

# COSMOLOGICAL STRUCTURE FORMATION WITH TOPOLOGICAL DEFECTS

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Structure formation with topological defects is described. The main difference from inflationary models are highlighted. The results are compared with recent observations. It is concluded that all the defect models studied so far are in disagreement with recent observations of CMB anisotropies. Furthermore, present observations do not support 'decoherence', a generic feature of structure formation from topological defects.

## 1 Introduction

Even if the big bang is an "irreproducible experiment", we want to learn from it as much as possible about the physics at high energies. We have reasons to hope that it may have left traces from energies much higher than those reached in any astrophysical event or terrestrial experiment. Therefore, even if it is irreproducible and hence not as controllable as we might want, we simply cannot afford to ignore the information it may have left.

The initial fluctuations in the cosmic matter density and geometry may represent one such trace. In fact, presently there are two relatively worked out ideas for cosmic initial fluctuations, both relying on the physics at very high energies. In the first model, cosmic initial perturbations are due to quantum fluctuations which 'freeze in' as classical fluctuations when they become super-horizon during an *inflationary era*.

The second possibility is that *topological defects* which may have formed during a phase transition in the early universe have induced structure formation. This second possibility is the topic of this talk.

Here, a pedagogical remark may be in order: Often these two alternatives have been represented as 'inflation versus defects'. This is of course not quite correct, as topological defects have nothing to say about the flatness, the horizon and the monopole or moduli (or whatever unwanted relicts) problems which inflation also solves. It is, however, easy to construct inflationary models where the amplitude of initial fluctuations is much too small to be relevant for structure formation. Therefore, in a model, where cosmic structure is due to topological defects, one needs either inflation prior to defect formation or another mechanism to solve the flatness, horizon and relict problems.

The remainder of this talk is organized as follows: In Section 2 I give a short overview on the formation of topological defects during cosmological phase transitions. In Section 3 I discuss the problem of structure formation with topological defects. I will first describe some generic insights and then discuss results for specific models. Conclusions are presented in Section 4.

## 2 Topological defects

During adiabatic expansion the universe cools down from a very hot initial state. It is natural to expect that the cosmic plasma undergoes several symmetry breaking phase transitions. In the process of such a transition an initial symmetry group  $G$  is broken down to a subgroup  $H$ . Depending on the topology of the vacuum manifold  $\mathcal{M}$ , which generically is topologically equivalent to the homogeneous space  $G/H$ , topological defects may form.

This is described by an order parameter or Higgs field,  $\phi$ , with a temperature dependent effective potential. The field values which minimize the potential form the vacuum manifold  $\mathcal{M}$ . After the phase transition the field will assume different values in  $\mathcal{M}$  in different positions of physical space, which are uncorrelated if, *e.g.* the spatial separation is larger than the present particle horizon,  $l_H \sim t$ . If the topology of the vacuum manifold is non-trivial, the Kibble mechanism<sup>1</sup> generically leads to the formation of topological defects: the field  $\phi$  may vary in space in such a way that there are points, where  $\phi$  has to leave the vacuum manifold by continuity reasons and assume values with higher potential energy. Such points have to form a connected sub-manifold of spacetime.

For example if  $\mathcal{M}$  is not connected,  $\pi_0(\mathcal{M}) \neq \{0\}$ , in different positions  $\phi$  can assume values which belong to disconnected parts of  $\mathcal{M}$  and therefore it has to leave the vacuum manifold somewhere in between. The sub-manifold of higher energy is in this case three dimensional in spacetime and is called **domain wall**. (Domain walls from high energy phase transitions are disastrous for cosmology.) Similarly, a non simply connected vacuum manifold,  $\pi_1(\mathcal{M}) \neq \{0\}$ , leads to the formation of two dimensional defects, **cosmic strings**. Domain walls and cosmic strings are either infinite or closed. If  $\mathcal{M}$  contains non shrinkable two spheres,  $\pi_2(\mathcal{M}) \neq \{0\}$ , one dimensional defects, **monopoles** form. Finally, if  $\pi_3(\mathcal{M}) \neq \{0\}$ , zero dimensional **textures** appear, which are events of higher energy. By Derrick's theorem one can argue that a scalar field configuration with non-trivial  $\pi_3$  winding number (*i.e.* a texture knot) contracts and eventually unwinds producing a space-time 'point' of higher energy. A summary of this is given in Table 1; more details can be found in Refs.<sup>2,3</sup>.

Homotopy $\pi_n$ , dimension in spacetime = $d = 4 - 1 - n$	appearance		
$\pi_0(\mathcal{M}) \neq 0$ , $\mathcal{M}$ is disconnected	walls	$d = 3$	sheets in space
$\pi_1(\mathcal{M}) \neq 0$ $\mathcal{M}$ contains non shrinkable circles	strings	$d = 2$	lines in space
$\pi_2(\mathcal{M}) \neq 0$ $\mathcal{M}$ contains non shrinkable 2-spheres	monopoles	$d = 1$	points in space
$\pi_3(\mathcal{M}) \neq 0$ $\mathcal{M}$ contains non shrinkable 3-spheres	texture	$d = 0$	events in spacetime

Table 1: Topological defects in four dimensional spacetime.

Topological defects are also very well known in solid state physics. For example the vortex lines in type II superconductors are nothing else than cosmic strings. Also in liquid crystals<sup>4</sup> (see Fig. 1) or super fluid Helium<sup>5</sup> a variety of topological defects form during symmetry breaking phase transitions.

The defects are called **local**, if a gauge symmetry is broken and **global** if they emerge from global symmetry breaking. In the case of local defects, gradients in the scalar field are 'compensated' by the gauge field and the energy density of the defect is confined to the defect manifold with very small transverse dimension of the order of the symmetry breaking scale. Soon after formation, local defects therefore cease to interact over distances larger than the inverse symmetry breaking scale.

The energy density of global defects is dominated by gradient energy and hence of the order of  $\rho_{\text{defect}} \sim T_c^2/t^2$  where  $T_c$  is the symmetry breaking temperature and  $t$  is the horizon scale, the typical scale over which the scalar field varies. As the energy density of the cosmic

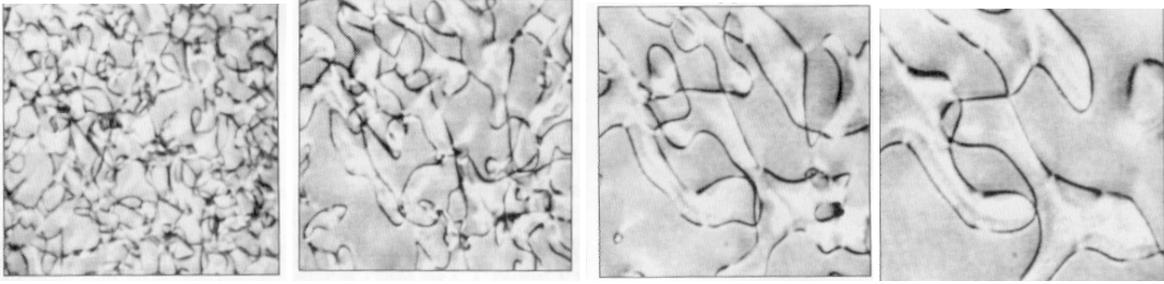


Figure 1: A scaling sequence of a string network in a nematic liquid crystal after the isotropic - nematic phase transition. Time runs from left to right. (From Ref.<sup>4</sup>.)

fluid also decays like  $1/t^2$ , global defects always scale<sup>a</sup> and lead to fluctuations with a typical amplitude of

$$\rho_{\text{defect}}/\rho \sim 4\pi GT_c^2 = \epsilon . \quad (1)$$

In the case of local defects only cosmic strings scale and obey (1). Local monopoles soon come to dominate the cosmic energy density and are therefore ruled out from observations. Local texture die out. To be relevant for structure formation, the defects have to induce scaling fluctuations with an amplitude  $\epsilon \sim 10^{-5}$  which implies

$$T_c \sim 10^{16} \text{GeV},$$

a grand unified energy scale. Topological defects which form at lower temperature are of no relevance for structure formation<sup>b</sup>.

### 3 Structure formation with topological defects

We discuss especially the differences of structure formation with topological defects from inflationary initial perturbations. I first highlight some very generic features, then we discuss results for specific models.

#### 3.1 Generics

The large scale fluctuations in the cosmic microwave background (CMB) are of the same order as the deviation of the cosmic metric from a Friedmann metric. Since these fluctuations are small, linear perturbation theory is justified. For a cosmic fluid consisting of radiation, massless neutrinos, baryons, cold dark matter, possibly hot dark matter and/or a cosmological constant, we obtain linear perturbation equations (in Fourier space). For each wave vector  $k$  they are of the form

$$DX = S , \quad (2)$$

where  $X$  is a long vector describing all the random perturbation variables,  $D$  is a deterministic linear first order differential operator and  $S$  is a random source term which consists of linear combinations of the energy momentum tensor of the defect network. More details can be found, *e.g.* in Ref.<sup>7</sup>.

For inflationary perturbations  $S = 0$  and the solutions are determined entirely by the random initial conditions,  $X(k, t_{\text{in}})$ . For most inflationary models  $X(k, t_{\text{in}})$  is a set of Gaussian

<sup>a</sup>Up to logarithmic corrections to the scaling law which are especially important in the case of global cosmic strings.

<sup>b</sup>With a possible exception of 'soft domain walls', see *e.g.*<sup>6</sup> or the contribution of M. Bucher to these proceedings.

random variables and hence their statistical properties are entirely determined by the spectra  $\mathcal{P}$  (the Fourier transforms of the two point functions),

$$\langle X_i((t_{\text{in}}, \mathbf{k}) X_j^*((t_{\text{in}}, \mathbf{k}')) \equiv \mathcal{P}_{ij}(\mathbf{k})\delta(\mathbf{k} - \mathbf{k}') . \quad (3)$$

Here the Dirac delta is a consequence of statistical homogeneity which we want to assume for the random process leading to the initial perturbations.

Be  $A_i(k, t)$  the solution with initial condition  $X_j(k, t_{\text{in}}) = \delta_{ij}$ . The spectra of the solution with initial 'spectrum' given by Eq. (3) is then just

$$\langle X_i((t_0, \mathbf{k}) X_j^*((t_0, \mathbf{k}')) = A_i(k, t_0)A_j^*(k, t_0)\mathcal{P}_{ij}(k)\delta(\mathbf{k} - \mathbf{k}') .$$

Tehrefore, if  $A_i$  is oscillating, *e.g.* as a function of  $kt$  so will  $\langle |X_i|^2 \rangle$ . This leads to a very important feature in the CMB anisotropy spectrum, the acoustic peaks: Prior to recombination, due to radiation pressure the photon/baryon plasma undergoes acoustic oscillations on subhorizon scales. At recombination the photons become suddenly free and 'stream' into our antennas without further interaction. Since the acoustic oscillations of a given wave number  $k$  are all in phase, they have a fixed amplitude at decoupling. This phenomenon imprints itself in the CMB anisotropy spectrum as a series of peaks. On very small scales, the finite thickness of the recombination shell and free streaming have to be taken into account which leads to an exponential damping of the peaks (Silk damping). As we shall see below, the acoustic peaks are very characteristic of inflationary perturbations.

If the source term  $\mathcal{S}$  does not vanish, the situation is different. Equation (2) can be solved by means of a Green's function (kernel),  $\mathcal{G}(t, t')$ , in the form

$$X_j(t_0, \mathbf{k}) = \int_{t_{\text{in}}}^{t_0} dt \mathcal{G}_{jl}(t_0, t, \mathbf{k}) \mathcal{S}_l(t, \mathbf{k}) . \quad (4)$$

Power spectra or, more generally, quadratic expectation values of the form  $\langle X_j(t_0, \mathbf{k}) X_l^*(t_0, \mathbf{k}) \rangle$  are then given by

$$\langle X_j(t_0, \mathbf{k}) X_l^*(t_0, \mathbf{k}) \rangle = \int_{t_{\text{in}}}^{t_0} dt \int_{t_{\text{in}}}^{t_0} dt' \mathcal{G}_{jm}(t_0, t, \mathbf{k}) \mathcal{G}_{ln}^*(t_0, t', \mathbf{k}) \langle \mathcal{S}_m(t, \mathbf{k}) \mathcal{S}_n^*(t', \mathbf{k}) \rangle . \quad (5)$$

The only information about the source random variable which we really need in order to compute power spectra are therefore the unequal time two point correlators

$$\langle \mathcal{S}_m(t, \mathbf{k}) \mathcal{S}_n^*(t', \mathbf{k}) \rangle . \quad (6)$$

This nearly trivial fact has been exploited by many workers in the field, for the first time probably in Ref. <sup>8</sup> where the decoherence of models with seeds has been discovered, and later in Refs. <sup>9,10,11,12,7</sup> and others.

To determine the correlators (6) one has to calculate the unequal time correlators of the defect energy momentum tensor by means of numerical simulations. To solve the enormous problem of dynamical range, 'scaling', statistical isotropy and causality have to be used.

Seeds from global topological defects and from cosmic strings are 'scaling' in the sense that their correlation functions  $C_{\mu\nu\rho\lambda}$  defined by

$$\Theta_{\mu\nu}(\mathbf{k}, t) = M^2 \theta_{\mu\nu}(\mathbf{k}, t) , \quad (7)$$

$$C_{\mu\nu\rho\lambda}(\mathbf{k}, t, t') = \langle \theta_{\mu\nu}(\mathbf{k}, t) \theta_{\rho\lambda}^*(\mathbf{k}, t') \rangle \quad (8)$$

are scale free; *i.e.* the only dimensional parameters in  $C_{\mu\nu\rho\lambda}$  are the variables  $t, t'$  and  $\mathbf{k}$  themselves. Here the energy scale  $M$  corresponds to the symmetry breaking scale. One can set

$M = T_c$ . Up to a certain number of dimensionless functions  $F_n$  of  $z = k\sqrt{tt'}$  and  $r = t/t'$ , the correlation functions are then determined by the requirement of statistical isotropy, symmetries and by their dimension. Causality requires the functions  $F_n$  to be analytic in  $z^2$ . A more detailed investigation of these arguments and their consequences is presented in Ref.<sup>13</sup>. There it is shown that statistical isotropy and energy momentum conservation reduce the correlators (8) for global defects to five such functions  $F_1$  to  $F_5$ . Since cosmic strings loose energy by gravitational radiation, which is crucial to ensure scaling, in this case 14 functions  $F_n$  are needed to fully describe the correlators. However, numerical simulations show that for cosmic strings the density-density correlator is significantly larger than all the other components of  $C_{\mu\nu\rho\lambda}$  which again simplifies the problem<sup>14</sup>.

Since analytic functions generically are constant for small arguments  $z \ll 1$ ,  $F_n(0, r)$  actually determines  $F_n$  for all values of  $k$  with  $z = k\sqrt{tt'} \lesssim 0.5$ . Furthermore, the correlation functions decay inside the horizon and we can safely set them to zero for  $z \gtrsim 40$  where they have decayed by about two orders of magnitude. In Fig. 2 I show one of these functions for global  $O(4)$ -texture (a) and for the large  $N$  limit of global  $O(N)$  models<sup>7</sup> (b).

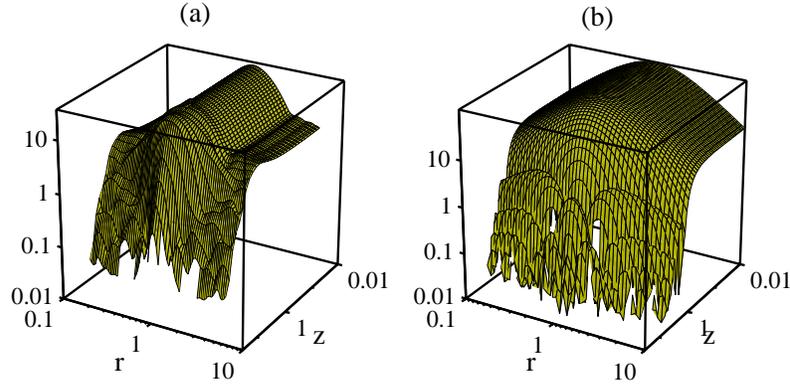


Figure 2: A two point correlation function of scalar perturbations is shown. Panel (a) represents the result from numerical simulations of the texture model; panel (b) shows the large- $N$  limit. For fixed  $r$  the correlator is constant for  $z < 1$  and then decays. Note also the symmetry under  $r \rightarrow 1/r$  (figure from Ref.<sup>7</sup>).

For the induced perturbations in the cosmic fluids, the presence of of this source term has several important consequences. First of all, as is clear from Eqn. (4), the randomness of the source term enters at all times (as long as the source term is non-zero). Therefore, fluctuations of a given wave number  $k$  are in general not in phase, and the distinctive series of acoustic peaks present in inflationary models is blurred into one 'broad hump'. This phenomenon has been termed 'decoherence'<sup>8</sup>. A key ingredient for decoherence to happen is the non-linearity of the time evolution of the source term<sup>7</sup>. Even though time evolution is deterministic, different Fourier modes mix due to non-linearity, and the randomness in one mode 'sweeps' into the other modes. In the case of topological defects,  $\mathcal{S}$  is given by linear combinations of the defect stress energy tensor,  $\theta_{\mu\nu}$ , quadratic in the defect field, which itself obeys non-linear evolution equations. Only in the large  $N$  limit, the evolution of the 'defect field' becomes linear and decoherence is much weaker. The non-linearity of the source evolution also leads to the non

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<sup>7</sup>If the source term would evolve linearly it could just be added to the components of  $X$  and we would obtain a new, somewhat longer linear system of equations where again randomness can enter only via the initial conditions.

Gaussianity of defect models. Even if the initial field configuration would be Gaussian (which it usually is not due to non-linear constraints), the non-linear time evolution renders the source term and therefore also the perturbations highly non Gaussian.

In Table 2 we highlight the similarities and differences of inflationary and defect models of structure formation.

Inflationary models	Topological defects
<b>Similarities</b>	
<ul style="list-style-type: none"> <li>• Cosmic structure formation is due to gravitational instability of small 'initial' fluctuations. → Gravitational perturbation theory can be applied.</li> <li>• GUT scale physics is involved in generating initial fluctuations.</li> <li>• The only relevant 'large scale' is the horizon scale. → Harrison-Zel'dovich spectrum.</li> </ul>	
<b>Differences</b>	
<ul style="list-style-type: none"> <li>• The fluctuation amplitude depends on details of the inflaton potential, fine tuning.</li> <li>• Homogeneous perturbations (passive).</li> <li>• Vector perturbations decay and become irrelevant.</li> <li>• Tensor perturbations can have arbitrary amplitudes.</li> <li>• Perturbations are usually adiabatic.</li> <li>• Perturbations are usually Gaussian.</li> <li>• For given initial perturbations, the problem is linear.</li> <li>• Randomness enters only via the initial conditions.</li> <li>• The phases of perturbations at a fixed scale are coherent.</li> <li>• Super Hubble scale correlations exist.</li> </ul>	<ul style="list-style-type: none"> <li>• The amplitude of fluctuations is fixed by the symmetry breaking scale.</li> <li>• Inhomogeneous perturbations (active).</li> <li>• Vector perturbations are sourced on large scales and are typically of the same order as scalar perturbations.</li> <li>• Scalar, vector and tensor modes are generically of the same order.</li> <li>• Perturbations are isocurvature.</li> <li>• Perturbations are non Gaussian.</li> <li>• The source evolution is non-linear at all times.</li> <li>• Randomness enters at all times due to the mixing of scales</li> <li>• The phases of perturbations become incoherent, <b>decoherence</b>.</li> <li>• No correlations on super Hubble scales.</li> </ul>

Table 2: Similarities and differences of inflationary perturbations versus perturbations induced by seeds.

### 3.2 Results

As we have seen, there are several important differences between defect models and inflationary models of structure formation. First of all, defect models generically predict scalar, vector and tensor perturbations with comparable amplitudes at horizon scale, whereas in inflationary models vector perturbations are absent (they simply have decayed from their initial values) and tensor perturbations are often significantly smaller than scalar modes. Furthermore, inflationary perturbations are usually adiabatic. This leads to an important cancellation in the temperature fluctuations due to gravity, given by  $\left(\frac{\Delta T}{T}\right)_{\text{grav}} = -2\Phi$ , where  $\Phi$  denotes the Newtonian potential, and the intrinsic temperature fluctuation on large scales, which is  $\left(\frac{\Delta T}{T}\right)_{\text{int}} = \frac{1}{4}\delta_{\text{rad}} = \frac{1}{3}\delta_{\text{mat}} = \frac{5}{3}\Phi$  in the adiabatic case. The net result becomes  $\left(\frac{\Delta T}{T}\right)_{\text{SW}} = -\frac{1}{3}\Phi$ , the ordinary Sachs-Wolfe effect for adiabatic perturbations<sup>15</sup>.

Both these effects lower the temperature fluctuations of inflationary models on very large scales if compared to those from defect models. This leads to the result that the amplitude of fluctuations on very large scales, the height of the 'Sachs-Wolfe plateau' is comparable to

the amplitude of intermediate scales, the acoustic peak(s). This has first been noted in Ref.<sup>16</sup>. Furthermore, the isocurvature nature of defect models leads to a shift of the first acoustic peak towards smaller angular scales. For flat cosmologies the peak position is around  $\ell_{\text{peak}} \sim 350 - 450$ , depending on the specific model (to be compared with  $\ell_{\text{peak}} \sim 220$  for inflationary models).

Thorough numerical simulations from two different groups<sup>9,7</sup> now show that CMB anisotropies from global  $O(N)$  models do not agree with present data see Fig. 3). These models also require a very high bias to fit the galaxy power spectrum and exhibit much too low bulk flows on large scales. For example the bulk velocity on  $50h^{-1}\text{Mpc}$  for the texture model is  $V_{50} \sim 60\text{km/s}$  whereas the measured value is more like  $V_{50} \sim 300 \pm 100\text{km/s}$ .

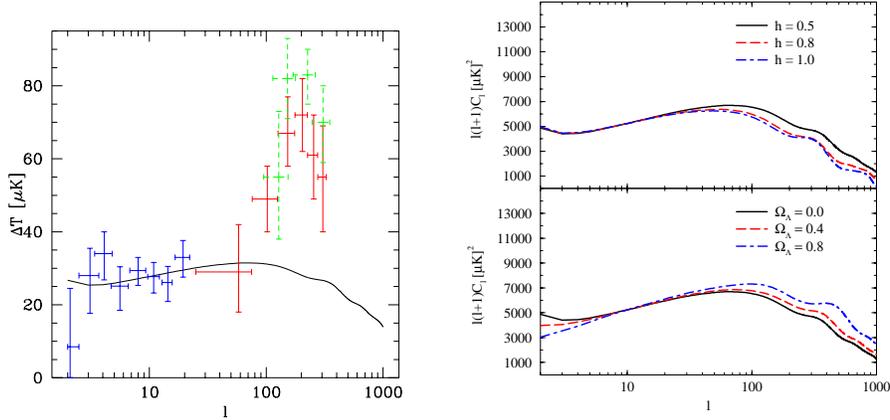


Figure 3: **Left:** The CMB anisotropies obtained from global  $O(4)$  texture are compared with data, COBE<sup>17</sup> (the 8 leftmost data points), BoomerangNA<sup>18</sup> (solid crosses) and Toco<sup>19</sup> (dashed crosses). The model clearly disagrees with the Toco and Boomerang data. **Right:** The CMB anisotropies obtained from global  $O(4)$  texture are shown for different values of cosmological parameters. It is clear that varying the cosmological parameters cannot save the model.

The results for cosmic strings are somewhat more promising due to a variety of effects. Most notably the following:

- The cosmic string energy density seems to be considerably higher in the radiation era than in the matter era, therefore boosting the fluctuations on scales which enter the horizon already in the radiation dominated era of the universe,  $\lesssim 50h^{-1}\text{Mpc}$ , just the scales where global  $O(N)$  models are missing power.
- $T_0^0 = \rho$  is much larger than the other components of the string energy momentum tensor. Being of scalar nature it induces only scalar perturbations so that vector and tensor perturbations are suppressed in the case of strings.
- Cosmic strings loose power on scales inside the horizon by inter-commutation and gravitational radiation. These processes are slower than the speed of light with which global defects decay. Therefore, the energy momentum tensor persists to later times, up to larger values of  $kt$  than for  $O(N)$  models. This induces larger fluctuations in the dark matter.

The induced fluctuations in the dark matter may even be too large on small scales, a problem which can be solved by introducing hot dark matter<sup>20</sup>. The persistence of the string energy momentum tensor induces even more decoherence<sup>8</sup> than for  $O(N)$  models.

Therefore, cosmic strings may lead to one broad 'acoustic hump' but certainly not to a series of peaks. The precise height of the hump depends sensitively on several unknowns, for example on how one models the string energy momentum non conservation<sup>21</sup> and on the small scale structure of the string network<sup>22</sup>, see, e.g., Fig. 4.

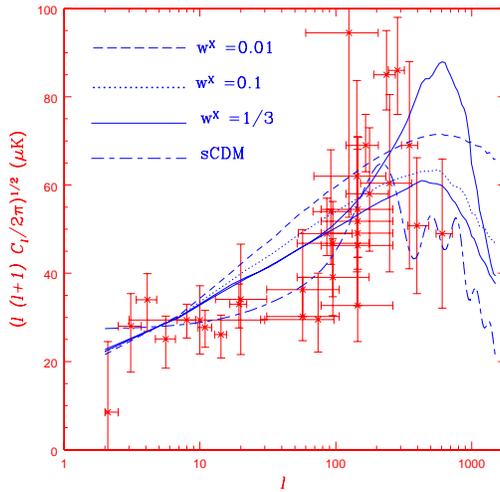


Figure 4: The CMB anisotropy spectrum for a model where cosmic string loops 'decay' into a fluid with equation of state  $p_X = w_X \rho_X$  are shown and compared with the standard inflationary CDM model (long-short dashed line). Clearly, the result depends very sensitively on  $w_X$ . Figure taken from Ref.<sup>21</sup>.

Decoherence which leads to a 'smearing' of acoustic peaks (if they are there) is one of the few features about which all results on cosmic strings agree. The height of the 'acoustic hump' may be about two to four times the height of the plateau at low  $l$ . The position of the hump is not very well defined and depends on the details of the modelling, but it is typically at  $l \gtrsim 400$  for a flat universe, which is in disagreement with the new data shown in Fig. 3. The bias factor needed in the dark matter spectra (maybe between 2 and 5) are still quite uncertain. Some recent work on this subject can be found in Refs.<sup>10,23,24,21</sup>.

#### 4 Conclusions

All the defect models studied in detail are in disagreement with current observations. They exhibit no acoustic peaks (global  $O(N)$  models) or only one broad hump on too small scales (cosmic strings). Decoherence, which is inherent to the non-linear evolution of the defect source term smears out the distinguished series of acoustic peaks expected in inflationary models. The width of the first peak measured by the the Toca<sup>19</sup> and BoomerangNA<sup>18</sup> experiments is relatively narrow, which already clearly disfavors a model where decoherence is important. Secondary peaks in the CMB anisotropy spectrum will finally be a unambiguous sign for a (quasi-)linear process of structure formation like, *e.g.*, inflation.

It has been shown, however, that linearly evolving causal scaling seeds might mimic an inflationary CMB and dark matter power spectrum<sup>25,12</sup>. Nevertheless, due to causality they differ from inflation in the CMB polarization spectrum<sup>26</sup>. Clearly, such seeds are not topological defects and there is so far no convincing physical motivation to introduce them.

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