East of the Sun, West of the Moon and a Little Bit Down to Earth

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XXI. **Experiments to determine the Density of the Earth.** By
Henry Cavendish, Esq. F.R.S. and A.S.

Read June 21, 1798.

Many years ago, the late Rev. John Michell, of this Society,
contrived a method of determining the density of the earth, by
rendering sensible the attraction of small quantities of matter;
but, as he was engaged in other pursuits, he did not complete
the apparatus till a short time before his death, and did not
live to make any experiments with it. After his death, the
apparatus came to the Rev. Francis John Hyde Wollaston,
Jacksonian Professor at Cambridge, who, not having conveni-
ences for making experiments with it, in the manner he could
wish, was so good as to give it to me.

The apparatus is very simple; it consists of a wooden arm,
6 feet long, made so as to unite great strength with little
weight. This arm is suspended in an horizontal position, by
a slender wire 40 inches long, and to each extremity is hung a
leaden ball, about 2 inches in diameter; and the whole is
enclosed in a narrow wooden case, to defend it from the wind.

As no more force is required to make this arm turn round
on its centre, than what is necessary to twist the suspending
wire, it is plain, that if the wire is sufficiently slender, the most
minute force, such as the attraction of a leaden weight a few
inches in diameter, will be sufficient to draw the arm sensibly
aside. The weights which Mr. Michell intended to use were....
# Relative accuracy in the constant of gravitation

<table>
<thead>
<tr>
<th>Year</th>
<th>Accuracy</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>1687</td>
<td>2%</td>
<td>Newton (density of the earth)</td>
</tr>
<tr>
<td>1775</td>
<td>20%</td>
<td>Maskelyne and Hutton (density of the earth)</td>
</tr>
<tr>
<td>1798</td>
<td>$10^{-2}$</td>
<td>Cavendish and Mitchel (density of the earth)</td>
</tr>
<tr>
<td>1895</td>
<td>$10^{-3}$</td>
<td>Boys</td>
</tr>
<tr>
<td>1942</td>
<td>$4 \times 10^{-4}$</td>
<td>Heyl and Chrzanowski</td>
</tr>
<tr>
<td>1982</td>
<td>$6 \times 10^{-5}$</td>
<td>Luther and Towler</td>
</tr>
<tr>
<td>2000</td>
<td>$1 \times 10^{-5}$</td>
<td>Gundlach and Merkowitz</td>
</tr>
</tbody>
</table>
THE CAVENDISH TORSION BALANCE
The Luther and Towler torsion balance
The wuppertaler experiment

Precept: determination of the change in the distance between two pendula due to the change in the gravitational force
Source Mass Geometry

Positional Insensitivity

Gravitational Field
Axial Component

Z
Laser System

Diagram showing the components of a laser system, including Laser Tube, Heater, AOM (Acousto-Optic Modulator), EOM (Electro-Optic Modulator), PBS (Polarizing Beam Splitter), Photodiode, and Cavity. The diagram also includes a Loop Filter.
Performance

Differential Displacement: 80 nm
or 120 Mhz

Pendulum Noise: just under 1 nm peak-to-peak
The difference of our value from the 2002 CODATA value in parts in $10^5$. The error bars are the statistical uncertainty from each data set RSS'ed with a 0.1 mil uncertainty in opposing mass separations.
Error Contributions

Source Mass Positions

Interferometer Alignment

Density Inhomogeneity
figure from Riley Newman, Matters of Gravity 21, spring 2003

[4] Heinrich Meyer et al. reported by Jim Fuller at the CPEM2002 metrology conference, Ottawa (June 2002)
McDonald and Grasse

McDonald, Texas   Grasse, France
Early History

• The Lunar Laser Ranging effort was conceived and proposed to NASA as an Apollo experiment in the late 1960s.
• The first ranges were in 1969, shortly after the Apollo 11 landing.
• I was involved with the experiment from the very beginning.
Lunar Laser Ranges

- Laser pulses are sent from stations on the Earth toward the Moon where they bounce off of retroreflector arrays and return to the Earth.
- Ranging started in 1969 and continues to present.
Goals in 1969 Apollo 11 Report

• Study of gravitation and relativity (secular variation in the gravitational constant),

• the physics of the Earth (fluctuation in rotation rate, motion of the pole, large-scale crustal motions),

• the physics of the Moon (physical librations, center-of-mass motion, size and shape).
Apollo 14 & 15 Retroreflectors

- Apollo 14
  February
  1971

- Apollo 15
  July 1971
A Proposed Lunar Package
(A Corner Reflector on the Moon)

This note describes what is felt to be both a useful and
at the same time a practical lunar package. The total weight
involved would be only 2 to 3 pounds, and it could be constructed
to withstand a rather hard landing. Once there, the only require-
ment for it to function successfully is that its landing leave
it free to bounce and roll until it comes naturally to rest.
This lunar package containing an optical corner has been built...

In addition to permitting the afore-mentioned experiment, a
group of corners on the moon would provide the long-desired marking
stripes on the moon and thereby permit precise period measurements
to be made. The moon would then be a precision gravity clock.

Proposed package (cross section)

Dress wires to prevent landing (and staying)
wrong side up

Note that the COM is arranged to be below the center of the ball
when the corner is properly positioned, and therefore the package
will position itself with the corner pointing up.
RMS Residual vs. Year

Lunar Laser Ranging
Data Accuracy

Year


Weighted Annual RMS Residual (cm)
Orbit — Gravitational Physics

- The equivalence principle test is sensitive to the gravitational/inertial mass ratio difference between the Earth and Moon.
  \((-1.0 \pm 1.4) \times 10^{-13}\)
- Equivalent to amplitude of 3±4 mm at 29.53 days.
- PPN beta-1 is \((1.2 \pm 1.1) \times 10^{-4}\)
- Geodetic precession confirmed with 0.64% (0.00012 "/yr) uncertainty compared to its value of 0.0192"/yr.
More Gravitational Physics

- Relative rate for gravitational constant.
  \[(4\pm9) \times 10^{-13} \text{ /yr}\]
- Results in accord with general relativity. The solar system does not share cosmic expansion.
Status of LLR Stations

- McDonald, Texas — operating 37 yr
- Grasse, France (OCA) — being upgraded after 2+ decades of operation
- Apache Point, New Mexico — new station has first ranges, being debugged
- Matera, Italy — first two days of data
- South Africa — installing former OCA LLR
Summary

- Analysis of lunar ranges gives information on orbit, gravitational physics, geodesy, geophysics, and lunar science.
- Einstein’s general relativity is confirmed. Earth rotation and station positions and motions are measured. Lunar tides are measured and fluid core is detected.
Summary Continued

- Lunar Laser Ranging continues to provide new results because of improving range and data analysis accuracies.
- The future offers improved accuracies and new results.
"Curse Con Edison! Another brownout!"
Gravity Illustrated.

A. Gravity

B. Lots of Gravity
ERECTOR

DEVELOPED AT THE
GILBERT HALL OF SCIENCE

THEY WHISTLE

THEY'RE ALL-ELECTRIC

THEY BUZZ WITH ACTION
CALVIN AND HOBBS

GRAVITY IS ARBITRARY!
That's funny—when Galileo did it, it worked perfectly.
Linear Motion Cam

Cam Equation: $R = R_0 - a\theta$
Spring height is 24 inches (61 cm), 14 inches in transport mode.

Interferometer height with legs attached is 22.5 inches.

Interferometer with laser weight 45 pounds.

Dropping chamber's height is 13.5 inches.

Dropper weight 31 pounds.

Spring weight 7.5 pounds.

Weight of 3 legs is 9.3 pounds.