EQUIVALENCE PRINCIPLE & QUANTUM TEST PARTICLES

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- IF DIFFERENT INTERACTIONS ARE DIFFERENT MANIFESTATIONS OF A UNIFIED DESCRIPTION OF NATURE WE EXPECT MERGING OF GRAVITY AND THE STANDARD MODEL

- THIS IS USUALLY RESTATED AS THE NEED FOR A CONSISTENT (RENORMALIZABLE) THEORY OF QUANTUM GRAVITY

- SOME ATTEMPTS IN THE LAST THREE DECADES, IN PARTICULAR IN THE FRAMEWORK OF STRING THEORIES
  \[ O(10^4) \text{ PAPERS, } O(10^3) \text{ RESEARCHERS,} \]
  BUT SCARCE PREDICTIONS
May be that studies of quantum gravity are premature. Another roadmap which could be data-driven consists of:

1. Further scrutiny of gravity and the standard model in their usual (disjoint) domains of application
   (⇒ to look for failures of one of the two, or both)

2. Study of phenomena in which gravity and quantum are forced to live together

   a) Quantum field theory in a classical curved space-time

   b) Experimental gravitation using microscopic/mesoscopic test particles

In particular, one should explore if geodesics can be operatively defined in terms of quantum test particles.
LIST OF EFFORTS TO EXPLORE "MICROSCOPIC GRAVITATION" IS QUITE RICH

- Free fall of electrons and positrons
- Neutron interferometry
- Free fall of antiprotons & antihydrogen
- Atomic interferometry
- Opto-gravitational atomic traps
- Bouncing neutrons
- Atom lasers in gravitational fields
- Yukawian contributions to gravity
- Gravitationally induced decoherence

All these experiments have in common:

α) Particles with internal dynamics determined by quantum laws

β) Particles which are "fundamental" (no need to use a sample of macroscopic matter defined arbitrarily, with finite tolerance)
Equivalence Principle in Classical Physics:

Test bodies used to determine geodesics are universal, i.e., it is impossible, looking at their free fall, to evidence any intrinsic feature (in particular their inertial mass).

Question: Is this true also by using quantum test bodies?

One can understand various features of this issue by means of a gedankenexperiment of free fall with quantum states

[l. viola & r.d., prd 55, 455 (1997)]
Initial Preparation of the Test Particles in Classical Mechanics in Quantum Mechanics

\[
\begin{align*}
\begin{cases}
\mathbf{z}_0^{(1)} = \mathbf{z}_0^{(2)} \\
\frac{p_0^{(1)}}{m_c^{(1)}} = \frac{p_0^{(2)}}{m_c^{(2)}}
\end{cases}
\Rightarrow
\begin{cases}
\langle \hat{\mathbf{z}} \rangle_{\psi_1} = \langle \hat{\mathbf{z}} \rangle_{\psi_2} \\
\frac{\langle \hat{p}_z \rangle_{\psi_1}}{m_c^{(1)}} = \frac{\langle \hat{p}_z \rangle_{\psi_2}}{m_c^{(2)}}
\end{cases}
\end{align*}
\]

where \( |\psi_1\rangle, |\psi_2\rangle \) are the initial state vectors for test particles 1, 2.

(1) In general \( |\psi_1\rangle \neq |\psi_2\rangle \) (as also in phase space).

(2) \( |\psi_1\rangle \) and \( |\psi_2\rangle \) are not univocally determined by the prescription above.

This prescription is motivated by the correspondence principle for all quantum states having a classical counterpart, and has to be postulated for all remaining states.
Non-classical states can be generated for instance considering linear superpositions of classical states:

\[ \psi_0(z) = \sum_{n=1}^{N} c_n \psi_n(z) \quad c_n \in \mathbb{C} \]

\[ \downarrow \]

\[ \varphi_0(p) \]

\[ \langle \hat{p}^2 \rangle_0 = \sum_{n} |c_n|^2 \langle \psi_n \mid \hat{p}^2 \psi_n \rangle + \sum_{m,m'} c_m^* c_{m'} \langle \psi_m \mid \hat{p}^2 \psi_{m'} \rangle \]

\[ \langle \hat{p}^2 \rangle_{\text{class}} + \langle \hat{p}^2 \rangle_{\text{purely quantum}} \]

Quantum \[\Rightarrow\] Classical

If \[\langle \psi_n \mid \hat{p} \psi_n \rangle = 0 \quad \forall n, \] then \[\langle \hat{p} \rangle_0 \] is purely quantum.

Schrödinger cat states:

\[ \psi_0(z) = N \left\{ c_+ e^{-\frac{(z-\xi_0+\Delta)^2}{2\Delta_0^2}} + c_- e^{-\frac{(z-\xi_0-\Delta)^2}{2\Delta_0^2}} \right\} \]

\[ |N|^2 = \left( \frac{\Delta_0^2}{\pi} \right)^{-\frac{1}{2}} \left\{ 1 + |c_+|^2 + |c_-|^2 + 2\Re(c_+ c_-) e^{-i\Delta_0^2} \right\}^{-1} \]
\begin{align*}
\langle \hat{\varepsilon}_c \rangle_{\text{ca}r} &= \varepsilon_0 - \Delta \quad \frac{1c_+^2 - 1c_-^2}{1c_+^2 + 1c_-^2 + \text{Re}(c_+ c_-^*) e^{-\Delta^2/\Delta_0^2}} \\
\langle \hat{p}_c \rangle_{\text{ca}r} &= -2 \frac{\hbar A}{\Delta_0^2} \quad \frac{\text{Im}(c_+ c_-^*) e^{-\Delta^2/\Delta_0^2}}{1c_+^2 + 1c_-^2 + \text{Re}(c_+ c_-^*) e^{-\Delta^2/\Delta_0^2}} \\
\text{Im}(c_+ c_-^*) &= |c_+| |c_-| \sin \Theta
\end{align*}

Various ways to match the Gaussian prescription:

1. \( |c_+| = |c_-| \Rightarrow \langle \hat{\varepsilon}_c \rangle = \langle \hat{\varepsilon}_c \rangle_2 \)

\[ \Theta = 0, \pi, 2\pi \ldots \quad \langle \hat{p}_c \rangle_1 \cdot \langle \hat{p}_c \rangle_2 = 0 \]

2. \( \cos \Theta = 0 \Rightarrow \frac{m_i^{(c, \text{in})}}{m_i^{(c, \text{out})}} = \frac{\Delta_i}{\Delta} e^{-\frac{A_i^2 - \Delta_i^2}{\Delta_0^2}} \]

3. \( \Delta_i / \Delta_0 = 1 \Rightarrow \frac{m_i^{(c, \text{in})}}{m_i^{(c, \text{out})}} = \frac{\sin \Theta_1 \cdot (1 + e^{-1} \cos \Theta_0)}{\sin \Theta_2 \cdot (1 + e^{-1} \cos \Theta_0)} \]

This means that the initial state can be prepared in a way for which the comparision of the free fall of the two test particles is fair and simple.
**Running the Gedankenexperiment**

\[ H = \frac{\hat{p}_z^2}{2m_i} + mg \hat{z} \]

Possible Observable: Time of Flight

From \( z_0 = \langle \hat{z} \rangle_{\psi(t=0)} \) to \( \phi = \langle \hat{z} \rangle_{\psi(t=Tof)} \)

Times of Flight in Q.D. can only be predicted in a probabilistic way.

\[ \langle z^{(k)}(t) \rangle = - \frac{1}{2} \frac{mg_{(z)}}{m_{(z)}} g t^2 + \frac{\langle \hat{p}_z \rangle_{\psi_0}}{m_{(z)}} t + \langle \hat{z} \rangle_{\psi_0} \]

Average Time of Flight

\[ Tof = \sqrt{2 \frac{m_{(z)}}{mg_{(z)}} z_0} \]

if \( \langle \hat{z} \rangle_{\psi_0} = 0 \) and \( \langle \hat{p}_z \rangle_{\psi_0} = 0 \)

Spreading of TOF

\[ 6_{Tof} = \frac{6_z(Tof)}{\sqrt{\overline{6}_z(Tof)}} \]

or

\[ 6_{Tof} = \frac{t_+ - t_-}{2} \]

where \( t_\pm \) such that

\[ \langle z(t) \rangle_{\psi_0} \pm 6_z(t) = 0 \]

All is analytically calculable for Gaussian and cat-like states.
\[ T_{of}^{(k)} = 6_{b.o.f.}^{(k)} = \sqrt{2} \left( \frac{m_i^{(k)}}{m_g^{(k)}} \right) \frac{\Delta \theta}{g} \pm \frac{\sqrt{2}}{2} \delta \frac{h}{\Delta m g} \]

\[
\delta = \begin{cases} 
1 & \text{GAUSSIAN STATE} \\
(e^{-1})^{1/2} & \text{MALE (+) / FEMALE (-) CAT STATES} 
\end{cases}
\]

Even if the experiment provides identical values for the average time of flight, the standard deviation is mass-dependent.

- Time of flight probability distributions depend on the state and the mass of the test particle.

- However, the same dependence is also expected if the time of flight is measured in an accelerated frame 'simulating' the free fall: the usual equivalence between free fall and non-inertial frames still holds.
FURTHER ISSUES ARISE IF ONE IMAGINE A HIGH-PRECISION CONTINUOUS MONITORING OF THE TEST MASS

\[
\frac{d \hat{\rho}}{dt} = -i \frac{\hbar}{\Delta} \left[ \hat{H}, \hat{\rho} \right] - \frac{K_z}{2} \left[ \hat{z}, [\hat{z}, \hat{\rho}] \right]
\]

(LINDBLOM EQUATION)

\(K_z\) IS THE COUPLING OF THE POSITION REGISTER (m^2/Hz^2)

BY INTRODUCING \(\rho(z, z', t) = \langle z | \hat{\rho}(t) | z' \rangle\) WE GET

\[
\frac{\partial \rho(z, z', t)}{\partial t} = \left\{ \frac{i \hbar}{2\Delta} \left( \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial z'^2} \right) - \frac{2i \Delta}{\hbar} (z - z') - \frac{K_z}{2} (z - z')^2 \right\} \rho(z, z', t)
\]

THE NET EFFECT OF THE CONTINUOUS MEASUREMENT TERM IS TO ADD SPREADING IN THE POSITION VARIANCE, AND THEN A FURTHER SPREADING IN THE TIME OF FLIGHT DISTRIBUTION

\[
\sigma_z^2(t; K_z) = \sigma_z^2(t; K_z = 0) + \frac{K_z}{3} \left( \frac{\hbar}{\Delta m_i} \right) t^3
\]

\(m_i\) - DEPENDENT

IN A GENERIC GRAVITATIONAL FIELD (WITH CURVATURE) THE MEASUREMENT TERM ALSO AFFECTS THE AVERAGE TIME OF FLIGHT (A SORT OF "GRAVITATIONAL" QUANTUM ZERO EFFECT)
Cesium Atoms Bouncing in a Stable Gravitational Cavity

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![Experimental setup](image)

**FIG. 1.** (a) Experimental setup. Atoms from a magneto-optical trap are released above a curved mirror formed by an evanescent light wave. The number of atoms present in this stable gravitational cavity is measured as a function of time using the fluorescence induced by a probe beam. (The shown beam radii are not to scale.) (b) Number of atoms in the probe beam, for different times after their release (points). Background pressure $3 \times 10^{-7}$ mbar, mirror power 800 mW, detuning 1.9 GHz, and waist 1 x 1.1 mm. The curve is a fit calculated by our Monte Carlo simulation of the experiment; the fitted parameters are loss per bounce, temperature, the radius of the cloud when it is first dropped, and the drop height. The values used here are 39% loss per bounce, temperature 4 μK, initial cloud radius 0.25 mm (1 standard deviation of the Gaussian profile), and drop height 2.91 mm. The simulation assumed a reflective parabolic surface, elliptical in the horizontal plane, of major axes 2.1 by 2.3 mm (diameter). These diameters were obtained from measurements of the profile of the elliptical Gaussian beam used to form the mirror, combined with a calculation from Eq. (1).

Phase Modulation of Atomic de Broglie Waves

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![Time-of-flight signals](image)

**FIG. 3.** Time-of-flight signals. (a) Nonmodulated potential; (b)–(d) modulated potential, with frequency $\nu = 950$ kHz (b), 880 kHz (c), and 800 kHz (d). The modulation depth is $\delta = 0.82$. The width of the points indicates the time window for digitization and averaging of the continuous signal.

Reflection from a vibrating mirror
TABLE I: Predicted standard deviations for the times of flight (in msec) of freely falling Gaussian states and Schrödinger cat (male, \(c_+ = c_- = 1\) and female, \(c_+ = -c_- = 1\)) states, starting at rest from an height \(z_0 = 3\) mm. The corresponding average time of flight is \(T_\text{av} = 24.74\) ms and the values \(\Delta = \Delta_0 = 100\) am have been chosen.

<table>
<thead>
<tr>
<th>Gaussian state</th>
<th>Male cat state</th>
<th>Female cat state</th>
</tr>
</thead>
<tbody>
<tr>
<td>He (^a)</td>
<td>10.86</td>
<td>7.38</td>
</tr>
<tr>
<td>Be</td>
<td>4.82</td>
<td>3.28</td>
</tr>
<tr>
<td>Na</td>
<td>1.89</td>
<td>1.29</td>
</tr>
<tr>
<td>Rb</td>
<td>0.36</td>
<td>0.24</td>
</tr>
<tr>
<td>Cs</td>
<td>0.33</td>
<td>0.22</td>
</tr>
</tbody>
</table>

\(^a\)S metastable helium.

- **The experiment is within reach with ultracold atoms prepared in non-classical states.**

- **In any event, the ideal experiment allows us to focus on possible conflicts between gravitation and quantum physics:**

  a) **Trajectories are not defined in quantum physics, so geodesics**

  b) **A test mass either is in free fall (unmeasured) or is perturbed by the measurement apparatus**