$F$-term uplifted racetrack inflation

Marcin Badziak

Institute of Theoretical Physics, University of Warsaw

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in collaboration with Marek Olechowski
Outline

1. Racetrack inflation
2. Constraints for the Kähler potential
3. $F$-term uplifted racetrack inflation
KKLT Moduli Stabilization

F-term potential in 4D SUGRA

\[ V = e^K \left( K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3 |W|^2 \right) \]

Kähler potential for the volume modulus

\[ K = -3 \ln (T + \bar{T}) \]

For fixed dilaton and CSM fluxes contribute a constant term to the superpotential

\[ W = A \]

Introducing non-perturbative correction (e.g. gaugino condensation) to the superpotential

\[ W = A + Ce^{-cT} \]

volume modulus can be stabilized at AdS SUSY minimum.

We live in dS space \(\Rightarrow \overline{D3}\)-branes introduced to uplift minimum to dS space:

\[ \Delta V = \frac{E}{(T + \bar{T})^2} \]
Moduli stabilization allow for constructing inflationary models inspired by string theory.

Moduli fields can be considered as candidates for the inflaton (moduli inflation).

KKLT potential is too steep (|\eta| > 1) and does not fulfill the slow-roll conditions \( \Rightarrow \) inflation cannot be realized.

Inflation can be realized with the racetrack superpotential:

\[
W = A + Ce^{-cT} + De^{-dT}
\]
Inflation in the vicinity of a saddle point
Axion $\tau$ is the inflaton

Fine-tuning
Flux parameter $A$ fine-tuned at the level of $10^{-4}$

CMB signatures
- $n_s \lesssim 0.95$
- $r \ll 1$

*Blanco-Pillado et al. '04*
**Racetrack Inflation - Inflection Point Model**

Inflation in the vicinity of an **inflection point**

$t$ is the inflaton

**Fine-tuning**

Fine-tuning of parameters related to the height of the barrier

*M.B., Olechowski '08*

Avoiding overshooting problem requires fine-tuning at the level of $10^{-8}$

**CMB signatures**

- $n_s \gtrsim 0.93$
- $r \ll 1$

*Linde, Westphal '07*
In both racetrack inflation models SUSY is broken explicitly by $D3$-branes. Most of the existing uplifting mechanisms have not been applied to inflationary models.

**Goal**

Construct racetrack inflation models in a fully supersymmetric framework with the hidden sector matter field as a source of uplifting and SUSY breaking.
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**Goal**

Construct racetrack inflation models in a fully supersymmetric framework with the hidden sector matter field as a source of uplifting and SUSY breaking.
The maximal value of $\eta$ is related to the curvature of the Kähler manifold spanned by the scalar fields appearing in the theory.

$MB, Olechowski '08; Covi et al. '08$

The necessary condition for $|\eta| \ll 1$:

$$R(f^i) < \frac{2}{\hat{G}^2} < \frac{2}{3}$$

where $G = K + \log |W|^2$ and $\hat{G}^2 \equiv \sqrt{G_i G_i} = 3 + e^{-G} V$

$R(f^i) \equiv R_{ijpq} f^i f^j f^p f^q$ is the sectional curvature along the direction of the SUSY breaking ($f_i \equiv G_i / \hat{G}^2$ is the unit vector defining that direction).

Note: $\hat{G}^2 = 3$ for Minkowski, $\hat{G}^2 > 3$ for de Sitter.

The above condition can be used to eliminate some models even without specifying the superpotential!
In the one field case the necessary condition simplifies:

\[ R_T < \frac{2}{G^2} < \frac{2}{3} \]

Kähler potential for the volume modulus:

\[ K = -3 \ln(T + \bar{T}) \]

The curvature scalar for the volume modulus takes the form:

\[ R_T = \frac{2}{3} \]

The trace of the \( \eta \)-matrix is constant and negative: \( MB, Olechowski '08 \)

\[ \text{Tr}(\hat{\eta}) = -\frac{4}{3} \]

\( \eta \leq -2/3 \Rightarrow \text{slow-roll conditions violated!} \)

How is it possible that racetrack inflation works?

Uplifting from \( \overline{D3} \)-branes is non-supersymmetric and gives additional, positive contribution to \( \text{Tr}(\hat{\eta}) \)
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For the no-scale Kähler potential

\[ K = -3 \ln(T + \bar{T} - |\Phi|^2) \]

the Kähler manifold is a maximally symmetric coset space with a constant curvature \( R(f^i) = 2/3 \)

\( \text{Gomez-Reino, Scrucca '06} \)

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For the separable Kähler potential:

\[ K = K^{(T)}(T, \overline{T}) + K^{(\Phi)}(\Phi, \overline{\Phi}) \]

the necessary condition for slow-roll inflation reduces to:

\[ R_T \Theta_T^4 + R_\Phi \Theta_\Phi^4 < \frac{2}{G^2} \]

\( R_i \) are the scalar curvatures of the one dimensional submanifolds associated with each of the fields

\( \Theta_i^2 \equiv G_{ii} f^i f^\dagger \) parameterize SUSY breaking and satisfy \( \sum_i \Theta_i^2 = 1 \).
Polonyi Uplifting

Volume modulus coupled to canonically normalized matter field:

\[ K = -3 \ln(T + \bar{T}) + \Phi \bar{\Phi} \]

\[ R_\Phi = 0 \] so the necessary condition for slow-roll inflation is:

\[ \Theta_T^4 < \frac{3}{\hat{G}^2} \]

If the matter field dominates SUSY breaking during inflation \((\Theta_T^2 \ll 1)\) then \(F\)-term uplifted racetrack inflation is possible.

Superpotential

\[ W = A + Ce^{-cT} + De^{-dT} - \mu^2 \Phi \]

is sufficient to uplift both racetrack inflation models!
Volume modulus coupled to canonically normalized matter field:

\[ K = -3 \ln (T + T) + \Phi \Phi \]

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If the matter field dominates SUSY breaking during inflation (\( \Theta^2_T \ll 1 \)) then \( F \)-term uplifted racetrack inflation is possible.

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Polonyi Uplifting of Inflection Point Inflation

\[ \Theta^2_\phi \gg \Theta^2_T \Rightarrow \Phi \text{ dominates SUSY breaking} \]
(during and after inflation)

\( \phi \) is the main component of the inflaton

**Fine-tuning**

Fine-tuning is not strictly related to the height of the barrier

Fine-tuning at the level of \( 10^{-3} \Rightarrow 5 \text{ orders of magnitude weaker than in the original model!} \)

**CMB signatures**

\( n_s \gtrsim 0.93 \) not altered by \( F \)-term uplifting

but for some sets of parameters isocurvature perturbations may be produced
Polonyi Uplifting of Saddle Point Inflation

All 4 fields are involved in the inflationary dynamics

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Fine-tuning at the level of \( 10^{-3} \)
(slightly weaker than in the original model)

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Summary of Polonyi uplifting

Volume modulus no longer the inflaton but fine-tuning reduced
Polonyi Uplifting of Saddle Point Inflation

All 4 fields are involved in the inflationary dynamics

\[ \Theta_\phi^2 \gg \Theta_T^2 \Rightarrow \Phi \text{ dominates SUSY breaking (during and after inflation)} \]

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\[ n_s \lesssim 0.95 \text{ not altered by } F\text{-term uplifting} \]

**Summary of Polonyi uplifting**

Volume modulus no longer the inflaton but fine-tuning reduced
The volume modulus coupled to quantum corrected O’Raifeartaigh model:

**Modified Kähler potential**

\[ K = -3 \ln(T + \bar{T}) + \Phi \bar{\Phi} - \frac{(\Phi \bar{\Phi})^2}{\Lambda^2}, \quad \Lambda \ll 1 \]

\[ R_\Phi = \frac{-4}{\Lambda^2(1 - 4|\Phi|^2/\Lambda^2)^3} < 0 \]

**Superpotential**

\[ W = A + Ce^{-cT} + De^{-dT} - \mu^2 \Phi \]

SUSY breaking minimum occurs at \( |\Phi| \sim \Lambda^2 \ll 1 \)
O’uplifting - Decoupling of the Matter Field

The mass matrix at $\tau = \theta = 0$ in the limit $\phi \ll \Lambda \ll 1$:

$$
\begin{pmatrix}
m^{2}_{tt} & m^{2}_{t\tau} & m^{2}_{t\phi} & m^{2}_{t\theta} \\
m^{2}_{t\tau} & m^{2}_{\tau\tau} & m^{2}_{\tau\phi} & m^{2}_{\tau\theta} \\
m^{2}_{t\phi} & m^{2}_{\tau\phi} & m^{2}_{\phi\phi} & m^{2}_{\phi\theta} \\
m^{2}_{t\theta} & m^{2}_{\tau\theta} & m^{2}_{\phi\theta} & m^{2}_{\theta\theta}
\end{pmatrix}
\sim
\begin{pmatrix}
\Lambda^{0} & 0 & \phi m^{2}_{\phi\phi} & 0 \\
0 & \Lambda^{0} & 0 & \Lambda^{0} \\
\phi m^{2}_{\phi\phi} & 0 & \Lambda^{-2} & 0 \\
0 & \Lambda^{0} & 0 & \Lambda^{-2}
\end{pmatrix}
$$

- the matter field is heavier than the volume modulus
- the mixing between the matter field and the volume modulus is strongly suppressed

The matter field is decoupled from the inflationary dynamics
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O’uplifting of Racetrack Inflation Models

- $\Phi$ is almost constant during inflation
- SUSY breaking dominated by the matter field ($\Theta_\Phi > \Theta_T$)
- matter field $F$-term provides effective uplifting term $|F_\Phi|^2 \sim 1/t^3$
- O’uplifted racetrack models resemble the original ones
- Volume modulus is the inflaton but SUSY is broken spontaneously by the matter field
Racetrack inflation can be realized in a fully supersymmetric framework with the matter field $F$-term as a source of SUSY breaking and uplifting.

Details of the inflationary scenario depend on the choice of the matter field sector:

- **Polonyi uplifting** - the volume modulus is no longer the inflaton but fine-tuning significantly reduced.
- **O’uplifting** - the matter field is decoupled from the inflationary dynamics even though it dominates SUSY breaking during and after inflation (i.e. $|F_\phi| \gg |F_T|$).