**Tests of gravity at short and long distances**

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**General Relativity (in two slides …)**

**Einstein 1907-1913 (with Grossman …)**
- applies the principle of relativity to gravitation
- introduces the equivalence principle as the geometrical base of the theory of gravitation
- ideal (atomic) clocks measure the proper time along their trajectory
- freely falling probes (test masses and light rays) follow geodesics

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu \]

\[ \delta \left[ \int ds \right] = 0 \]

**Gravitational field ↔ metric in Riemannian space-time**

One of the most accurately ever tested principles of physics: free fall of test masses with \# compositions coincide within a few parts in \(10^{13}\); accuracy attained in lab tests (torsion pendulums) as well as in space tests (Lunar Laser Ranging)

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**General Relativity (in two slides …)**

**Einstein 1912-1915 (with Grossman, Hilbert …)**
- curvature tensors = (non linear) differential expressions built up on the metric tensor
- one curvature tensor has a null divergence (Bianchi identities)
- the stress tensor too (conservation laws)

\[ E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \]

\[ D^\nu E_{\mu\nu} = 0 \]

\[ D^\nu T_{\mu\nu} = 0 \]

**General Relativity (GR) in the solar system**

**Solution for the metric**
- with the Sun treated as a point-like motionless source
- using spatially isotropic coordinates
- with the Newton potential

\[ \phi = -\frac{G_N M}{r c^2}, \quad |\phi| \ll 1 \]

**GR usually tested through its confrontation with the larger family of PPN metrics**

\[ g_{00} = 1 + 2\phi + 2\phi^2 + \ldots \]

\[ g_{jk} = -(1 - 2\phi + \ldots) \delta_{jk} \]

\[ \alpha = 1 \text{ fixes } G_N \]

\[ \beta = \gamma = 1 \text{ in GR} \]

- Motions predicted as the geodesics of this metric
- Comparisons between observations and predictions expressed in terms of anomalies of the PPN parameters \( \beta - 1, \gamma - 1 \)
After 30 years of PPN tests of GR

Living Reviews in Relativity, C.F. Will
(2001, regularly updated)

The best test to date:
Doppler velocimetry on Cassini during its cruise from Jupiter to Saturn

\[ \gamma - 1 = (2.1 \pm 2.3) \times 10^{-5} \]

Using also Lunar Laser Ranging:

\[ |\beta - 1| \lesssim 3 \times 10^{-4} \]

Tests select GR out of the PPN family

Good reasons to going on testing GR

Theoretical reasons:
- GR is a classical theory which show inconsistencies with quantum field theory
- All unification models predict (small) deviations of gravitation laws from GR

Observational reasons:
- “Dark matter” and “dark energy” are observed as gravitational anomalies; as long as they are not also observed through independent means, they may as well be interpreted as modifications of gravity laws at galactic and cosmic scales
- A few measurements in the solar system show deviations from the predictions of GR

Scale dependent tests of GR

“Fifth-force” tests of the Newton law

Search for a deviation

\[ g_{00} = [g_{00}]_{GR} + \delta g_{00} \]

in particular under the form of a Yukawa correction

\[ \delta g_{00}(r) = 2\phi(r) \alpha \exp \left( -\frac{r}{\lambda} \right) \]

Windows remain open for deviations at short ranges

\[ \lambda < 1 \text{ mm} \]

or long ranges

\[ \lambda > 10^{16} \text{ m} \]

Searches in the short-range window

Gravity measurements at short distances

\[ \lambda > \text{ a few } 10 \mu m \]

At shorter ranges, tests consist in theory-experiment comparisons for the Casimir force

No deviation reported to date in the short-range window!

**Eöt-Wash experiment (Adelberger et al)**

"Missing-mass" torsion balance
Two disks with holes, the attractor is rotated uniformly, the produced torque is extracted at harmonic frequencies. The systematic errors are carefully controlled.

Newtonian signals used to calibrate hypothetical new forces.

**The Casimir force**

\[
F_{\text{Cas}} = -\frac{\hbar c \pi^2}{240 L^4} A
\]

\[
E_{\text{Cas}} = -\frac{\hbar c \pi^2}{720 L^3} A
\]

Assumptions
- plane parallel mirrors
- perfect reflection
- zero temperature
- perfectly flat surfaces

Order of magnitude of the (negative) pressure

\[
L = 1 \mu m \rightarrow \frac{F_{\text{Cas}}}{A} \approx 10^{-3} \text{Pa}
\]

More realistic description needed to compare experiments to theory.

**Eöt-Wash experiment**

The strongest constraint obtained at small distances:
At 95% confidence, a Yukawa interaction with gravitational strength ($\alpha=1$) must have a range $\lambda<56\mu m$.


**Casimir vs Newton : the challenge**

For two Cu plates (1cm x 1cm x 1mm), Casimir dominates Newton at $L<10\mu m$.

Gravity tests rely on comparisons of Casimir force measurements with QED predictions. The theory has to be independent and as accurate as the experiments.

Focus issue in the New Journal of Physics (Oct 2006)
- A. Lambrecht, P.A. Maia Neto, S. Reynaud,
- R. Onofrio
New Dynamic Casimir Measurement: Experimental Setup
(Decca et al., IUPUI)

Sphere Radius: $R = 150 \, \mu m$
MTO Dimensions: $500 \, \mu m \times 500 \, \mu m \times 3.5 \, \mu m$

Force Gradient Measurement:
- Change in Oscillator
- Resonance Frequency
- Proximity Force Approximation

Current Limits on Yukawa Forces: Casimir Force Experiments

R. S. Decca, D. López, E. Fischbach, G. L. Klimchitskaya,

Searches in the long-range window

Best example to date:
NASA approval to extend Pioneer 10/11 missions after their primary planetary goals have been met with the aim (among other ones) of testing Celestial Mechanics at large heliocentric distances

Agency: NASA
Pioneer 10
Launch: 2 March 1972
Planetary fly-bys: Jupiter: 4 Dec 1972
Last data point received: 27 Apr 2002
distance: ~40.2 AU
Pioneer 11
Launch: 5 April 1973
Planetary fly-bys: Jupiter: 2 Dec 1974
Saturn: 1 Sep 1979
Last data point received: 1 Oct 1995
distance: ~30 AU

The most precisely ever navigated deep-space vehicles:
- Spin-stabilization and design permitted acceleration sensitivity $\sim 10^{-10} \text{ m/s}^2$, unlike a Voyager-type 3-axis stabilization that were almost 50 times worse

The Pioneer “gravity test”:
the largest scaled test ever carried out...

...and it failed to confirm the known laws of gravity!
The Doppler observable

A radio signal is sent from a station on Earth to the probe, sent back (transponded) by the probe, and finally received by a station on Earth (the same=2-way; another one=3-way).

The observable is the frequency ratio, interpreted as a relative velocity (Minkowskian formula), but it accounts for all relativistic and gravitational effects.

\[ \frac{f_+}{f_-} = \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \]

It must also be corrected for propagation effects, station motions, etc.

Annual variation

Doppler shift Pioneer 11 – 2 Way (red) and 3 Way (blue)
Compared to NASA Horizon ephemeris (black)

Orbital motion of Earth + mean motion of P11

\(~ 860000 \text{ Hz}\)

Diurnal variation

Doppler shift Pioneer 11 – 2 way station DSN 43 Camberra
Compared to NASA Horizon ephemeris (black)

Diurnal motion of the station ~ 6000 Hz

\[ \Delta f_4 \]

\(~ 1 \text{ Hz is equivalent to 65 mm/s velocity} \)

Anomalous acceleration

Deviation of the observed velocity from the modeled one varying ~ linearly with time

\[ \frac{\Delta v_{\text{obs}}}{\Delta v_{\text{model}}} \sim -a_p(t - t_{\text{in}}) \]

Interpreted as an anomalous acceleration

\[ a_p \sim 0.9 \times 10^{-9} \text{ m/s}^2 \]

\(~ 3 \text{ Hz}\)

The anomaly has been registered on the two deep space probes with the best navigation accuracy.

The two probes were identical and had similar trajectories: one experiment performed twice with the same result. Might be an artefact? Satisfactory explanation actively looked for, not yet found.

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- On-board systematic & other hardware-related mechanisms:
  - Precessional attitude control maneuvers and associated "gas leaks"
  - Nominal thermal radiation due to $^{238}$Pu decay [half life 87.75 years]
  - Heat rejection mechanisms from within the spacecraft
  - Hardware problems at the DSN tracking stations

- Examples of the external effects:
  - Solar radiation pressure, solar wind, interplanetary medium, dust
  - Drag force due to mass distributions in the outer solar system
  - Gravity from the Kuiper belt; gravity from the Galaxy
  - Gravity from Dark Matter distributed in halo around the solar system
  - Errors in the planetary ephemeris, in the Earth Orientation Parameters, precession, and nutation

- Phenomenological time models:
  - Drifting clocks, quadratic time augmentation, uniform carrier frequency drift, effect due to finite speed of gravity, and many others

All the above were rejected as explanations


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A challenge ahead of us

Will we be able to solve the discrepancy and to show that Pioneer gravity test once more confirmed GR?

Or to confirm the existence of a gravity anomaly?

Any of these two conclusions would be of great value for fundamental physics, astrophysics and cosmology.

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Recent Pioneer Data Recovery Effort


- Pioneer 10: 11.5 years; distance = 40–70.5 AU
- Pioneer 11: 3.75 years; distance = 22.4–31.7 AU

Pioneer data recently recovered (Planetary Society & JPL):

- Telemetry & Doppler data recovered from launch to the last data point
  - V. Toth and S. Turyshev, arXiv:gr-qc/0603016

Doppler Data now available:

- Pioneer 10:
  - 1973-2002: ~30 years
  - Distance range: 4–87 AU
  - Jupiter encounter
  - ~60,000 data points, ~20GB
  - Maneuvers, spin, initial cond.

- Pioneer 11:
  - 1974-1994: ~20 years
  - Distance range: 4–33 AU
  - Jupiter & Saturn encounters
  - ~50,000 data points, ~15GB
  - Maneuvers, spin, initial cond.

Ongoing Pioneer data re-analysis planned as an international effort:

- More informations on the website "Investigation of the Pioneer Anomaly" Team @ ISSI
  - http://www.issi.unibe.ch/teams/Pioneer; results expected within the next year
Motivations and lessons for a future probe:
arXiv:gr-qc/0506139

It is unlikely that the equivalence principle be violated at the level of the Pioneer anomaly

\[ a_N \sim 1 \mu m \, s^{-2} \quad a_P \sim 1 \text{nm} \, s^{-2} \quad \frac{a_P}{a_N} \sim 10^{-3} \]

We keep the description of gravitation as a Riemannian metric theory with motions identified as geodesics

But the Einstein-Hilbert equation can be modified, leading to modifications of geodesic motions in the solar system

These metric extensions of GR define a phenomenology larger than the PPN framework

Pioneer observations as well as other gravity tests have to be re-analyzed in this new framework