Searching for New Forces over Ultrashort Distances

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Ephraim Fischbach

and

Ricardo Decca
Galina Klimchitskaya
Dennis Krause
Daniel Lopéz
Vladimir Mostepanenko
Overview

• Was Newton Really Right?
  – Tests of $1/r^2$ law
  – Tests of the equivalence principle
  – Importance of scale
• Motivations for Non-Newtonian Gravity
  – New light quanta $\Rightarrow$ new forces
  – Unification and extra dimensions
• Experimental Tests
  – Problems with short-distance experiments
  – The “iso-electronic” effect
  – The IUPUI Casimir experiment
• Outlook
Summary of Newtonian Gravity

- **Newtonian Gravity**
  \[ V(r) = -\frac{G m_1 m_2}{r} \]
  \[ \vec{F}(r) = -\frac{G m_1 m_2}{r^2} \hat{r} = m_1 \vec{a}_1 \]
  \[ \vec{a}_1 = -\frac{G m_2}{r^2} \hat{r} \]
  
  a) independent of the nature of \( m_1 \)
      (Equivalence Principle)
  b) varies as \( 1/r^2 \)

- **Non-Newtonian Gravity**
  \[ V(r) = -\frac{G m_1 m_2}{r} \left\{ 1 + \alpha_{12} e^{-r/\lambda} \right\} \]
  \[ F(r) = -\frac{G(r) m_1 m_2}{r^2} \hat{r} \]
  
  doesn’t vary as \( 1/r^2 \) *
  
  \[ G(r) = G[1 + \alpha_{12} e^{-r/\lambda} (1 + r/\lambda)] \]
  
  not independent of 1 or 2 *

* Evidence for either of these would point to a new fundamental force in nature.
Theory of New Forces

A new force can arise from the exchange of a light quantum with appropriate properties.

\[ V_{12}(r) = \pm f_1 f_2 e^{-r/\lambda} \]

Bulk Matter

\[ V_{12}(r) = \pm Q_1 Q_2 f^2 e^{-r/\lambda} \]

\( \lambda = \text{Range} = \hbar/\mu c \)

if \( \mu = 10^{-9} \text{ eV}/c^2 \Rightarrow \lambda = 200 \text{ m} \)

\[ \text{scalar}(J^P = 0^+) \]

\[ \text{vector}(J^P = 1^-) \]

p, e, n, (\( \pi^0 \), ...) p, e, n, (\( \pi^0 \), ...)
Limits on Extra Dimensions and New Forces:
Long-Range Limits

Limits on Extra Dimensions and New Forces: Short-Range Limits

Mini-Review #1

• The existence of ultra-light particles can produce long-range fields which behave somewhat like gravity

• The presence of such fields would show up as apparent deviations from the predictions of Newtonian gravity

  $1/r^2$ violations
  equivalence principle violations

• To search for such violations, experiments must be carried out over many distance scales

• Newtonian gravity is OK down to $\sim 0.1$ mm. Below $\sim 0.1$ mm we know relatively little about the behavior of Newtonian gravity
Long-Range Forces Over Sub-mm Scales

- Various mechanisms lead to potentials with and inverse power law behavior

\[ V_n(r) = -\alpha_n \left( \frac{G_N m_1 m_2}{r} \right) \left( \frac{r_0}{r} \right)^{n-1} \]

- Also: String Theory Models …

- Such forces can best be detected in experiments where the interacting bodies are very close to each other, as in the Casimir Effect

- Problems
  - How to separate out the new forces from the known electromagnetic effects
  - Forces are intrinsically small
  - \( V(r) \neq 1/r \)

- For example:
  - \( n=2 \) 2-photon exchange; 2-scalar exchange
  - \( n=3 \) 2-pseudoscalar exchange
  - \( n=4 \)
  - \( n=5 \) \( \forall \) -exchange; 2-axion exchange
Neutrino Exchange Force

\[ V_{\nu\bar{\nu}}(r) = \frac{G_F^2}{4\pi^3} \frac{1}{r^5} \]
Gravity in $3 + n$ Non-Compact Dimensions

Potential Energy: \[ V_{\text{Gravity}}(r) = -\frac{GM_1 M_2}{r^{1+n}} \]

Force: \[ \vec{F}_{\text{Gravity}}(r) = -\vec{\nabla} V(r) \]

Observations \( \Rightarrow n = 0 \)
Compact Spatial Dimensions

\[ r \gg R \rightarrow \text{Space appears 3-D} \]
\[ r < R \rightarrow \text{Extra dimensions appear} \]

**Experiment:**
- All matter sees extra dims \( \Rightarrow R < 10^{-15} \text{ m} \)
- Only gravity sees extra dims \( \Rightarrow R < 10^{-4} \text{ m} \)
Gravity with $n$ Compact Extra Dimensions

\[ V_{\text{Gravity}}(r) = \begin{cases} 
-\frac{GM_1M_2}{r}(1 + \alpha e^{-r/\lambda}), & r \gg R \\
-\frac{G'M_1M_2}{r^{1+n}}, & r \leq R
\end{cases} \]

Range of Yukawa: $\lambda \sim R$

Strength Constant $\alpha$:

- Models of extra dimensions: $\alpha \sim 1-10$
- New Force Models: $\alpha \sim ???$
Numerical Estimates

- It is convenient to introduce an energy scale set by the usual Newtonian constant $G \equiv G_4$. This is the Planck mass $M_{pl} \equiv M_4$
  
  $M_4 \equiv M_{pl} = (\hbar c/G_4)^{1/2} = 2.18 \times 10^{-5} \text{ g} = 1.22 \times 10^{19} \text{ GeV/c}^2$

- In natural units ($\hbar = c = 1$) $M_4^2 = 1/G_4$

- The analog of the Planck mass in higher dimensions is called $M_{4+n}$

- When expressed in terms of $M_4$ and $M_{4+n}$ the previous result $G_4 = G_5/4R$ generalizes to

  $M_4^2 \approx R^n(n) M_{4+n}^{n+2}$

- This forms the basis for current experiments
How Big is $M_{4+n}$? How big is $n$?

1) The usual Planck mass $M_4$ and the associated length scale $\frac{\hbar}{M_4 c} \approx 10^{-33}$ cm are the scales at which gravitational interactions become comparable in strength to other interactions, and hence can be unified with these interactions.

2) It would be nice if this happened at a smaller energy scale $\Rightarrow$ bigger length scale. A natural choice is $\sim 1$ TeV $= 10^{12}$ eV where supersymmetry breaks down.

3) This can happen in some string theories with extra spatial dimensions if ordinary matter is confined to 3-dimensional walls ("branes"), and only gravity propagates in the extra dimensions.
How Big is $M_{4+n}$? How big is $n$?

4) In such theories $M_{4+n} \approx 1$ TeV by assumption.

We then solve:

$$M_4^2 \approx R^n(n)M_{4+n}^{n+2}$$

$$(10^{19} \text{ GeV})^2 \quad (1 \text{ TeV})^{n+2}$$

$$R(n) \approx 2 \times 10^{\left(\frac{32}{n-17}\right)} \text{ cm}$$

$n = 1 \Rightarrow R(1) \approx 2 \times 10^{15} \text{ cm} \quad \text{Excluded}$

$n = 2 \Rightarrow R(2) \approx 0.2 = 2 \text{ mm} \quad \text{ Highly Constrained}$

$n = 3 \Rightarrow R(3) \approx 9 \times 10^{-7} \text{ cm} \quad \text{Unconstrained}$
Forces acting between two 1 cm × 1 cm × 1 mm copper plates separated by distance $d$ ($T = 300$ K)

Gravity dominates when $d \gg 10^{-5}$ m

Casimir force dominates when $10^{-10}$ m < $d$ < $10^{-5}$ m
Casimir Review

- Casimir force, arising from quantum fluctuations of vacuum, dominates gravity at short separations.
- Large Casimir background hinders efforts to detect new forces and extra dimensions at short distances.
- Experiments utilizing the iso-electronic effect should be able to negate this Casimir background.
Casimir Force for Ideal Metals at \( T = 0 \)

Zero-Point Fluctuations of Radiation Field

\[ F(d) = -\frac{\pi^2}{240} \frac{\hbar c}{d^4} A \]
Casimir Force for Real Metals at $T \neq 0$: 
Lifshitz Formula

\[ F(d) = \frac{k_B T}{c^3} \int_0^\infty \frac{p^2 dp}{1 + (i \cdot \langle l \rangle)_p^2} e^{2d(\langle a_1 \rangle = c)_p} \]

where

\[ \langle a_1 \rangle = \frac{2 \cdot k_B T l}{\hbar} \]

\[ K(\langle i \cdot \langle a_1 \rangle \rangle) = \frac{2}{4} p^2 1 + (i \cdot \langle a_1 \rangle)_p^2 \]
Isotopic Dependence of the Casimir Force?

Our strategy

\[ F_X = F_{\text{Isotope # 2}}^{\text{Measured}} - F_{\text{Isotope # 1}}^{\text{Measured}} \]

assumes that the Casimir force is the same for two isotopes:

\[ F_{\text{Casimir Isotope # 1}} = F_{\text{Casimir Isotope # 2}} \]

Is this true?
Isotopic Dependence of the Casimir Force: A First Look

1. Casimir Force depends on dielectric properties of interacting bodies

2. Frequency-dependent dielectric constant $\varepsilon(\omega)$ depends on the lattice spacing $a$

3. Lattice spacing $a$ depends on isotopic mass through:
   - Anharmonicity of Interatomic Potentials
   - Zero-Point Motion of Ions
Isotopic Dependence of the Casimir force and New forces and extra dimensions: Logic

Isotopic Mass → Lattice Constant

Dielectric Constant ← Plasma Frequency

Lifshitz Formula → Casimir Force

Casimir Force ↓

Limits on Extra Dimensions and New Forces
Dielectric Constant of a Metal

The optical properties of a material depend on its dielectric constant \( \varepsilon(\omega) \).

**Free Electron Metal:**

\[
\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}
\]

Plasma Frequency:

\[
\omega_p^2 = \frac{4\pi Ne^2}{m_e V}
\]

\( N/V = \# \) of valence electrons/volume

If \( \omega < \omega_p \), light is reflected by the metal

If \( \omega > \omega_p \), light is transmitted through the metal.

For copper and gold, \( \omega_p = 9.3 \times 10^{16} \) rad/s (\( \lambda = 136 \) nm)
Anharmonicity of Interatomic Potentials: Thermal Expansion of Lattice

\[ V(r - r_0) \approx \frac{1}{2} k (r - r_0)^2 - \frac{1}{6} b (r - r_0)^3 \]

Thermal average interatomic spacing:

\[ \langle r - r_0 \rangle = \langle u \rangle = \frac{\int du \ u e^{-V(u)/k_B T}}{\int du \ e^{-V(u)/k_B T}} \approx \frac{b}{2k^2} k_B T \]

\[ a(T) \equiv \langle r \rangle \approx r_0 + \frac{b}{2k^2} k_B T \]
Zero-Point Motion of Atoms:  Isotopic Dependence of Lattice Constant

Thermal Oscillations $\rightarrow$ Quantum Zero-Point Motion

$$k_B T \rightarrow \frac{1}{2} \hbar \omega$$

$$a(T) \equiv \langle r \rangle \approx r_0 + \frac{b}{2k^2} k_B T$$

$$a_{ZP} \equiv \langle r \rangle \approx r_0 + \frac{b}{4k^2} \hbar \omega$$

$$\omega \sim \sqrt{\frac{k}{M}} \quad (M = \text{Atomic Mass})$$

$$a_{ZP} \equiv \langle r \rangle \sim r_0 + \frac{b\hbar}{4k^{3/2}} \frac{1}{\sqrt{M}}$$

Relative difference in $a$ for two different isotopes #1 and #2

$$= \frac{\Delta a}{a} = \frac{a_2 - a_1}{a_1} \sim - \frac{b\hbar}{4k^{3/2}a_1\sqrt{M_1}} \left( \frac{\Delta M}{M_1} \right)$$
Isotopic Dependence of $a$: Experiment

Observed dependence is more complicated!

\[
\frac{\Delta a}{a} \sim - (T \text{- dependent constant}) \frac{\Delta M}{M}
\]

<table>
<thead>
<tr>
<th>Isotopes</th>
<th>$\Delta a/a$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{58}\text{Ni}$, $^{64}\text{Ni}$</td>
<td>$-1.4 \times 10^{-4}$ (T = 78\text{ K}), $5.7 \times 10^{-5}$ (T = 300\text{ K})</td>
<td>Kogan and Bulatov, 1962</td>
</tr>
<tr>
<td>$^{12}\text{C}$, $^{13}\text{C}$ (Diamond)</td>
<td>$-1.5 \times 10^{-4}$ (T = 298\text{ K})</td>
<td>Holloway, et al., 1991</td>
</tr>
<tr>
<td>$^{6}\text{Li}$, $^{7}\text{Li}$</td>
<td>$-4 \times 10^{-4}$ (T = 293\text{ K})</td>
<td>Covington and Montgomery, 1957</td>
</tr>
<tr>
<td>$^{20}\text{Ne}$, $^{22}\text{Ne}$</td>
<td>$-1.9 \times 10^{-3}$ (T = 3\text{ K}), $-1.6 \times 10^{-3}$ (T = 24\text{ K})</td>
<td>Batchelder, Losee, and Simmons, 1968</td>
</tr>
<tr>
<td>$^{70}\text{Ge}$, $^{76}\text{Ge}$</td>
<td>$-5.3 \times 10^{-5}$ (T = 30\text{ K}), $-2.2 \times 10^{-5}$ (T = 300\text{ K})</td>
<td>Sozontov, et al., 2001</td>
</tr>
</tbody>
</table>
Isotopic Dependence of the Casimir Force: Plasma Model

If $d \ll \frac{2^\circ c}{!_p}$ and $T \phi \frac{\hbar c}{2k_B d}$, $F(d) \approx -\frac{\pi^2}{240} \frac{\hbar c A}{d^4} \left(1 - \frac{16}{3} \frac{c}{\omega_p d}\right)$

Since $\omega_p^2 \propto \frac{1}{a^3}$ and $\frac{\Delta a}{a} \sim -10^{-4}$,

Relative difference in $F_{\text{Casimir}}$ for isotopes #1 and #2

\[
\frac{\Delta F}{F} = \frac{F_2 - F_1}{F_1} \sim -\frac{8c}{\omega_{p,1} d} \left(\frac{\Delta a}{a}\right) \\
\sim \frac{8c}{\omega_{p,1} d} \times 10^{-4} \ll 10^{-4}
\]

IUPUI Experiment
(Decca, Lopéz, Fischbach, Krause)
IUPUI Experiment
(Decca, Lopéz, Fischbach, Krause)

Experimental Parameters:

Plate: 500 μm × 500 μm × 3.5 μm
Sphere Radius: 300 μm
Coatings:
   Sphere: 1 nm Cr with 200 nm Au
   Plate: Cu and Ni
Sphere/Plate Separation: 200-1200 nm
Minimum Force:
   Current: 6 fN /Hz^{1/2}
   (Note: E. Coli bacterium weight ~ 10 pN)
Experimental Setup for Casimir Force Measurement between Dissimilar Materials
Electrostatic Force Measurements

![Graph showing electrostatic force measurements with 
\( \Delta V = 0.3 \text{ V} \) and 
\( \Delta V = 0.24 \text{ V} \).]
Casimir Force: Static Measurement
Casimir Pressure: Gradient Measurement
Limits on Extra Dimensions and New Forces:
Sub-micron Limits

Reference: Decca, Lopéz, Fischbach, Krause (unpublished)
Future Work

1. Comparing the Casimir force between an Au sphere and a Au-Ge substrate

2. Comparing the Casimir force between an Au sphere and a $^{58}\text{Ni}$ and $^{64}\text{Ni}$ substrate