CONSTRAINTS ON ANISOTROPY OF DARK ENERGY

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MOTIVATION

Dark energy → Maybe they are related ?

CMB anomalies

With Moss I have been developing a framework for

describing lattice network of defects (cf Pearson talk)

- Anisotropic dark energy, Battye & Moss, PRD 2005
- Cosmological perturbations in elastic dark energy models, Battye & Moss, PRD 2007
- CMB anomalies & dark energy, Battye & Moss, PRD 2009

This work is a derivative of this, but the limits apply to any DE models

PROPERTIES OF DARK ENERGY

- Equation of state parameter :

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) \longrightarrow w = \frac{P}{\rho} < -\frac{1}{3}$$

for acceleration

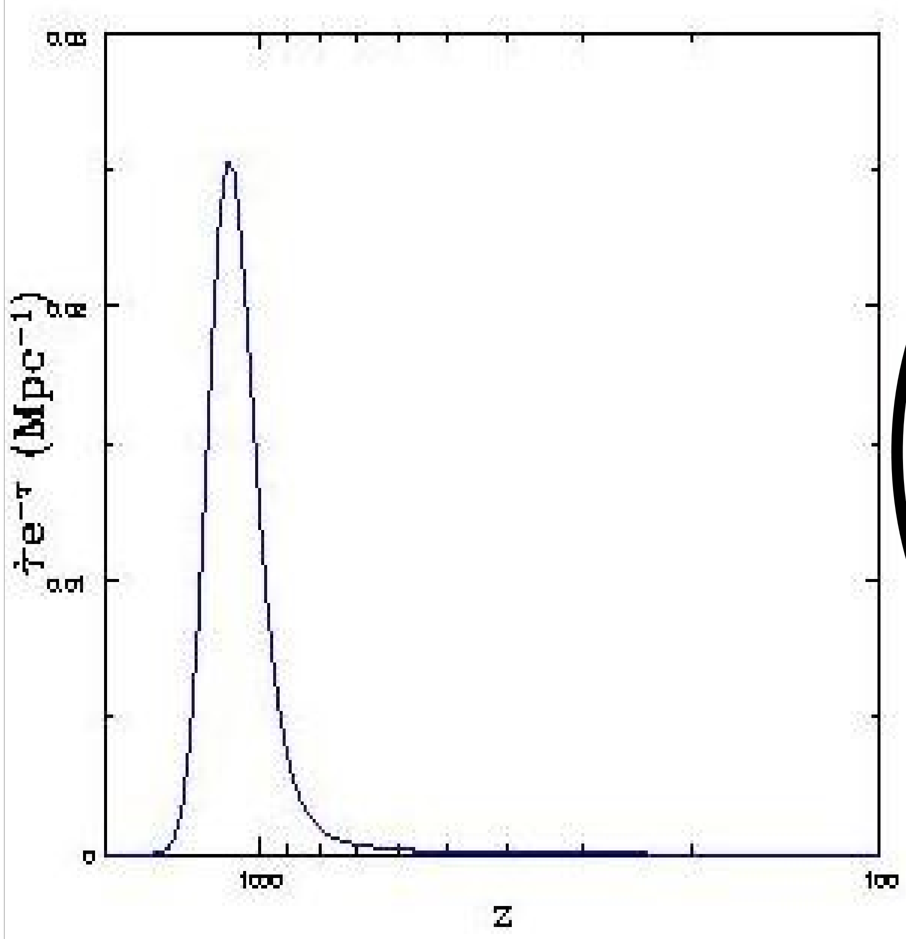
- Speed of sound :

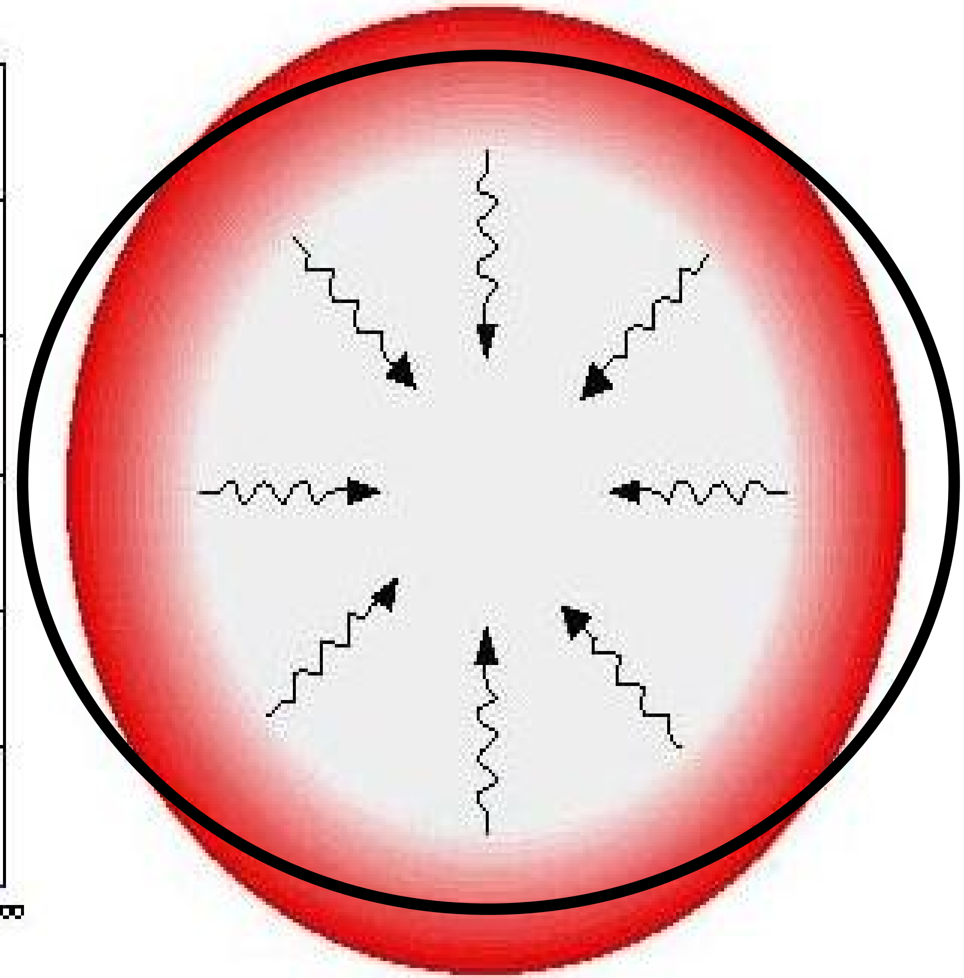
$\lambda_J \sim c_S t$ ~< size of cluster $\longrightarrow$ $c_s \sim 1$

- Anisotropic equation of state :

$\Delta w \sim< 0.1$ from SNe (eg. Cooke & Lynden-Bell 2009)

ELLIPTICAL LAST SCATTERING SURFACE





Distance to the last scattering surface will be different in different directions -

- this will create a quadrupole anisotropy contribution

FRAMEWORK

Use a Bianchi I spacetime metric :

$$ds^2 = a^2 \left(-d\eta^2 + \gamma_{ij}(\eta) dx^i dx^j \right)$$

& define $$\sigma_{ij} = \frac{1}{2} \frac{d}{d\eta} \gamma_{ij}$$

Shear tensor

- a Universe with 3 independent scale factors eg a,b,c

CONSTRAINTS ON ELLIPTICITY

Campanelli et al studied a model for ellipticity of the LSS:

$$\sigma_{ij} = -e_{\mathrm{rec}}^2 \delta_{i3}\delta_{j3}$$

$$\longrightarrow \quad e_{\mathrm{rec}} < 6.4 \times 10^{-3}$$

strong limit but no connection to something more physical

They also claimed that this could lead to a low quadrapole

- I don't think that this is correct unless there are correlations (cf comments by Bunn on Tuesday!)

ANISOTROPIC DARK ENERGY

Traceless

Consider the case : $P_{i}^{\cdot j} = \rho_{\mathrm{de}}\left(w\delta_{i}^{\cdot j} + \Delta w_{i}^{\cdot j}\right)$

- Time-varying, spatially independent anisotropic stress
- growing with time
- model independent ie the constraint should apply to any DE

We will ignore perturbations in the dark energy

$$10^{-5} \sim h_{ij} < \gamma_{ij} \ll 1$$

EQUATIONS OF MOTION

Einstein & conservation equations :

$$3\mathcal{H}^2 = 8\pi G a^2 \rho_{\rm tot} + \boxed{\frac{1}{2}\sigma^2}$$

$$\rho'_{\rm de} = -3\mathcal{H}(1+w)\rho_{\rm de} \boxed{- \sigma_i^{\ j}\Delta w_j^{\ i}\rho_{\rm de}}$$

$$\boxed{\sigma_i^{\ j\prime} = -2\mathcal{H}\sigma_i^{\ j} + 8\pi G a^2 \Delta w_i^{\ j}\rho_{\rm de}}$$

where $\rho_{\rm tot} = \rho_{\rm m} + \rho_{\rm de}$

$$\sigma^2 = \sigma_i^{\ j}\sigma_j^{\ i}$$

A SIMPLE SOLUTION

Take the isotropic case : $\Delta w_i^{\ j} = 0$

$$\rho_{\rm de} = \frac{\rho_{\rm de}(t_0)}{a^{3(1+w)}} \qquad \sigma_{ij} = \frac{\sigma_{ij}(t_0)}{a^2}$$

Expected dependence on a !

$$3\mathcal{H}^2 = 8\pi G a^2 \left(\frac{\rho_{\rm m}(t_0)}{a^3} + \frac{\rho_{\rm m}(t_0)}{a^{3(1+w)}} \right) + \frac{\sigma_{ij}(t_0)\sigma^{ij}(t_0)}{2a^4}$$

Differential equation for a

SOLUTION IN PRESENCE OF ANISO DE

Now consider the case of small Δw

$$\rho_{de} = \frac{\rho_{\rm de}(t_0)}{a^{3(1+w)}}\left[1 + \mathcal{O}[\Delta w^2]\right]$$

$$\frac{\sigma_i{}^j}{H_0} = \frac{3(1-\Omega_{\rm m})\Delta w_i{}^j}{a^2} F(a) + \mathcal{O}[\Delta w^3]$$

where

$$[E(a)]^2 = \frac{\Omega_{\rm m}}{a^3} + \frac{1-\Omega_{\rm m}}{a^{3(1+w)}} \quad \& \quad F(a) = \int_0^a \frac{db}{b^{1+3w}E(b)}$$

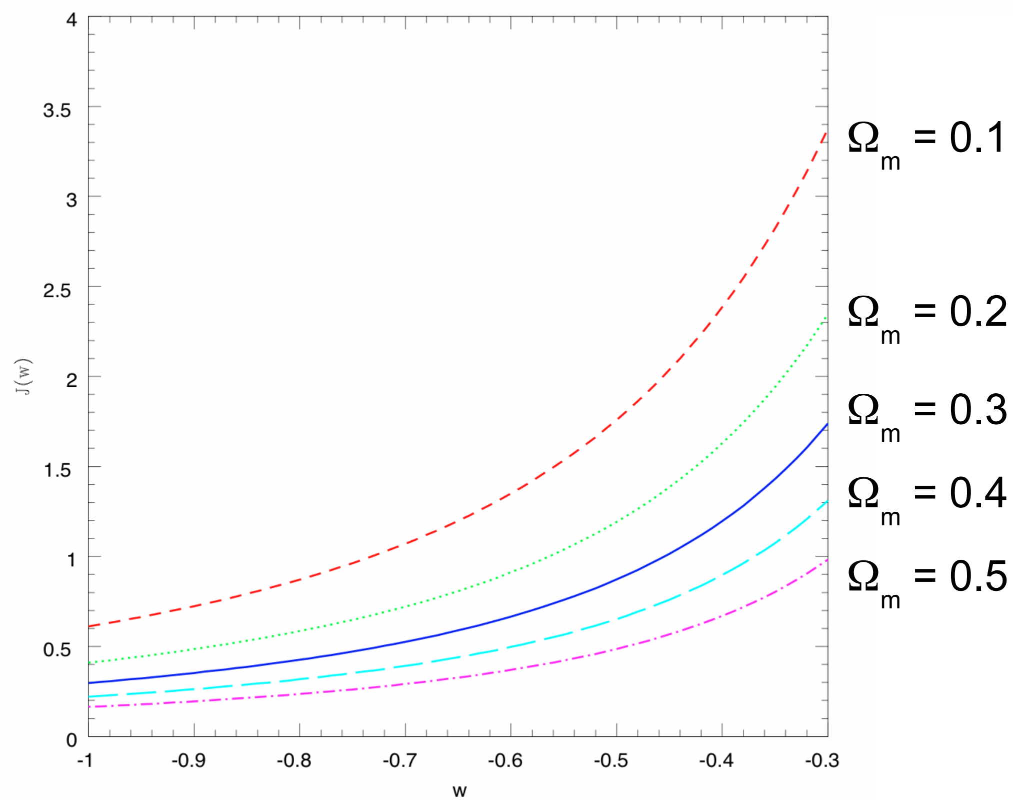
$$\longrightarrow \quad F(a) = \frac{a^{\frac{3}{2}-w}}{\Omega^{\frac{1}{2}}\left(\frac{3}{2}-w\right)}$$ for small a

CMB ANISOTROPY

$$\frac{\Delta T}{T}(\hat{n}) = -\int_{\eta_{\rm dec}}^{\eta_0} \sigma_{ij}\hat{n}^i\hat{n}^j d\eta$$
$$= -\Delta w_{ij}\hat{n}^i\hat{n}^j J(\Omega_{\rm m}, w)$$

- quadrupole anisotropy

where $J(\Omega_{\rm m}, w) = 3(1-\Omega_{\rm m})\int_{a_{\rm dec}}^{1}\frac{da}{a^4E(a)}\int_0^a\frac{db}{b^{1+3w}E(b)}$



a simple function of cosmological params

(essentially integrals over the comoving distance)

CONSTRAINT FROM C_2

Can choose : $\Delta w_{ij} = \mathrm{diag}\left(\Delta w_1, \Delta w_2, -(\Delta w_1 + \Delta w_2)\right)$

- rotation of coordinate system to make diagonal

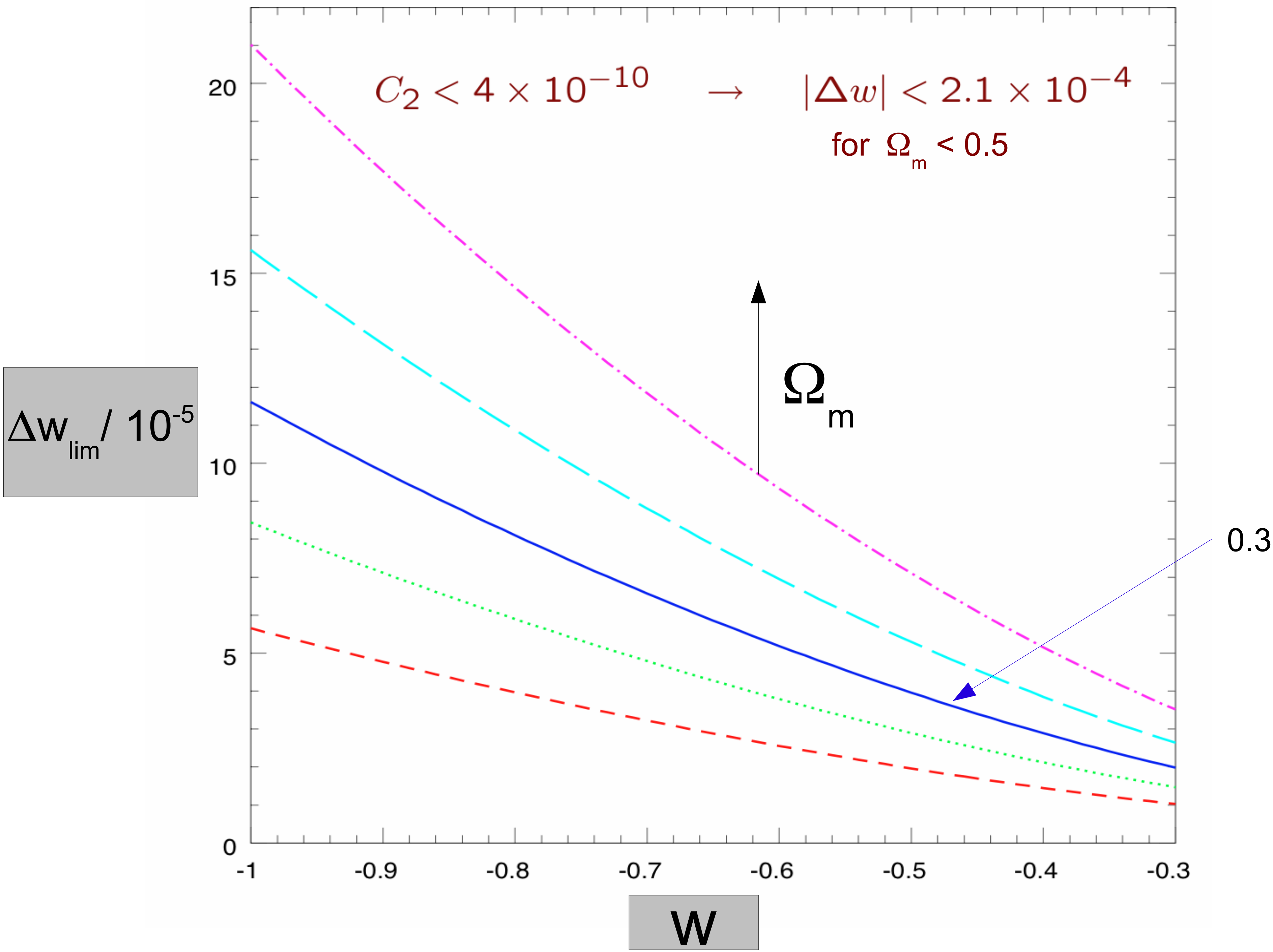
$$\longrightarrow \quad C_2^A = \frac{8\pi}{75}[J(\Omega_\mathrm{m}, w)]^2(\Delta w)^2$$

where $(\Delta w)^2 = \Delta w_{ij}\Delta w^{ij} = 2\left(\Delta w_1^2 + \Delta w_2^2 + \Delta w_1 \Delta w_2\right)$

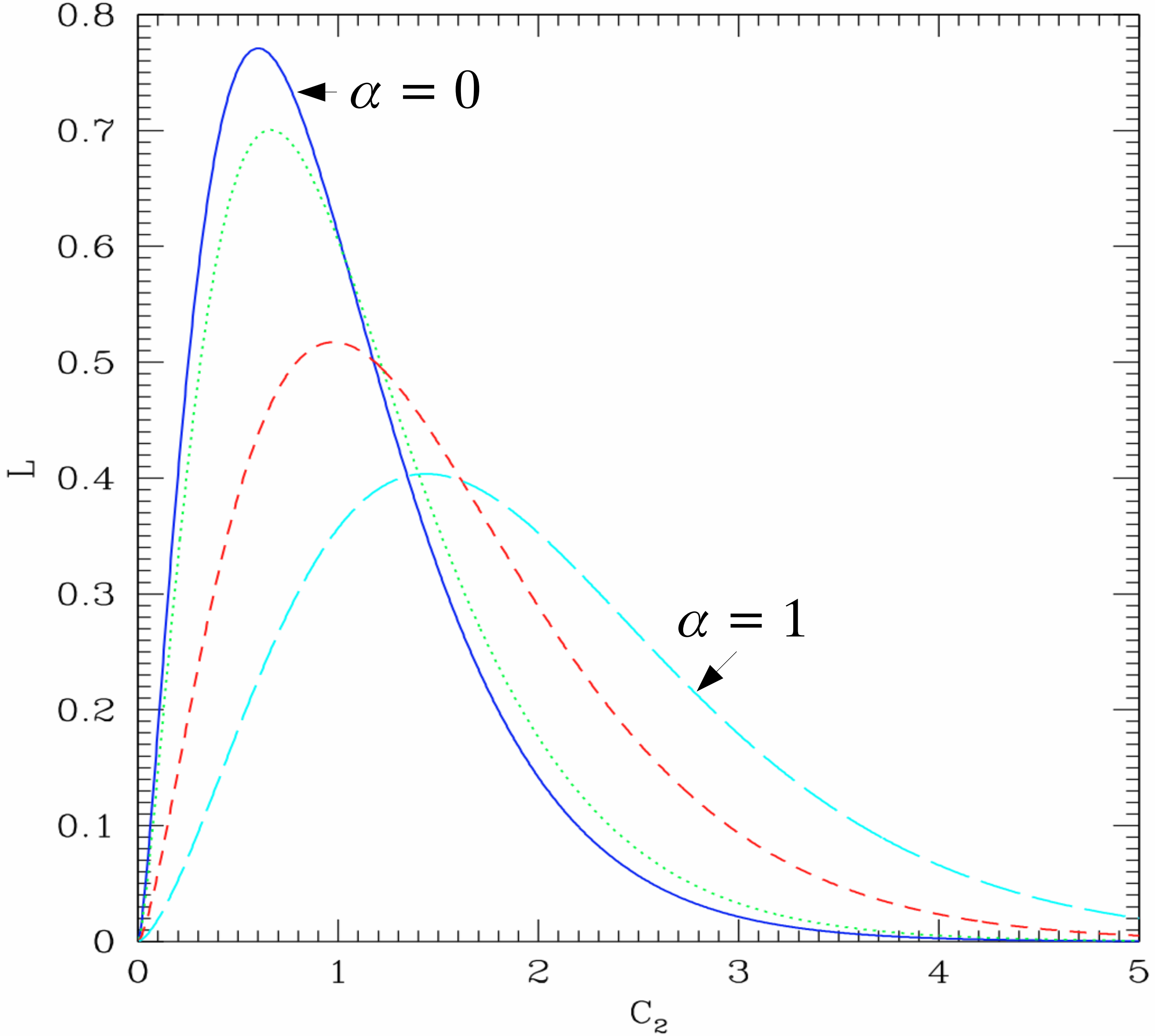
NB there will also be the isotropic component :

$$C_2 = C_2^I + C_2^A$$

LIMITS ON ANISOTROPY



LOW QUADRUPOLE ? NO!



Assuming the 2 components are indep then

$$(\Delta C_2)^2 = \frac{2(1+2\alpha)}{5(1+\alpha)^2} C_2^2$$

where

$$\alpha = \frac{C_2^A}{C_2^I}$$

CONCLUSIONS

- Limit on the anisotropy of dark energy

$$C_2 < 4 \times 10^{-10} \quad \rightarrow \quad |\Delta w| < 2.1 \times 10^{-4}$$

- Model independent

- Significantly stronger than present limits from SNe

- Cannot explain the low quadrapole since it additive !

    - unless it is correlated with primordial initial condition