)^\psi \quad (1)$$

where T_0 , α and ψ are our three free parameters. M_{15} is the cluster mass in $h^{-1} 10^{15} M_\odot$ units. With the $T - M$ relation and the PS mass function we can build the temperature function, simply by doing:

$$\frac{dN(T, z)}{dT} = \frac{dN(M, z)}{dM} \frac{dM}{dT} \quad (2)$$

Our aim is to use as many data sets as possible and so, we also want to include in our analysis other relevant data sets like the X-ray luminosity and flux functions. To do that we need another scaling law, the $L_x - M$ relation in order to relate the mass function with the X-ray luminosity and flux functions similarly as we did with the temperature function in eq. (2). As in the $T - M$ relation, the $L_x - M$ is not well established yet and for this reason we have adopted a parametrization similar to that given in eq. (1):

$$L_x^{Bol} = L_0 M_{15}^\beta (1+z)^\phi \quad (3)$$

where we added three additional free parameters: L_0 , β and ϕ . From this relation is possible to build the $S_x - M$ relation. Just taking,

$$S_x^{Bol} = \frac{L_x^{Bol}}{4\pi D_l(z)^2} \quad (4)$$

Summarising, the final number of free parameters are 9 : Ω , σ_8 , Γ , T_0 , α , ψ , L_0 , β and ϕ . We now want to play with these parameters and look for the best fitting model to the different data sets.

4 Best fit

To fit the five data sets we must decide which estimator will pick up our best fitting model. Because of the scaling relations assumed between the mass and temperature ($T - M$) and the

Table 1: Best model.

Parameter		Parameter	
σ_8	0.8	Γ	0.1
Ω	0.3	$T_0(10^8 K)$	1.0
α	0.7	ψ	1.0
$L_0(10^{45} h^{-2} \text{erg/s})$	0.6	β	3.1
ϕ	2.0		

mass and luminosity in the X-ray band ($L_x - M$), then there must be some correlations among the five data sets predicted by the model. Therefore we should start by considering an estimator like the standard likelihood estimator.

In our case, the model depends on 9 free parameters and if we consider a grid of, let's say 5 values per parameter, then we should compute the correlation matrix for $5^9 \sim 1$ million different models. This process would take many years. A faster technique would require a search method that avoids exploring all the parameter space. This can be the solution if we are interested just in the best model but we want also the error bars of our model, or in other words the marginalized probability distribution of the parameters. In order to do that, we need to know the probability in a given regular grid.

To simplify the problem, the most simple approach is to consider as our estimator the standard χ_{joint}^2 :

$$\chi_{joint}^2 = \chi_M^2 + \chi_{M(z)}^2 + \chi_T^2 + \chi_{L_x}^2 + \chi_{S_x}^2 \quad (5)$$

where χ_x^2 represents the corresponding ordinary χ^2 for the five different data sets and we are supposing that the correlation matrix is in this case diagonal.

By doing this, we know that we are forgetting the correlations between the curves and that there will be some bias in our estimation. For this reason we want to check other more elaborated estimators.

We have considered as a second estimator of the best model the next one based on bayesian theory (Lahav et al, 2000);

$$-2\ln P_L = \chi_L^2 \quad (6)$$

where,

$$\chi_L^2 = \sum_i^5 N_i \ln(\chi_i^2) \quad (7)$$

In this estimator χ^2 is the same as before and N_i represents the number of data points for the data set i . The authors have shown that this estimator is appropriate for the case when different data sets are combined together, as is our case. The factor N_i plays the role of a weight factor. Those data sets with more measures (more data points) are considered as more realistic.

We have checked both estimators by doing a bias test. In this test we have simulated the data for a known model with the corresponding error bars. The input model was selected according to the criterium that it would be as close as possible to the data (for instance the model which minimises χ_{joint}^2). In the simulations we have taken into account all the characteristics of the data, that is, sky coverage, limiting flux, maximum redshift etc. Then we compare each one of these realizations with the mean value of the different models and for each realization we get the best model using both estimators.

We have concluded from this test that the second estimator works better than the standard χ_{joint}^2 . There is still some bias with the second estimator but the agreement between the input model and the recovered one is very good.

Using the second estimator we have computed the probability distribution in our 9 parameter

space. We have used a grid with about 1 million different models and for each of them we have computed its P_L (eq. 6). Knowing that, we can obtain the best model (maximum probability, see table 1).

5 Conclusions

In this work we have shown that combining different data sets and using the Lahav's et al. estimator is a powerful tool to constrain the cosmology and cluster parameters. In a future paper (Diego et al, in preparation) we will present the full analysis taking into account both, open ($\Lambda = 0$) and flat ($\Lambda > 0$) CDM models. The marginalized probability distributions for the parameters of the models will be also included.

Additional data coming from high redshift clusters (CHANDRA, XMM-Newton, PLANCK) will improve this result.

Particularly interesting is the work that can be done with future CMB surveys. The PLANCK satellite will explore the whole sky at nine different frequencies (from 30 Ghz to 800 Ghz) and with resolutions between 5 arcmin and 30 arcmin. At these frequencies and with those resolutions we have shown (Diego et al. in preparation) that several clusters are expected to be observed at high redshift ($z > 2$) through the Sunyaev-Zel'dovich effect. The information that these clusters will provide will be decisive to definitely exclude many models.

Acknowledgments

We would like to thank to Piero Rosati for kindly providing his data for the differential flux function. JMD acknowledges the DGES for a fellowship. JMD, EM, JLS, & LC thanks CFPA/Astronomy Dept. Berkeley for the facilities given during this work.

References

1. Bahcall, N.A. Cen, R. ApJ, 407, L49, 1993.
2. Bahcall, N.A., Fan, X., ApJ, 504, 1, 1998.
3. Bryan, G.L., Norman, M.L., ApJ, 495, 80, 1998.
4. De Grandi, S., Böhringer, H., Guzzo, L., Molendi, S., Chincarini, G., Collins, C., Cruddace, R., Neumann, D., Schindler, S., Schuecker, P., Voges, W., ApJ, 514, 148, 1999.
5. Diego, J.M., Martínez-González, E., Sanz, J.L., Cayón, L., Silk, J., in preparation.
6. Ebeling, H., Edge, A.C., Fabian, A.C., Allen, S.W., Crawford, C.S., ApJ, 479, L101, 1997.
7. Efstathiou G., Frenk, C.S., White, S.D.M., Davis, M., MNRAS, 235, 715, 1988.
8. Henry, J.P., Arnaud, K.A., ApJ, 372, 410, 1991.
9. Lacey, C., Cole, S., MNRAS, 271, 676, 1994.
10. Lahav, O., Bridle, S.L., Hobson, M.P., Lasenby, A.N., in preparation, 2000. astro-ph/9912105.
11. Navarro, J.F., Frenk, C.S., White, S.D.M., MNRAS, 275, 720, 1995.
12. Press, W.H., Schechter, P., ApJ, 187, 425, 1974.
13. Rosati, P., Della Ceca, R., Norman, C., Giacconi, R., ApJ, 492, L21, 1998
14. Voit, G.M., Donahue, M., ApJ, 500, L111, 1998.
15. White, S.D.M., Efstathiou, G., Frenk, C.S., MNRAS, 262, 1023, 1993.