Experiments for Gravitation in the Solar System

Hansjörg Dittus
ZARM, University of Bremen, Germany,

I acknowledge support from:

C. Lämmerzahl,
ZARM, University of Bremen, Germany

Slava G. Turyshev
Jet Propulsion Laboratory, California Institute of Technology, USA,
Background

Many aspects of General Relativity are well tested and confirmed:

Foundations:
- Universality of Free Fall
- Local Lorentz Invariance
- Universality of Gravitational Redshift

Predictions:
- Solar System Effects
  - Perihelion shift
  - Gravitational Redshift
  - Light deflection
  - Time delay
  - Gravitomagnetic effects
- Strong field observations
  - Binary systems
  - Black holes
- Gravitational waves
- Cosmology
Experimental confirmation of GR

- Tests within PPN frame

\[
g_{00} = -1 + 2\alpha \frac{U}{c^2} - 2\beta \frac{U^2}{c^4}
\]
\[
g_{0i} = 4\mu \frac{(\vec{J} \times \vec{r})}{c^3 r^3}
\]
\[
g_{ij} = (1 + 2\gamma) \frac{U}{c^2}
\]

| perihelion shift | astronomical observations | \(\left| \frac{2}{3}(\alpha + \gamma) - \frac{1}{3}\beta - 1 \right| \leq 10^{-4}\) |
|------------------|---------------------------|----------------------------------|
| light deflection | Very Long Baseline Interference | \(\left| \gamma - 1 \right| \leq 10^{-4}\) |
| time delay       | Cassini S/C                | \(\left| \gamma - 1 \right| \leq 2 \cdot 10^{-5}\) |
| gravitational redshift | Gravity Probe A          | \(\left| \alpha - 1 \right| \leq 1.4 \cdot 10^{-4}\) |
| Lense-Thirring effect | LAGEOS satellites       | \(\leq 0.1\) |
| Schiff effect    | Gravity Probe B            | \(\leq 5 \cdot 10^{-3}\) (not yet confirmed) |

*Moriond-Meeting, 17.3.2007*
Space experiments for Eddington parameters

Turyshnev et al., 2006

Moriond-Meeting, 17.3.2007
Fundamental quests

- All approaches to quantum gravity predict deviations from principles underlying GR

- Tests for
  - violations of the UFF
  - violations of the UGR
    - time–dependence of the fine structure constant
    - time–dependence of the gravitational constant
  - violations of LLI in many aspects:
    - non–isotropy of light propagation
    - non–constancy of velocity of light
    - fundamental dispersion of light propagation
    - non–isotropy of elementary particle parameters like mass
    - search for anomalous spin–interactions
  - non–Einsteinian effects like
    - Yukawa–like gravitational potential
    - Modification of gravity at large scale
    - Modification of weak gravity
    - Nordtvedt–effect
    - time–variation of the gravitational constant $G$
Fundamental quests

- Additional tests
  - linearity of quantum physics
  - entanglement
  - Casimir force
  - physics of Bose–Einstein condensates
  - search for a fundamental decoherence
  - test of renormalization group theory

- Quantum gravity characterized by

\[ E_P \approx 10^{28} \text{ eV} \]

\[ 10^2 \text{ GeV} \]

- Laboratory experiments:

- But, all predictions are merely hypotheses (not based on complete theories):

  *Any improvement of the accuracy of experimental results is of great value.*
New interactions?

- Violation of the UFF at the $10^{-13}$ level predicted from dilaton scenarios and at the $10^{-14}$ level from quintessence theories
- Deviation from GR in terms of the PPN parameters $\gamma$ and $\beta$, again predicted within dilaton scenarios
- Violations of LLI at Planck scale predicted from non–commutative geometry approaches
- Time–dependence of fundmental constants
- Additional Yukawa part of the gravitational potential at small distances predicted from higher–dimensional theories
- Gravitational interaction at large distances or for weak gravitational fields (Yukawa-potential for large ranges or MOND for small gravitational accelerations, maybe relevant to explain the Pioneer anomaly
Yukawa potential tests
Space conditions

- Infinitely long and periodic free fall:
  - High precision tests for UFF

- Long interaction times:
  - Atomic and molecular interferometry / laser cooling
  - Long exposure to certain interactions
  - Space clocks
  - BEC

- Large gravitational potential differences along a S/C orbit
  - Tests of Gravitational Redshift

- Large velocity (wrt the cosmic background) changes along the S/C orbit
  - Kennedy-Thorndike tests
  - $\Delta v$ 30 times higher than on Earth

- Long distance experiments
  - Huge interferometric arm lengths (observation of gravitational waves)
  - Large distance tests of Newton’s Law

- Low noise environment (microgravity)
  - Drag free environment important for frequency range below $10^{-2}$ Hz

- Observations in the grav. fields of large spinning objects
  - Gravitomagnetic effects
Disadvantages of space tests

- Huge financial effort
- Long time for preparation and development
- Unforeseeable time lines
- No direct access to the experiment during operation
- Post–mission analysis of the experimental payload not possible
Structure of experimental exploration of gravity

- Yukawa potential
  - Gravity at large scales
  - Weak gravity
  - Nordtvedt effect
  - Time dependence of $G$

- Einstein’s theory of gravity
  - Metric theory of gravity
    - Einstein’s Equivalence Principle
      - Universality of Free Fall
        - MICROSCOPE, STEP, GG, HYPER, DSGE
      - Universality of Gravitational Redshift
        - ACES / PHARAO, SPACETIME, OPTIS, DSGE
      - Local Lorentz Invariance
        - ACES, OPTIS, PARCS, RACE
  - Gravitational redshift
    - Perihelion shift
    - Light deflection
    - Gravit. time delay
    - Lense-Thirring effect
    - Schiff effect
    - Gravitational waves
      - GP-A, Cassini, LAGEOS, GP-B, LISA, HYPER, DSGE, ASTROD, LATOR, OPTIS
Mission objectives to be discussed
Space experiments so far
Lunar Laser Ranging

- Mirrors placed on moon surface between 1969 and 1973: Apollo 11, 14, 15 and Luna 17, 21
- Resolution: ca. 1 cm
- Laser pulse width: 150 – 300 ps
- Pulse frequency: 10 Hz
- "Illuminated" area on moon surface: 20 km²
- 1 of 10¹⁹ photons is observed (1 photon per 10 pulses)
- Observatories: McDonald Obs. Fort Davis TX (60 cm telescope), Grasse, and Mt. Haleakala
Lunar Laser Ranging

Earth and Moon are freely falling masses within the sun’s gravitational field.

\[ \zeta_E \leq 10^{-13} \quad \text{for} \quad a_d = \zeta_E \frac{Gm_S}{r_{ES}^2} \]

depending on the validity of the strong gravitational Equivalence Principle.

Selfgravitation \( \Omega \) contributes the same to inertial and gravitational mass.

Nordtvedt parameter

\[ \zeta \leq 4 \dot{\alpha} - \ddot{\alpha} - 3 \frac{10}{3} \dot{i} - \ddot{i} \dot{\alpha}_1 + \frac{2}{3} \ddot{\alpha}_2 - \frac{2}{3} \dot{\alpha}_1 - \frac{1}{3} \alpha_2 \leq 10^{-3} \]
Cassini-Exp. / Shapiro Time Delay

Einstein-Infeld-Hoffmann equation
- Numerical models based on isotropic PPN n-body metric metric
- Planets and asteroids treated as point masses
- Accelerations calculated wrt their barycentric position

\[
\ddot{\vec{r}}_i = \sum_{j \neq i} \frac{G m_j (\vec{r}_j - \vec{r}_i)}{\left| \vec{r}_j - \vec{r}_i \right|^3}
\]

\[
\begin{align*}
&\left[ 1 - \frac{2(\dot{\alpha} + \ddot{\alpha})}{c^2} \sum_{k \neq i} \frac{G m_j}{|\vec{r}_i - \vec{r}_k|} - \frac{2\ddot{\alpha} - 1}{c^2} \sum_{k \neq i} \frac{G m_j}{|\vec{r}_j - \vec{r}_k|} + \frac{\ddot{\alpha}}{c^2} + (1 + \dot{\alpha}) \frac{\dot{\vec{r}}_j}{c^2} - \frac{2 + 2\ddot{\alpha}}{c^2} \dot{\vec{r}}_i \cdot \dot{\vec{r}}_j - \frac{3}{2c^2} \left( \frac{\dot{\vec{r}}_i - \dot{\vec{r}}_j}{|\vec{r}_j - \vec{r}_i|} \right)^2 + \frac{1}{c^2} \left( \vec{r}_j - \vec{r}_i \right) \ddot{\vec{r}}_j \right] \\
+ \frac{1}{c^2} \sum_{j \neq i} \frac{G m_j}{|\vec{r}_j - \vec{r}_i|} \left( (2 + 2\ddot{\alpha}) \dot{\vec{r}}_i - (1 + 2\ddot{\alpha}) \dot{\vec{r}}_j \right) \right) + \frac{3 + 4\ddot{\alpha}}{2c^2} \sum_{j \neq i} \frac{G m_j}{|\vec{r}_j - \vec{r}_i|}
\end{align*}
\]

Time delay for curved space-time due to grav. fields of Sun and Earth

\[
\dot{\Delta t} = \frac{\left| \vec{r}_i^C - \vec{r}_s^C \right|}{c} + \frac{(1 + \ddot{\alpha}) G m_S}{c^3} \ln \left( \frac{\vec{r}_s^S + \vec{r}_t^S + \vec{r}_t^S - \vec{r}_s^S}{\vec{r}_s^S + \vec{r}_t^S - \vec{r}_t^S - \vec{r}_s^S} + \frac{(1 + \dot{\alpha}) G m_S}{c^2} \right) + \frac{(1 + \ddot{\alpha}) G m_E}{c^3} \ln \left( \frac{\vec{r}_s^E + \vec{r}_t^E + \vec{r}_t^E - \vec{r}_s^E}{\vec{r}_s^E + \vec{r}_t^E - \vec{r}_t^E - \vec{r}_s^E} + \frac{(1 + \dot{\alpha}) G m_S}{c^2} \right)
\]

Cassini Conjunction Experiment  2002:
- Satellit - Earth distance > 10^9 km
- Ranging: X~7.14GHz  &  Ka~34.1GHz (dual band)
- Result: \( \gamma = 1 + (2.1 \pm 2.3) \times 10^{-5} \)

Moriond-Meeting, 17.3.2007
**Gravity Probe B** *(launched April 2005)*

- Measurement of gyro precession due to space curvature of the rotating Earth
- 6.6 arcsec per year from geodetic precession
- 0.042 arcsec per year from Lense-Thirring effect (frame dragging, Schiff effect)
- Experiment has been proposed already 1959 *(G. Pugh)*
Future Plans
Micro-satellite à Trainée Compensée pour l’Observation du Principe d’Equivalence (MICROSCOPE)

Missions parameters:

- Sunsynchronous orbit: 660 km
- Orbit exzentricity: < 5 \cdot 10^{-3}
- Spin rate: variable, modulates the orbit frequency
- Signal frequency: \((\pi + 1/2) f_{orb}\) and \((\pi + 3/2) f_{orb}\)
- Mission duration 6 to 12 months
- Satellite mass: < 120 kg
- CNES project (with DLR- and ESA-contributions)
OPTIS - Advanced tests of Special and General Relativity

- Michelson-Morley
- Kennedy-Thorndike
- Univ. grav., red-shift

U(x₁) to sun

perigee 10000 km

apogee 40000 km

U(x₂)

Laser-tracking

Laser-link

Univ. grav.

1

frequency comparison

2

frequency comb

atomic clock(s)

laser

cavity
Moriond-Meeting, 17.3.2007

**LATOR-Mission Concept**

*Turyshev et al.*

---

**Reference spacecraft** $t_1$

- $D_{S-Earth} \geq 2$ AU $\approx 300$ million km

**Target spacecraft** $t_2$, $t_3$

- $D_{R-T} \approx 5$ million km

\[ \cos \theta \neq \frac{t_1^2 + t_2^2 - t_3^2}{2t_1t_2} \]

\[ \theta \approx 1^\circ \]

**Accuracy needed:**
- Distance: $\sim 1$ cm
- Angle: 0.1 picorad

Geometric redundancy enables a very accurate measurement of curvature of the solar gravity field

---

**Accurate test of gravitational deflection of light to 1 part in $10^9$**
LISA: Joint ESA/NASA Mission

- Cluster of 3 S/C in heliocentric orbits at 1 AU
- S/C form an equilateral triangle with 5 mio km arm length
- Earth trailing orbit: 20° behind the Earth
- Leaned 60° with respect to the ecliptic
- S/C contain laser and inertial test masses
- System forms a Michelson Interferometer
- Designed for galactic and cosmological sources
- Launch 2014

K.Danzmann, MPI-AEI
Moriond-Meeting, 17.3.2007
Everything outside the Solar System refuses to follow the laws of General Relativity.

Joao Magueijo, Imperial College
Open questions and motivation

- Unexplained phenomena within GR
  - Dark Matter (Zwicky 1933):
    to describe galactic rotation curves, gravitational lensing effects and early structure formation in cosmological models
  - Dark Energy (Turner 1999):
    to describe the accelerated expansion of the universe seen from supernovae observations and CMB anisotropy measurements
  - Increase of the Astronomical Unit (Pitjeva 2005, Krasinski 2005):
    length scale related to the earth-sun distance increases by $7 \pm 1$ cm per 100 years (confirmed by astronomical observations); solar mass loss only explains ca. 1 m per century
  - Quadrupole/Octopole Anomaly (Tegmark et al. 2005, Schwarz et al. 2005):
    quadrupole and octopole of CMB are correlated with solar system eclipse
  - Pioneer Anomaly (Anderson et al. 1998, 2002/04)
    confirmed for 3 satellites
    satellite trajectory velocities are too high by some [mm/s] after planetary fly-bys / require non-conservative gravitational potential

Moriond-Meeting, 17.3.2007
Indication, from Pioneer 10/11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration

John D. Anderson,1* Philip A. Laing,2,+ Eunice L. Lau,1,‡ Anthony S. Liu,3,§
Michael Martin Nieto,4,∥ and Slava G. Turyshev1,¶

1Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109
2The Aerospace Corporation, 2350 E. El Segundo Boulevard, El Segundo, California 90245-4691
3Astrodynamics Sciences, 2393 Silver Ridge Avenue, Los Angeles, California 90039
4Theoretical Division (MS-B285), Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545
(Received 10 June 1998)

Radio metric data from the Pioneer 10/11, Galileo, and Ulysses spacecraft indicate an apparent anomalous, constant, acceleration acting on the spacecraft with a magnitude $\sim 8.5 \times 10^{-8}$ cm/s$^2$, directed towards the Sun. Two independent codes and physical strategies have been used to analyze the data. A number of potential causes have been ruled out. We discuss future kinematic tests and possible origins of the signal. [S0031-9007(98)07300-1]

PACS numbers: 04.80.Cc, 95.10.Eg, 95.55.Pe
Pioneer Anomaly

- Pioneers 10/11: most precisely navigated deep space satellites (Jet Propulsion Lab., Pasadena CA)

- Observation of a small, anomalous, blue-shifted Doppler frequency drift (Anderson et al. 1998, 2002), uniformly changing with the rate of

\[ \dot{f}_p = (5.99 \pm 0.01) \cdot 10^{-9} \text{ Hz/s} \]

- Drift can be interpreted as a sunward constant acceleration of

\[ a_p = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2 \]

- This interpretation has become known as the Pioneer Anomaly:
  - Constant acceleration of the spacecraft toward the Sun
  - Anomaly occurs when satellites have set to hyperbolic (escape) orbits
  - No real indication of how far out the anomaly goes.
  - Temporal and spatial variations are less than 3%
The orbits of Pioneer 10 and 11

- Elliptical (bound) orbits before last fly-by
- Hyperbolic (escape) orbits after last fly-by
Detection of the Anomaly

- Search for unmodeled accelerations with Pioneers started in 1979:
  - Motivation: search for Planet X – initiated when Pioneer 10 was at 20 AU;
  - The solar-radiation pressure away from the Sun became $< 5 \times 10^{-10}$ m/s$^2$

- Original detection of the anomaly by JPL orbit determination in 1980:
  - The analysis found the biggest systematic error in the acceleration residuals is a constant bias $a_p \sim (8 \pm 3) \times 10^{-10}$ m/s$^2$ directed towards the Sun
Observed Anomalous Doppler Drift

frequency received at S/C:

\[ f' = \frac{1}{\sqrt{1-v^2/c^2}} \left( 1-\frac{v}{c} \right) \cdot f \]

frequency sent back and received on earth:
(neglecting the transponder shift)

\[ f'' = \frac{1}{\sqrt{1-v^2/c^2}} \left( 1-\frac{v}{c} \right) \cdot f' \]

\[
\frac{f'' - f}{f} = -2 \frac{v/c}{1 + v/c} \approx -2 \frac{v}{c} \\
 v_{\text{observed}} - v_{\text{modelled}} = -a_p t
\]

The two-way Doppler residuals for Pioneer 10 vs time
[1 Hz is equal to 65 mm/s range change per second].

Anderson et al.

Moriond-Meeting, 17.3.2007
## On-board Systematics: Power and Heat

### Error Budget Constituents

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Bias $[10^{-10} \text{ m/s}^2]$</th>
<th>Uncertainty $[10^{-10} \text{ m/s}^2]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio beam reaction force</td>
<td>+1.10</td>
<td>±0.11</td>
</tr>
<tr>
<td>Thermal and propulsion effects from RTGs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>→ RTG heat reflected off the S/C</td>
<td>-0.55</td>
<td>±0.55</td>
</tr>
<tr>
<td>→ Differential emissivity of the RTGs</td>
<td></td>
<td>±0.85</td>
</tr>
<tr>
<td>→ Non-isotropic radiative cooling of S/C</td>
<td></td>
<td>±0.16</td>
</tr>
<tr>
<td>→ Expelled He produced within the RTGs</td>
<td>+0.15</td>
<td>±0.16</td>
</tr>
<tr>
<td>Mass expulsion / gas leakage</td>
<td></td>
<td>±0.56</td>
</tr>
<tr>
<td>Variation between S/C determinations</td>
<td>+0.17</td>
<td>±0.17</td>
</tr>
</tbody>
</table>

### Diagram

- Radioisotope Thermoelectrical Generator (SNAP-19)

- $^{94}\text{Pu}^{238} \rightarrow ^{92}\text{U}^{234} + 2\text{He}^4$
  - Half life: 87.74 years

- 1987 [97 W] ~32.8% reduction


---

*Moriond-Meeting, 17.3.2007*
Yukawa modification?

- Ansatz
  \[ V(r) = G \frac{M_{\text{Sun}}}{r} \left(1 + \alpha \cdot e^{-r/\lambda}\right) \]

- with Taylor extension and
  \[ G_0 = (1 + \alpha)G \] as observed grav. constant for \( r \to \infty \)

- \[ a(r) = -G_0 \frac{M_{\text{Sun}}}{r^2} + \frac{\alpha}{1 + \alpha} G_0 \frac{M_{\text{Sun}}}{2\lambda^2} - \frac{\alpha}{1 + \alpha} G_0 \frac{M_{\text{Sun}}}{3\lambda^2} \frac{r}{\lambda} + \ldots \]

  \[ = a_p (\text{anomalous Pioneer acceleration}) \]

- Next order term smaller by \( 2/3(r/\lambda) \leq 0.06 \) (could account for small decrease observed during missions)

- Strong \( \alpha \approx 1 \)
  \[ \to \text{long range coupling} \]
  \[ \to \text{acceleration plateau} \]
  \[ \text{between ca. } 1 - 100 \text{ AU.} \]
Yukawa modification? (2)

\[ a_p = \frac{\alpha}{1 + \alpha} G_0 \frac{M_{\text{Sun}}}{2\lambda^2} \quad \Rightarrow \quad \alpha = \frac{2\lambda^2 a_p}{G_0 M_{\text{Sun}} - 2\lambda^2 a_p} \]

\[ \Rightarrow \lambda \geq \sqrt{\frac{G_0 M_{\text{Sun}}}{2a_p}} = 2.8 \cdot 10^{14} \text{ m} \]

A viable model?

- **Pioneer anomaly**
  \[ \log_{10} (\lambda) > 16, \alpha + 1 \leq 10^{-5} \]

- **Galactic rotation curves**
  \[ \log_{10} (\lambda) > 16, \alpha + 1 \leq 10^{-1} \]

- **Local strength:**
  modification by „Yukawa in Yukawa“

\[ \log_{10} (\lambda) > 16 \text{ for } \log_{10} |\alpha| = 1 \text{ compatible with present experimental results in the solar system (including planetary orbits)} \]

(Lämmerzahl, 2005)

Moriond-Meeting, 17.3.2007
Fly-by Anomaly

- 2-way S-band Doppler and range residuals during fly-bys at Earth show an exit (asymptotic) velocity greater than expected reported by several authors (Antreasian & Guinn 1998, Anderson & Williams 2001, Morley 2005, Preuss 2006)
Fly-by / gravity assist manoeuver

\[ \Delta E_{\text{kin}} / m_{\text{sat}} = \left( v_0^2 - v_i^2 \right) / 2 \]
\[ = v_{\text{earth}} (v'_0 - v'_i) \]

Anderson, Campbell, Nieto, 2006
Earth fly-by’s analyzed

<table>
<thead>
<tr>
<th></th>
<th>Galileo (1st fly-by)</th>
<th>NEAR</th>
<th>Cassini</th>
<th>Rosetta</th>
<th>Messenger</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_\infty$ [km/s]</td>
<td>8.949</td>
<td>6.851</td>
<td>16.01</td>
<td>3.863</td>
<td>4.056</td>
</tr>
<tr>
<td>$h$ [km]</td>
<td>956</td>
<td>532</td>
<td>1,172</td>
<td>1,954</td>
<td>2,336</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>2.47</td>
<td>1.81</td>
<td>5.86</td>
<td>1.31</td>
<td>1.13</td>
</tr>
<tr>
<td>$\Theta$ [°]</td>
<td>47.67</td>
<td>66.92</td>
<td>19.66</td>
<td>99.396</td>
<td>94.7</td>
</tr>
<tr>
<td>$i$ [°]</td>
<td>142.9</td>
<td>108.0</td>
<td>25.4</td>
<td>144.9</td>
<td>133.1</td>
</tr>
<tr>
<td>$\Delta v_\infty$ [mm/s]</td>
<td>3.92 ± 0.08</td>
<td>13.46 ± 0.13</td>
<td></td>
<td>1.82 ± 0.05</td>
<td></td>
</tr>
<tr>
<td>$\Delta v_F$ [mm/s]</td>
<td>2.56 ± 0.05</td>
<td>7.24 ± 0.07</td>
<td>[-0.2] (?)</td>
<td>0.67 ± 0.02</td>
<td>O (0)</td>
</tr>
</tbody>
</table>

- to be implemented: Hayabusa fly by 05/2004 ($h = 3,725$ km)
- Cassini data not reliable due to perigee manoeuver
- 2nd Galileo fly by too deep / large atmospheric influence
## Error analysis

### Error budget constituents

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric drag</td>
<td>- 0.0001</td>
</tr>
<tr>
<td>Ocean tides</td>
<td>± 0.1</td>
</tr>
<tr>
<td>Solid earth tides</td>
<td>«</td>
</tr>
<tr>
<td>S/C charging (modeled / analyzed for LISA; for charging Q &lt; 10(^{-7}) C)</td>
<td>± 0.0001</td>
</tr>
<tr>
<td>Magnetic moments (&lt; 2 \cdot 10(^{-7}) G/m)</td>
<td>± 10(^{-10})</td>
</tr>
<tr>
<td>Earth albedo (1 t S/C)</td>
<td>± 0.00024</td>
</tr>
<tr>
<td>Solar wind</td>
<td>± 0.0003</td>
</tr>
<tr>
<td>Relativistic corrections (U \cdot v^2 / c^2 \approx 10^{-20})</td>
<td>not affecting</td>
</tr>
<tr>
<td>Spin rotation coupling (coupling of the helicity of radio waves with S/C spin and Earth rotation (only effective for 2-way Doppler ranging))</td>
<td>not affecting</td>
</tr>
</tbody>
</table>
Phenomenological observations

- $\Delta v$ decreases with increasing eccentricity and perigee height
- $\Delta v$ disappears at $e = 1$ (as expected for bound orbits)

\[ a_{\text{Fly-by}} = 10^{-4} \text{ m/s}^2 \quad \frac{a_{\text{Fly-by}}}{a_{\text{Newton}}} \approx 10^{-5} \approx \frac{a_{\text{PA}}}{a_{\text{Newton}}} \]
Technology
Key technologies / Drag free control of S/C

- Air-drag
- Radiation pressure
- Magnetic fields
- Solar wind, etc.

- Internal perturbations
  - Patch effects
  - Radiometer effect
  - Non-perfect shielding etc.

$\Rightarrow$ Huge complexity of signal!
Precision of attitude control / Drag-free control

- Satellite follows a freely falling test mass
- Movement of mass is controlled permanently (closed-loop control)
- Micro propulsion thrusters direct the satellite

Residual acceleration: ca. $10^{-14}$ m / s$^2$
Force increment resolution: ca. 0.1 µN
**Importance of directional resolution**

<table>
<thead>
<tr>
<th>Central forces from the Sun:</th>
<th>Sun-pointing</th>
</tr>
</thead>
<tbody>
<tr>
<td>“fifth force”, modified gravity, cosmological influence</td>
<td></td>
</tr>
<tr>
<td><strong>Other Central forces</strong></td>
<td>Pointing towards the object</td>
</tr>
<tr>
<td>– e.g. Galactic centre</td>
<td></td>
</tr>
<tr>
<td><strong>Blue shifting of light:</strong></td>
<td>Along Earth-s/c vector</td>
</tr>
<tr>
<td>– varying speed of light, cosmology</td>
<td></td>
</tr>
<tr>
<td><strong>Drag:</strong></td>
<td>Along velocity vector of dragging material</td>
</tr>
<tr>
<td>– conventional, i.e. dust, gas;</td>
<td></td>
</tr>
<tr>
<td>– other, coupling to dark matter</td>
<td></td>
</tr>
<tr>
<td><strong>Modification of inertia</strong></td>
<td>Along velocity vector? Sun-pointing?</td>
</tr>
<tr>
<td><strong>On-board systematics (heat, leaks)</strong></td>
<td>Along spin axis</td>
</tr>
<tr>
<td><strong>DSN hard- and software:</strong></td>
<td>Along Earth-s/c vector?</td>
</tr>
<tr>
<td>– clock drift (varying constants?), ephemeris, Earth Orientation Par.</td>
<td></td>
</tr>
</tbody>
</table>
Ranging accuracy

- 1 % PA resolution as function of measurement interval ("sampling rate")

<table>
<thead>
<tr>
<th>Measurement interval</th>
<th>Relative velocity</th>
<th>Relative distance</th>
<th>Relative Doppler (Ka)</th>
<th>Required ranging accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 day</td>
<td>70 µm / s</td>
<td>6 m (25 cm / h)</td>
<td>7 mHz</td>
<td>6 cm, 0.7 µm / s, 0.07 mHz</td>
</tr>
<tr>
<td>1 month</td>
<td>2,100 µm / s</td>
<td>5,300 m</td>
<td>200 mHz</td>
<td>50 m</td>
</tr>
</tbody>
</table>

Figures apply to the complete two step process:
radio link to earth and local laser ranging

Lateral coordinates can be determined with less precision, because S/C orbit is a sun-radial outbound trajectory

Chasing of proof mass necessary; but very low Δv manoeuvres are kept on a minimum
### Present radio link accuracies

- **DSN radio link capabilities for S/C tracking from earth**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measures</th>
<th>Accuracy (1σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Doppler</strong></td>
<td>range rate</td>
<td>0.03 mm / s</td>
</tr>
<tr>
<td><strong>range</strong></td>
<td>range</td>
<td>1 m</td>
</tr>
<tr>
<td><strong>angle</strong></td>
<td>lateral angular resolution (rectascension declination)</td>
<td>170 μrad (0.01°)</td>
</tr>
<tr>
<td><strong>ΔDOR</strong></td>
<td>lateral angular resolution (rectascension declination)</td>
<td>2.4 nrad</td>
</tr>
</tbody>
</table>
Key technologies

- Star trackers
- Gyroscopes
- Resonators

- Frequency combs
- SQUID-based sensing
  - (successfully tested at GP-B)
Clocks to explore gravity in deep space

- Redundant measurement
  - Measuring acceleration of S/C on geodesic via ranging and Doppler tracking
  - Measuring redshift of clocks on-board S/C

for Pioneer Anomaly

$$\frac{\Delta \nu}{\nu} = \frac{1}{c^2} \int_{20 \ AU}^{90 \ AU} a_{PA} \, dx \approx 10^{-13}$$

- Clock exploration does not depend on geodesic motion, independent from acceleration
- Clock exploration is cumulative
- Clocks automatically isolate the pure gravity sector
- Clocks represent an absolute DC-accelerometer

DSGP requirement

Challenge: long term stability as well as mass and size reduction
Atom interferometers

Sagnac-phase

\[ \ddot{\phi} = \frac{4\partial}{\bar{\epsilon}V} \bar{u} \cdot \bar{A} \]

de-Broglie wave length

\[ \tilde{\epsilon} = \frac{\tilde{h}}{p} \approx \frac{\tilde{h}}{\sqrt{2Em_0}} \]

Ernst Rasel, IQO, Hannover

Moriond-Meeting, 17.3.2007
Summary

- At least, some gravitational space tests are under preparation.
- GR only seems to be well tested in the Solar System
  - Newton’s law?
  - Dark Matter?
  - Pioneer Anomaly
  - Fly-by-Anomaly
  - CMB quadrupole and octopole anomaly
  - Increase of AU
- Space is the ideal place to carry out gravitational experiments
- Need of improved technology