Cosmological models involving an interaction between dark matter and dark energy have been proposed in order to solve the so-called coincidence problem. Different forms of coupling have been studied, but there have been claims that observational data seem to narrow (some of them down to something annoyingly close to the $\Lambda$CDM model, thus greatly reducing their ability to deal with the problem in the first place. The smallness problem of the initial energy density of dark energy has also been a target of cosmological models in recent years. Making use of a moderately general coupling scheme, this paper aims to unite these different approaches and shed some light as to whether this class of models has any true perspective in suppressing the aforementioned issues that plague our current understanding of the universe, in a quantitative and unambiguous way.

1 Introduction

Several sets of observational indicate that we live in a nearly flat, low-matter-density universe whose expansion is speeding up at present. Possible candidates discussed in the literature as a driving source of such an acceleration usually fall into two categories: exotic components with negative pressure (dark energy or simply DE, henceforth also denoted by the subindex “x”) or proper modifications of general relativity which become relevant at cosmological scales. In either case different observational and theoretical problems arise, and this made the soil fertile for a plethora of models to bloom. On the theoretical side we identify three possible issues which we will dub: the coincidence problem (CP), the DE energy density initial condition problem (DEICP) and the sensitivity to initial conditions (SIC). These can be strongly related (as in the case of any $\Lambda$-like dark energy model, defined below) or even dependent on the physical interpretation underlying a particular model (as opposed to being a problem of the model itself), as we will clarify below. Another important point is that all three problems often impose fine-tuning to observationally viable models.

The CP can be stated as: why only recently (in terms of redshift) has the DE energy density become comparable to the matter one, since both components are usually assumed independent and thus scale in different ways? The DEICP (which is present only when one interprets DE as a proper field, but not when one works in modified gravity theories) is phrased as: why is the initial (by which we mean just after inflation, $z \sim 10^{26}$) value of the DE energy density much smaller than what we would expect from equipartition considerations? It seems natural to expect that after inflation the different fields in nature would have energy densities with the same order of magnitude. Thus a reasonable value for $\rho_x/\rho_\tau$, would be $\sim 10^{-2}$ or $10^{-3}$. The SIC is a measure of the robustness of a model to different initial conditions, which usually can be thought in terms of basins of attraction. Models with a larger SIC require a more fine-tuned initial condition than those with a smaller one. The relevance of each of these three issues (CP, DEICP and SIC) can be disputed, but our main intention here is to clarify their distinction, as is not uncommon for ambiguity among them to arise in the literature.

Here we shall address the CP by requiring a duo-scaling cosmology. We seek models whose phase space exhibit two fixed points: one responsible for the matter dominated era (henceforth
MDE) and the other for the present dark energy dominated acceleration. The former needs to be a saddle point and the latter an attractor. The CP could be solved by a model in which our present universe has reached (by which we mean it is close enough for a given criterium) such an accelerated attractor, as this would mean that the current cosmological energy distribution is not a transient phase but rather an unavoidable and permanent regime. The existence of such stable and accelerated fixed point requires a coupling between matter and DE, as otherwise the DE energy density would have to decrease with $a^{-3}$.

In order to quantify both the DEICP and SIC, we will define two quantities: $\zeta \equiv \rho_{\text{max}}/\rho_{\text{ri}}$ and $\Delta \equiv (\rho_{\text{max}} - \rho_{\text{min}})/|\rho_{\text{max}}|$, where $\rho_r$ stands for the radiation energy density and $\rho_{\text{max/min}}$ denotes the maximum and minimum values of $\rho_{xi}$ that evolve to the present day observed values within $1\sigma$. Models which have small SIC are characterized by large values of $\Delta$ and vice-versa. One should bear in mind that newer and improved observations will probably narrow our present uncertainties on $\rho_x$ and this could reflect on the initial range $\Delta$, which should depend only on the model and not on the quality of our observations. This is avoided by computing the ratio $\Delta/\Delta_{\Lambda\text{CDM}}$. Furthermore, our definition of $\Delta$ is not a good one in the cases where $\rho_{\text{max}}$ is very close to zero and $\rho_{\text{min}}$ is not. Finally, for the CP we will use the ratio $R_z$ between DE and dark matter energy densities, where $z$ stands for the redshift, as a measure of how close we are to the accelerated attractor. More specifically, a solution to the CP requires $R_z \sim R_0 \approx 3.4$.

Making use of a three-parameter model with a coupling scheme that generalizes some previous ones in the literature, we analyse the phase space and apply two different cosmological tests: SNIa, as given by a combined catalog, and the so-called CMB shift parameter, as inferred from WMAP3. We look at three aspects: what is the parametric region that allows such duo-scaling cosmology; how does the model cope with the DEICP inside that parametric region; and what is the SIC. We also propose a two-parameter toy model for which there exists analytic solutions and which has small SIC, alleviate the CP and, in some cases, also the DEICP. Finally, we note that by choosing freely all three parameters we can actually solve the CP.

These proceedings are essentially a short version of the paper by Quartin et al.1.

2 The Dark Interactions Model

We consider the universe as filled by four components: radiation, baryons, and two coupled barotropic fluids. One, a cold, pressureless, dark matter (CDM) and the other a negative pressure DE. We will denote them, respectively, by the subindexes $r$, $b$, $c$, and $x$. For the sake of simplicity, our analysis will be restricted to models with constant equation of state parameter $w_x \equiv p_x/\rho_x$. We neglect any possible interactions with baryons and focus on a coupling with dark matter alone. An interaction with radiation is also discarded on the grounds that any such reaction would not affect the dynamics of the system near the sought two scaling regimes, required to address the CP. On the other hand, a coupling with radiation might be desirable if one wants to address also the DEICP. Since this would introduce many difficulties of its own, we will not consider it further in this work.

Our interest is in cosmological scaling solutions in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) background metric, in which the Friedmann equation in General Relativity is given by

$$3H^2 = M_P^{-2}(\rho_x + \rho_c + \rho_b + \rho_r),$$

where $M_P^{-2} = 8\pi G$ with $G$ being the gravitational constant, and where the different $\rho_i$ make up the energy density of the universe. In what follows we shall set $M_P = 1$. Different forms of coupling have been considered in the literature. We may take the coupling between CDM and
Figure 1: Evolution of all four energy densities as a function of redshift. From left to right, the dominant component is, in order: radiation, dark matter, baryons and dark energy. The dashed lines correspond to the $\Lambda$CDM model and the solid ones to $\lambda_x = 0.23, \lambda_c = -0.14, w_x = -0.85$. In this scenario, we have already reached the accelerated attractor ($R_{-1}$ within 4% of $R_0$) and thus, by our definition, solved the CP.

DE to be such that

$$\frac{d\rho_x}{dN} + 3(1 + w_x)\rho_x = -3Q(\rho_c, \rho_x),$$

(2)

$$\frac{d\rho_c}{dN} + 3\rho_c = 3Q(\rho_c, \rho_x),$$

where $N \equiv \ln a$. Note that positive values of $Q(\rho_c, \rho_x)$ indicate that energy is being transferred from dark energy to dark matter, meaning that the latter will dilute slower than in the case without interactions.

Our analysis will be restricted to the case

$$Q(\rho_c, \rho_x) = \lambda_x \rho_x + \lambda_c \rho_c,$$

(3)

which is a more general form than some others found in the literature. It is, however, in a different class than the ones used in interacting scalar field models. In particular, it is not equivalent to the couplings used in scalar fields models and therefore it might circumvent the tough challenges regarding the solution of the CP imposed to them. In what follows we will always consider $w_x$ to be a negative constant, whose value is the third free parameter of our model. The system (2) does not admit analytic solutions in the general case but a noteworthy exception exists for $w_x = -1 - \lambda_x$. This allows simultaneously $w_x$ and $\lambda_x$ to be close to $-1$ and 0, respectively, while still exhibiting a distinct behaviour compared to $\Lambda$CDM.

In order to probe the usefulness of such a model, we restrict the values of the coupling constants to those which allow a duo-scaling regime and which are within the 2$\sigma$ contours obtained by applying two observational tests: type Ia supernovae and the so-called CMB shift parameter. The present values of the baryonic energy density $\Omega_b$ and the radiation energy density $\Omega_r$ were held fixed respectively at 0.042 and $4.2 \times 10^{-5}$, which are the best fit values of the combined WMAP3 and SDSS observations. The value of $h$ was marginalized analytically. For $\Omega_c$ we assumed a gaussian prior equivalent to WMAP3+SDSS observations, i.e., with a mean at 0.22 and a standard deviation of 0.03.

Guided by the analytic solution discussed above, we found a good candidate model with $\{\lambda_x = 0.23, \lambda_c = -0.14, w_x = -0.85\}$. Figure 1 depicts the evolution of the model for these parameter values, which give $R_{-1} = 3.48$. This is within 4% of the present (WMAP3+SDSS) value, and one could argue that this solves the CP. Note that $\rho_x$ is negative throughout the entire matter dominated era, which helps to increase its duration. Another important feature of this example is that the radiation-matter equipartition occurs much earlier, at $z \simeq 20000$, and therefore it is possible that different observational tests such as a full CMB analysis would rule
out such an anticipation of the MDE. In fact, fitting the observed matter power spectrum would be a challenge since its peak is roughly estimated by $k_{\text{eq}}$, the wave number of perturbations which enter the horizon at matter-radiation equality, and that would be shifted to smaller scales by a factor of $\approx 6.6$ (in units of $h \times \text{distance}^{-1}$).

3 Conclusions

With the goal of clarifying some of the different aspects of cosmological fine-tuning and their relation to dark interactions, we proposed the use of three variables to quantify each of most common issues of cosmological models: the CP, the DEICP and the SIC. By restricting our parameters through the use of both supernovae and the CMB shift parameter and through the requirement of a duo-scaling cosmology, we greatly limited our parameter space.

Applying the proposed variables to our Dark Interactions Model we found that each coupling constant is related to a different problem. The CP can only be solved for large values of $\lambda_x$, while larger values of $\lambda_c$ guarantee higher amounts of DE in the early universe, and thus relate to the DEICP. We also found that nonzero values of $\lambda_c$ also give rise to smaller SIC for any value of $\lambda_x$. We thus investigated examples for which analytic solutions exist so as to gain some intuition on how to address these different problems and were able to find a candidate for which the CP was solved and DEICP alleviated. The latter because the higher value of $\zeta$ obtained was $\sim 10^{-26}$. Even if not a proper solution, the latter represent a 74 order-of-magnitude improvement over the $\Lambda$CDM scenario, for which $\zeta \sim 10^{-100}$. It is possible that a solution to the DEICP requires a third scaling solution: a scaling between dark energy and radiation. Such regime would only be achieved if we shed light on the dark interactions, including radiation in our coupling scheme.

It is also noteworthy that, based on the example shown and other examples studied, solutions to the CP (in our class of models) seem to require DE with negative energy density in the past. This may be a hint that modified gravity theories, which can more easily accommodate negative (effective) energy densities, might have an upper hand at explaining this issue.

Finally, we conclude that $R_{-1} \approx R_0$ forces an early equivalence between matter and radiation, and coping with the shift in the peak of the matter power spectrum will be a challenge for such a model. Should the perturbation equations turn out to be considerably different than those of the concordance model and the matter power spectrum fitted, then as sensitive as the model might be to its parameters in this region, compared to $\Lambda$CDM or quintessence no real fine-tuning will be necessary in order to solve the coincidence problem.

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References