LARGE-SCALE MODIFICATIONS OF GRAVITY, GRAVITATIONAL WAVES AND BLACK HOLES

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OUTLINE

- Motivations
- Overview of existing approaches
- The model of massive gravity
- Cosmological solutions
- Massive gravity waves
- Black holes
- Conclusions
MOTIVATIONS TO MODIFY GR

* Hints from cosmology: dark energy/dark matter.
  Taken at face value, GR is not consistent with observations at largest scales
  Is it missing matter or non-Newtonian gravitational interaction?

* Theoretical challenge
  Learning by trying to break

* Ongoing/planned gravity tests (gravitational waves, astrophysical black holes, gravity probe B, table top, . . . ) require alternative models
SOME OF THE EXISTING APPROACHES

- Fierz-Pauli model of massive graviton
  Fierz & Pauli 1939

- Brans-Dicke scalar-tensor theory
  Brans & Dicke 1961, Dicke 1962

- MOND; relativistic version of MOND by Bekenstein
  Milgrom 1983; Bekenstein, Milgrom 1984; Bekenstein 2004

- f(R) gravity

- Brane-world models
  Gregory, Rubakov, Sibiryakov 2002
  Kogan, Mouslopoulos, Papazoglou 2001; Damour & Kogan 2002

- Brane-world inspired DGP model
  Dvali, Gabadadze, Porrati 2002
**APPROACHES**

- **Ghost condensate model**

- **Massive gravity ("Higgs phase" of gravity)**

Models with massive gravitons have generic problems:

- ghosts
- van Dam-Veltman-Zakharov (vDVZ) discontinuity
- strong coupling at low scale

One way to escape these problems – to break the Lorentz symmetry (for instance, spontaneously). Lorentz symmetry is deeply encoded in the QFT formalism. Models breaking it

- are subject to strong constraints
- have too much freedom

Both these problems may be solved in the case of the **spontaneous braking**.
THE MODEL OF MASSIVE GRAVITY

Simply add 4 scalar fields $\phi^0, \phi^i$ with peculiar derivative coupling to gravity:

$$S = \int d^4x \sqrt{-g} \left\{ -M_{Pl}^2 R + \Lambda^4 F(\phi) + \text{(ordinary matter)} \right\}$$

where $F$ depends on $\phi$ through the following combinations:

$X = g^{\mu\nu} \partial_\mu \phi^0 \partial_\nu \phi^0$ and $W_{ij} = (g^{\mu\nu} - \partial^\mu \phi^0 \partial^\nu \phi^0 / X) \partial_\mu \phi^i \partial_\nu \phi^j$.

One may take, for instance, a generic $O(3)$-invariant function $F(X^\gamma W^{ij})$, $\gamma$ being a constant.

$\Lambda$ is a cutoff scale of the effective theory.
Flat Minkowski space is a solution (with no fine-tuning):

\[
\begin{align*}
g_{\mu\nu} &= \eta_{\mu\nu} \\
\phi^0 &= \Lambda^2 \cdot t \\
\phi^i &= \Lambda^2 \cdot x^i
\end{align*}
\]
Flat Minkowski space is a solution (with no fine tuning):

\[
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}
\]
\[
\phi^0 = \Lambda^2 \cdot t + \pi^0
\]
\[
\phi^i = \Lambda^2 \cdot x^i + \pi^i
\]

At the quadratic level

\[
\sqrt{-g} F \rightarrow M_{Pl}^2 \left\{ m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m^2 h_{ij}^2 + m_3^2 (h_{ii})^2 - 2m_4^2 h_{00} h_{ii} \right\}
\]

For some particular regions in the parameter space:
- No ghosts or instabilities
- No vDVZ discontinuity
- The cutoff scale of the effective theory is \( \Lambda = \sqrt{m M_{Pl}} \)

Rubakov 2004, Dubovsky 2004
Relation between the cutoff scale and the graviton mass:

$$\sqrt{g}(-M_{Pl}^2 R + \Lambda^4 F) \rightarrow$$

$$\rightarrow M_{Pl}^2(\partial h)^2 - \Lambda^4 h^2 \equiv M_{Pl}^2 [(\partial h)^2 - m^2 h^2]$$

$$\Rightarrow \Lambda^4 = M_{Pl}^2 m^2$$

To summarize: There exists a full non-linear effective theory where

* gravitons have finite mass $m$

* everything is calculable up to a scale $\Lambda \sim \sqrt{M_{Pl} m}$
Linear perturbations

“Unitary gauge”: perturbations of Goldstone fields are zero. What remains is:

\[
\begin{align*}
\delta g_{00} & = 2\varphi \\
\delta g_{0i} & = S_i - \partial_i B \\
\delta g_{ij} & = -h_{ij} - \partial_i F_j - \partial_j F_i + 2(\psi \delta_{ij} - \partial_i \partial_j E)
\end{align*}
\]

* Tensor perturbations:

\[
(-\partial_0^2 + \partial_i^2 - m^2)h_{ij} = 0
\]

\[\implies\] Two massive tensor modes with the mass \(m\)

* Vector perturbations: Like in GR: no propagating degrees of freedom.
Scalar perturbations: Gauge-invariant potential $\Phi = \varphi + \partial_0 B - \partial_0^2 E$:

$$\Phi = \Phi_E = \frac{1}{\partial_i^2} \frac{T_{00} + T_{ii}}{4M_{P1}^2} - 3\frac{\partial_0^2}{\partial_i^4} \frac{T_{00}}{4M_{P1}^2}$$

$\Rightarrow$ the standard Newton’s potential

- no extra scalar particles

- no modification of Newton’s potential (in particular, no Yukawa suppression $e^{-mr}$)
BOUNDS ON GRAVITON MASS

What the graviton mass could be?

* Unmodified Newton potential $\implies$ No constraints from Solar system or Cavendish-type experiments

* Emission of gravitational waves in binary pulsars is consistent with GR $\implies$ graviton mass must be smaller than inverse period

$$\frac{m}{2\pi} \equiv \nu < 3 \cdot 10^{-5} \text{Hz} = (1 \text{hr})^{-1} = (10^{15} \text{cm})^{-1} = 2 \cdot 10^{-20} \text{eV}$$

Note: this implies for the cutoff scale

$$\Lambda \lesssim 10 \text{ keV}$$
Cosmological ansatz:

\[ ds^2 = dt^2 - a^2(t)dx_i^2 \]
\[ \phi^0 = \phi(t) \]
\[ \phi_i = \Lambda^2 x^i \]

The Friedman equation reads:

\[ H^2 = \frac{1}{M_{Pl}^2} \left( \rho_m + \frac{\rho_1}{a^{3-1/\gamma}} + F_0 \right) \]

⇒ Extra matter component with the equation of state \( w = -1/(3\gamma) \).
⇒ Unconventional source of the cosmic acceleration
Cosmological perturbations

The presence of the background scalar fields may change the behavior of perturbations of ordinary matter during cosmological evolution and thus affect the structure formation.

Redo the linear analysis in the expanding Universe

Cosmological perturbations grow exactly as in the Einstein theory
MASSIVE GRAVITY WAVES

Can massive gravitons be produced in amounts sufficient for DM?

\[
\left\{-\partial^2_{\eta} - 2\frac{\dot{a}}{a}\partial_{\eta} + \partial^2_{i} - m^2 a^2\right\} h_{ij} = 0
\]

This is the same equation as for a minimally-coupled scalar field

\[\Rightarrow \text{massive gravitons are efficiently produced during inflation}\]

\[
\Omega_g \simeq 3 \cdot 10^3 \left(m \cdot 10^{15}\text{ cm}\right)^{1/2} \cdot \left(\frac{H_i}{\Lambda}\right)^4
\]

\[\Rightarrow \text{can (in principle) constitute all of the dark matter}\]
Can massive gravitons be detected?

* No difference with the detection of massless gravitational waves except monochromatic signal \((\Delta \nu/\nu \sim 10^{-6})\)

* Maximum signal: assume massive gravitons comprise all DM in our halo,

\[
\rho_{\text{halo}} \sim M_{\text{Pl}}^2 m^2 h_{ij}
\]

\[
\langle h_{ij} \rangle \sim 10^{-10} \cdot \left( \frac{3 \cdot 10^{-5} \text{Hz}}{\nu} \right)
\]

This is by many orders of magnitude above the sensitivity of LISA
Why black holes are so interesting?

- Exact non-linear solutions of GR
- Have very unusual properties (no-hair theorems, BH thermodynamics, Hawking radiation, information paradox, ...)
- Lab for quantum gravity
BLACK HOLES

Why black holes are so interesting?

- Exact non-linear solutions of GR
- Have very unusual properties (no-hair theorems, BH thermodynamics, Hawking radiation, information paradox, ...)
- Lab for quantum gravity
- They exist in Nature ...
- ... and will soon be studied quantitatively

Are there models which predict non-standard black holes?

Naively one would expect that any modification of gravity will change the black hole solution, but this is not the case because of the no-hair theorems.

For instance, black holes in the Brans-Dicke theory are identical to those of GR!

The best theories to look for alternative black hole solutions are those with the spontaneous Lorentz breaking, because they escape the no-hair theorems.
Schwarzschild black hole in massive gravity

One can find an exact solution in the case of non-rotating black hole (the same as in the ghost condensate model):

\[
ds^2 = (1 - \frac{R_g}{r}) dt^2 - (1 - \frac{R_g}{r})^{-1} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]

\[
\phi^0 = t + 2\sqrt{rR_g - R_g} \ln \frac{\sqrt{r} + \sqrt{R_g}}{\sqrt{r} - \sqrt{R_g}}, \quad \phi^i = x^i
\]

Note: this metric is identical to the conventional Schwarzschild black hole.
Kerr black hole

Too complicated to solve exactly.

Simplified approach: take the standard Kerr metric and see if the scalar fields have a solution such that $T_{\mu \nu} = 0$.

This amounts to finding a 3d conformally-flat slicing of the metric

$$g_{\mu \nu} = \Omega^2 \left( c^2 - v^2 \begin{pmatrix} v^i & \nu^i \\ \nu^i & - \delta_{ij} \end{pmatrix} \right), \quad c = \Omega^{\frac{1-\gamma}{\gamma}}$$

Still a very complicated problem...
Fortunately, the problem has been solved before (for a different purpose), as there exists a

**Theorem:** There does not exist conformally flat slicing of the asymptotically flat vacuum metric with non-zero angular momentum due to the presence of the non-vanishing obstruction of the quadrupole nature.


Rotating black holes are different in massive gravity

Since no-hair theorems do not hold, one should expect $O(1)$ deviations from standard relations between metric multipoles for rotating supermassive black holes in this model
SUMMARY

Coming back to the original motivations:

* No success (yet) in explaining dark energy/matter (except a new DM candidate – massive graviton), but interesting possibilities exist and should be explored

* There is a consistent effective theory of massive gravity where everything is (in principle) calculable – paradise for a theorist

* Alternatives to GR: massive gravity waves and hairy black holes
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Message for experimentalists: If you see strange bumps and wiggles on your curves, think of (spontaneously) broken theory as an alternative to broken apparatus. If you do not see strange bumps – put a bound on the graviton mass!