In Search of the Spacetime Torsion

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Gravitation and Universality

At the classical level, gravitation shows a quite peculiar property

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Particles with different masses and different compositions feel it in such a way that all of them acquire the same acceleration

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Universality of Free Fall
$\Updownarrow$
Weak Equivalence Principle

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Since, to be universal, the *inertial* and the *gravitational* masses must coincide

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The weak equivalence principle is also identified with this equality

GENERAL RELATIVITY

Einstein's theory for the gravitational field,
is fundamentally based on the Weak Equivalence Principle

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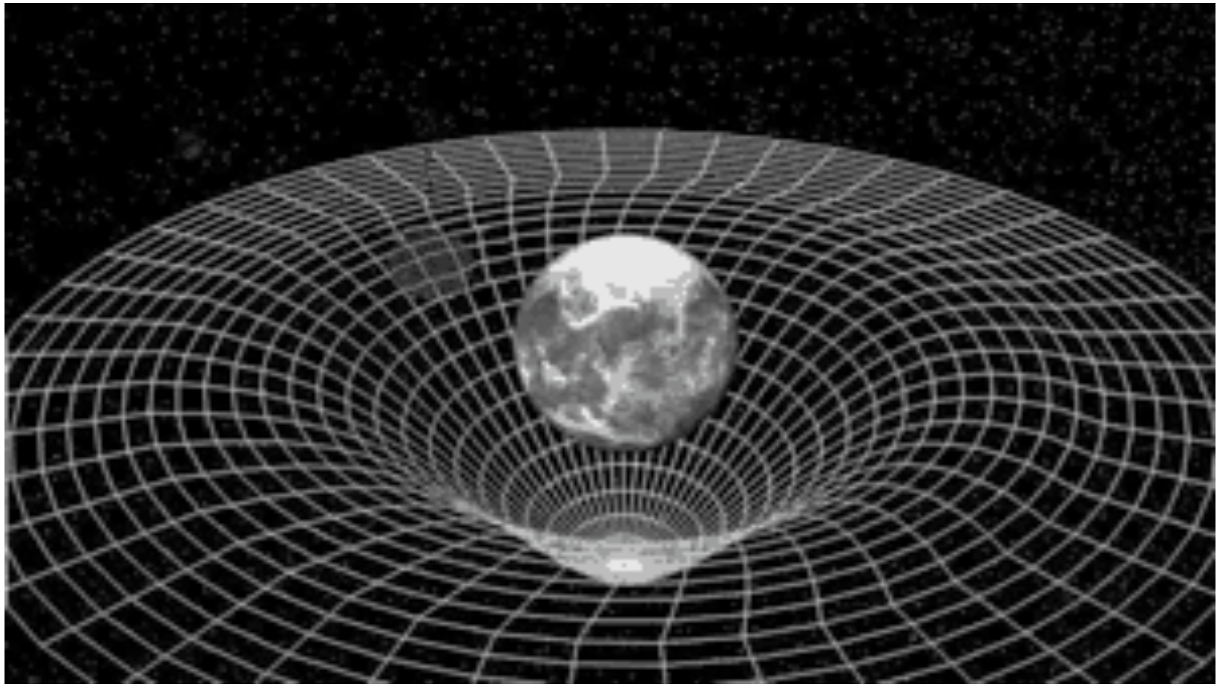
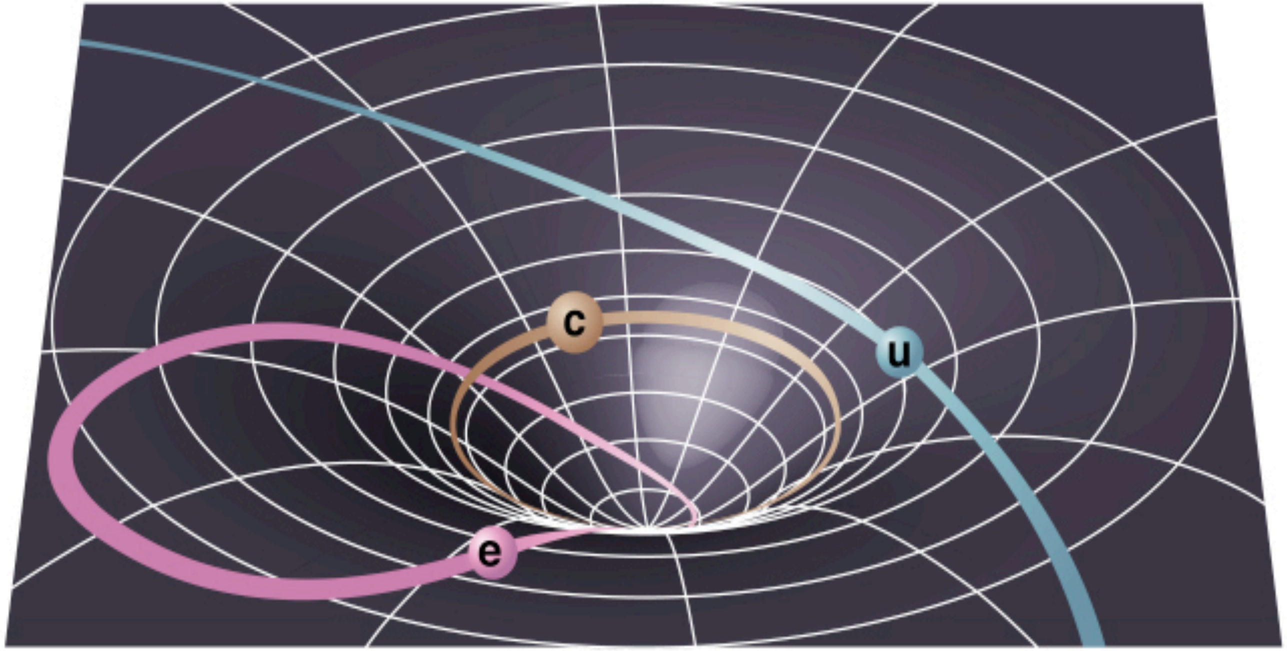
To comply with universality, the presence of a gravitational field
is supposed to produce a *curvature* in spacetime



The gravitational interaction ...

... is obtained by simply letting particles to follow the *curvature* of spacetime



In general Relativity, there is no the concept of "gravitational force"

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The responsibility of describing the gravitational interaction is transferred to spacetime

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Geometry replaces the concept of force

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General Relativity "geometrizes" the gravitational interaction

Equation of Motion

The fundamental connection of **general relativity** is the **Christoffel Connection**

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \tfrac{1}{2}\, g^{\rho\lambda}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu})$$

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It is a connection which has vanishing torsion, but non-vanishing curvature:

$$\overset{\circ}{T}{}^{\rho}{}_{\nu\mu} \equiv \overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} - \overset{\circ}{\Gamma}{}^{\rho}{}_{\nu\mu} = 0$$

$$\overset{\circ}{R}{}^{\rho}{}_{\lambda\nu\mu} \equiv \partial_\nu \overset{\circ}{\Gamma}{}^{\rho}{}_{\lambda\mu} - \partial_\mu \overset{\circ}{\Gamma}{}^{\rho}{}_{\lambda\nu} + \overset{\circ}{\Gamma}{}^{\rho}{}_{\eta\nu} \overset{\circ}{\Gamma}{}^{\eta}{}_{\lambda\mu} - \overset{\circ}{\Gamma}{}^{\rho}{}_{\eta\mu} \overset{\circ}{\Gamma}{}^{\eta}{}_{\lambda\nu} \neq 0$$

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In general relativity, torsion is present, but it is assumed to vanish

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The equation of motion of a test particle is given by the

Geodesic Equation

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$$\boxed{\frac{du_\mu}{ds} - \overset{\circ}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = 0}$$

Geodesics: curves that follow the curvature of spacetime

WHAT ABOUT TORSION?

A general spacetime can, in principle, present two different properties:

Curvature and Torsion

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Why then should matter (energy and momentum) produce **only curvature** in spacetime?

Was Einstein wrong when he made this assumption?

Does torsion play any role in gravitation?

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Cartan was the first to ask these questions, soon after Einstein published his theory

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He then constructed a new theory, called **Einstein-Cartan theory**

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The underlying spacetime of this theory is a Riemann-Cartan spacetime

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This spacetime presents both Curvature and Torsion

EINSTEIN-CARTAN THEORY

Energy and momentum $\longrightarrow$ Source of Curvature

Spin of matter $\longrightarrow$ Source of Torsion

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According to this theory, **curvature and torsion** represent independent degrees of freedom of the gravitational field

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This theory presupposes new physics associated to torsion

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At the macroscopic level, where spins vanish, it coincides with general relativity

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At the microscopic level, where spins are relevant it shows different results from general relativity

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Because of this theory, there is a widespread belief that torsion is important only at the microscopic level, where spins become important

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However, this is not the only possibility ...

Another Possibility is ...

TELEPARALLEL GRAVITY

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Gauge Theory for the Translation Group

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The field strength associated with translation is **Torsion**

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- In general relativity, **curvature** represents the gravitational field
- In teleparallel gravity, **torsion** represents the gravitational field

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In spite of this difference, the two theories are found to yield

Equivalent Descriptions of Gravitation

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Torsion appears as an alternative to curvature

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Gravitation can be described *alternatively* in terms of curvature, as in general relativity, or in terms of torsion, in which case we have teleparallel gravity

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Teleparallel gravity does not presupposes new physics associated with torsion

A Glimpse on Teleparallel Gravity

Basic field: Tetrad $h^a{}_\mu \quad \rightarrow \quad g_{\mu\nu} = \eta_{ab}\, h^a{}_\mu\, h^b{}_\nu$

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The fundamental connection of **teleparallel gravity** is the **Weitzenböck Connection**

$$\overset{\bullet}{\Gamma}{}^\rho{}_{\mu\nu} = h_a{}^\rho\, \partial_\nu h^a{}_\mu$$

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It is a connection with non-vanishing torsion, but vanishing curvature:

$$\overset{\bullet}{T}{}^\rho{}_{\nu\mu} \equiv \overset{\bullet}{\Gamma}{}^\rho{}_{\mu\nu} - \overset{\bullet}{\Gamma}{}^\rho{}_{\nu\mu} \neq 0$$

$$\overset{\bullet}{R}{}^{\lambda\rho}{}_{\nu\mu} \equiv \partial_\nu \overset{\bullet}{\Gamma}{}^\rho{}_{\lambda\mu} - \partial_\mu \overset{\bullet}{\Gamma}{}^\rho{}_{\lambda\nu} + \overset{\bullet}{\Gamma}{}^\rho{}_{\eta\nu} \overset{\bullet}{\Gamma}{}^\eta{}_{\lambda\mu} - \overset{\bullet}{\Gamma}{}^\rho{}_{\eta\mu} \overset{\bullet}{\Gamma}{}^\eta{}_{\lambda\nu} = 0$$

Fundamental Relation

The Weitzenböck and the Christoffel connections are related by

$$\boxed{\overset{\bullet}{\Gamma}{}^\rho{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^\rho{}_{\mu\nu} + \overset{\bullet}{K}{}^\rho{}_{\mu\nu}}$$

$$\overset{\bullet}{K}{}^\rho{}_{\mu\nu} = \tfrac{1}{2}\left(\overset{\bullet}{T}{}_\mu{}^\rho{}_\nu + \overset{\bullet}{T}{}_\nu{}^\rho{}_\mu - \overset{\bullet}{T}{}^\rho{}_{\mu\nu}\right) \quad \rightarrow \quad \text{Contortion Tensor}$$

Geodesics Versus Force Equation

Let us consider the geodesic equation of general relativity

$$\boxed{\frac{du_\mu}{ds} - \overset{\circ}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = 0}$$

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Using the relation between the Weitzenböck and the Christoffel connections

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \overset{\bullet}{\Gamma}{}^{\rho}{}_{\mu\nu} - \overset{\bullet}{K}{}^{\rho}{}_{\mu\nu}$$

the geodesic equation can be written in the alternative form

$$\boxed{\frac{du_\mu}{ds} - \overset{\bullet}{\Gamma}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu = \overset{\bullet}{T}{}^{\theta}{}_{\mu\nu}\, u_\theta\, u^\nu}$$

This is a **force equation**, with torsion playing the role of **gravitational force**

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The geodesic equation of general relativity and the force equation of teleparallel gravity yields the same physical trajectory

Although equivalent ...

... there are conceptual differences between GR and TG

In general relativity, curvature is used to **geometrize** the gravitational interaction

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In teleparallel gravity, torsion accounts for gravitation, not by geometrizing the interaction, but by acting as a **force**

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As a consequence, there are no geodesics in teleparallel gravity, but only force equations

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This is similar to Maxwell's theory, in which the interaction of a charged particle with the electromagnetic field is described by the Lorentz force

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This is not surprising because, like Maxwell's theory, teleparallel gravity is a gauge theory

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Gauge theory for the translation group

Why Gravitation Presents Two Alternative Descriptions?

Like any other interaction of nature, gravitation has a description in terms of a gauge theory

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Teleparallel Gravity

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On the other hand, **universality of gravitation** allows an alternative description in terms of a geometrization of spacetime

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General Relativity

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Universality of the gravitational interaction is then the responsible for the existence of the geometric description

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Only a universal interaction can present a geometric description

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As the solely universal interaction, gravitation is the only one to present two alternative descriptions

SUMMARY

According to **generalizations of general relativity**, like for example Einstein-Cartan theory, torsion is supposed to represent additional degrees of freedom of gravity, and consequently there might be new physics associated to its presence

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If this is true, Einstein made a mistake when he did not include torsion in general relativity

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However, from the point of view of **teleparallel gravity**, torsion does not represent additional degrees of freedom, but simply an alternative to curvature in the description of gravitation

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In this case, no new physics is associated with torsion, which means essentially that general relativity is a complete theory

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If this is true, then Einstein was right when he did not include torsion in general relativity

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Which of these interpretations is the correct one?

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The answer to this question can only be given by experiment

EXPERIMENTAL STATUS

Concerning the search for those new effects supposedly attributed to torsion, there are no experimental data

- Concerning **new gravitational effects** related to the coupling of torsion with **rotation**, there has been recently a proposal to look for these effects using the data of Gravity Probe B

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Yi Mao, Max Tegmark, Alan Guth and Serkan Cabi
Constraining Torsion with Gravity Probe B (*Preprint* gr-qc/0608121)

- Concerning **new gravitational effects** related to the coupling of torsion with **spin**, they could probably be tested near a neutron star — like a binary pulsar, for example — where a macroscopic spin might be present

Up to now, no evidences for these phenomena have ever been reported

On the other hand ...

in teleparallel gravity, curvature and torsion are simply
alternative ways of describing the very same gravitational field

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From this point of view: torsion has already been detected!

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It is the responsible for all gravitational effects, including the physics of the solar system,
which can be reinterpreted in terms of a force equation, with torsion playing the role of force

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According to teleparallel gravity, there are no new effects associated with torsion.
The search for these effects, therefore, is a nonsense

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We can say that the existing experimental data
favour the teleparallel point of view, and consequently general relativity

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Of course, this is not a definitive result.
The ultimate answer can only be given by further experiments

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