Estimating the distribution of the White Dwarf - White Dwarf galactic binary background with LISA

Andrzej Królak
Institute of Mathematics
Warsaw

Joint work with J. Edlund, M. Tinto and G. Nelemans
WD-WD binaries with GW frequency within LISA band are observed. These sources are GUARANTEED

Distribution of WD binaries (Nelemans et al)

Total number of detached binaries: 208736473
Total number of interacting binaries: 34291253

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Simulation of the background signal

Background signal

\[ s(t) = \sum_{n=1}^{N} X_n [t, T_0 = 3\text{yr}; (\varphi_0, \psi, \iota), (D, \beta, \lambda), (M_c, f)] \]

\[ \varphi_0, \quad \psi, \quad \cos \iota \quad \text{uniformly distributed} \]

Sources uniformly distributed in the Galactic disc

\[ \rho(R, z) = \rho_0 \exp(-R / H) \text{sech}^2 \left( \frac{z}{z_0} \right) \]

where \( H = 2.5\text{kpc}, \quad z_0 = 200\text{pc} \)
Spectrum of the LISA noise vs. WDWD background noise

WDWD simulation - Nelemans
LISA noise
WDWD - Bender

$\times \sqrt{3/2} \times 1/2$
Variance of the background signal

\[ \sigma^2(t) = \int_{0}^{\infty} P(f)df \times \sum_{k=-8}^{8} A_k \exp(ik\Omega t) \]

where \( \Omega = \frac{2\pi}{1 \text{ year}} \)

\[ P(f) = \frac{3}{4} \frac{1}{5} N(2\pi f)^{4/3} 2 \int_{M_c} (4M_c)^{10/3} p(M_c, f) dM_c \]

\[ A_k = \int_{V_D} \frac{b_k(\beta, \lambda)}{D^2} p(D, \beta, \lambda) dV_D \]

\( A_k = 0 \) for \( k \neq 0 \) when sources isotropically distributed around the detector

For a galactic disc distribution \( A_k \neq 0 \) for all \( k \)
Cyclostationary random processes

Random process $X(t)$ is cyclostationary if there exists a period $T$ such that

$$E[X(s) X(t)] = C(s, t) = C(s + T, t + T)$$

$T = 1$ year

Spherical harmonics expansion


\[ \sigma^2 = \int d\Omega B(\beta, \lambda) F(\beta, \lambda) \]

\[ B(\beta, \lambda) = \sum_{l,m} B_{lm} Y_{lm}(\beta, \lambda) \]

\( \sigma^2_A, \sigma^2_E, \sigma^2_{AE} \) depend only on \( l = 0 \) (monopole), \( l = 2 \) (quadrupole), \( l = 4 \) (hexadecapole) moments.

Moreover there is degeneracy of \( B_{00} \) and \( B_{02} \) coefficients
Combining the two expansions

\[ C = M \cdot B \]

- Coefficients of Fourier expansion of sample variances
- 51 x 14 matrix
- Coefficients of spherical harmonic expansion

Spherical harmonics coefficients obtained by least squares fit
Sample variances

\[
s_A, s_E, s_{AE}
\]
Harmonics of the sample variance of data – least squares fit

see also, Giampieri, Polnarev, 1997
Coefficients of spherical harmonic expansion and error bars

[ Bd(B00,B02) B21r B21i B22r B22i B40 B41r B41i B42r B42i B43r B43i B44r B44i ]