The String Revolver: How rotation of background galaxies could be a smoking gun for the existence of cosmic strings

Daniel Thomas
with Carlo Contaldi & João Magueijo
Imperial College London

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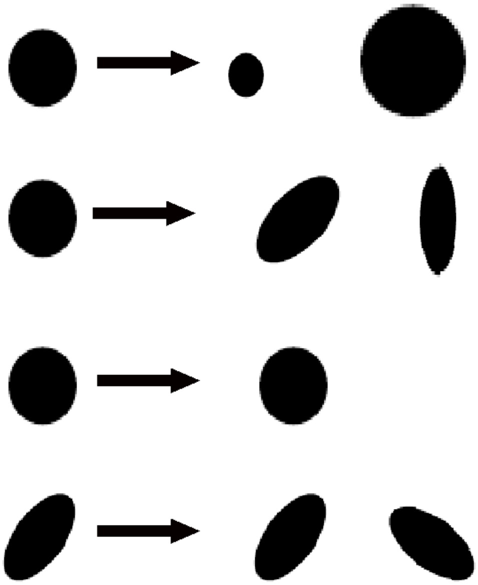
Quick review of weak lensing

- Only one image of the source, but distorted.
- Four ways that images can be distorted; convergence (κ), two components of shear (γ_1, γ_2) and rotation (ρ).
- Single source useless $\Rightarrow$ Weak lensing carried out as analysis of large surveys covering a large fraction of the sky.



The Why?

- "Normal" things=planets, stars, black holes, dust, galaxies, galaxy clusters..., predominantly source *scalar* perturbations.
- Scalar perturbations do not source the rotation component ρ
- Question: Could *vector* perturbations source this rotation component?

Contents

- Lensing effect of general vector perturbations
- Cosmic strings as source of vector perturbations
- Calculation of signal from network of strings in weak lensing survey

Vector perturbations

- Normally vector perturbations ignored: rapid decay due to expansion
- General FRW metric with vector perturbations:

$$
\begin{aligned}
g_{00} &= -a^2 \\
g_{0i} &= -a^2 V_i \\
g_{ij} &= a^2(\delta_{ij} + F_{i,j} + F_{j,i})
\end{aligned}
$$

- V_i and F_i divergenceless: "pure" vector modes
- Gauge choice $F_i = 0$

$$
\begin{aligned}
g_{00} &= -a^2 \\
g_{0i} &= -a^2 V_i \\
g_{ij} &= a^2 \delta_{ij}
\end{aligned}
$$

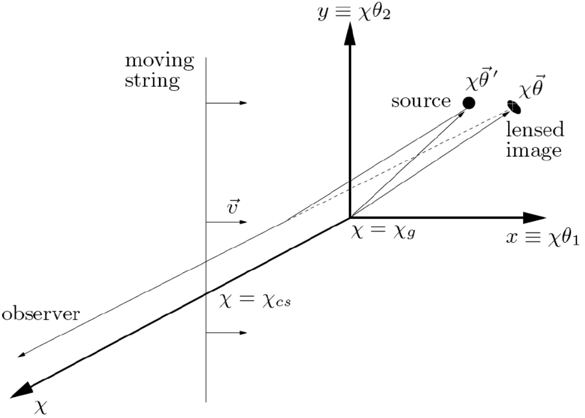
- We will take $\vec{V} \ll 1$

Setup

Consider lensing of a background galaxy by vector perturbations

We will use comoving coordinates such that $\vec{x} = \chi\left(\theta^1, \theta^2, 1\right)$

χ is the comoving radial coordinate and $\theta \ll 1$



- The trajectory of the photons comes from the spatial components of the geodesic equation

$$\frac{d^2x^i}{d\lambda^2} = -\Gamma^i_{\alpha\beta}\frac{dx^\alpha}{d\lambda}\frac{dx^\beta}{d\lambda}$$

- The perturbed metric gives

$$\frac{d^2(\chi\theta^i)}{d\chi^2} = \frac{\dot{V}}{a^2} + \frac{V_{3,i}}{a^2} - \frac{V_{i,3}}{a^2}$$

note: i,j = 1,2 only

- Integrating twice gives

$$\theta^i_S = \theta^i + \int_0^\chi d\chi^{'} \left(\frac{\dot{V}_i + V_{3,i} - V_{i,3}}{a^2}\right)\left(1 - \frac{\chi^{'}}{\chi}\right)$$

θ^i_S is the actual position of the source, θ^i is the observed position of the source

- Conventional to define:

$$A_{ij} = \frac{\partial \theta_S^i}{\partial \theta^j} = \delta_{ij} + \psi_{ij}$$

- So, for a single galaxy located at χ

$$\psi_{ij} = \int_0^{\chi} d\chi^{'} \left(\frac{\dot{V}_{i,j} + V_{3,ij} - V_{i,3j}}{a^2} \right) \chi^{'} \left(1 - \frac{\chi^{'}}{\chi} \right)$$

ψ_{ij}

- Distortion tensor ψ_{ij} has simple interpretation as 2x2 matrix

$$\psi_{ij} = \begin{pmatrix} -\kappa - \gamma_1 & -\gamma_2 + \rho \\ -\gamma_2 - \rho & -\kappa + \gamma_1 \end{pmatrix}$$

- κ is the convergence

  γ_1 and γ_2 are the two components of shear

  ρ is the rotation

There can be more than one

- Effects are measured via surveys across a patch of the sky, so insert source function $W(\chi)$, representing density of galaxies at different redshifts. Upper limit of integral now χ_∞, depth of survey
- $$\psi_{ij} = \int_0^{\chi_\infty} d\chi \; g(\chi) \left(\frac{\dot{V}_{i,j} + V_{3,ij} - V_{i,3j}}{a^2} \right)$$
  $$\text{with } g(\chi) \equiv \chi \int_\chi^{\chi_\infty} d\chi' \left(1 - \frac{\chi}{\chi'} \right) W(\chi')$$
- This is the expression for the components of the distortion tensor in terms of vector metric perturbations that happen to be floating around between background galaxies and us

So far, looked at general vector perturbations in metric, but need a source

Enter cosmic strings:

- Formed from phase transitions in early universe or inflated superstrings
- Moving cosmic strings have relatively large vector perturbations

Energy momentum tensor

Use the Einstein equations to relate cosmic strings to metric perturbations

$$
\begin{aligned}
\delta\tilde{T}^0_j &= \tilde{\omega}_j \\
\delta\tilde{T}^i_j &= -\frac{1}{2k}(\tilde{\Pi}_{i,j} + \tilde{\Pi}_{j,i}) \\
\tilde{V}_i &= \frac{16\pi G a^2}{k^2}\tilde{\omega}_i \\
\dot{\tilde{V}}_i &= -\frac{8\pi G a^2 \tilde{\Pi}_i}{k} - \frac{2\dot{a}}{a}\left(\frac{16\pi G a^2}{k^2}\right)\tilde{\omega}_i
\end{aligned}
$$

$$
\psi_{ij} = \frac{2G}{\pi^2}\int_0^{\chi_\infty} d\chi\; g(\chi)\int_{-\infty}^{\infty} d^3k e^{i\vec{k}\cdot\vec{x}}\hat{k}_j\left(\hat{k}_i\tilde{\omega}_3 - \hat{k}_3\tilde{\omega}_i - \frac{2\dot{a}}{ak}\tilde{\omega}_i - \frac{\tilde{\Pi}_i}{2}\right)
$$

So far, we have ψ_{ij}, i.e. how the image of a background galaxy is distorted, in terms of quantities characterising a cosmic string.

Now, need to calculate the effect that a network of cosmic strings would generate on a weak lensing survey consisting of many background galaxies.

Powerspectra

Observables are power spectra, quantifies signal over large patch of sky

- 2d power spectrum of ψ_{ij} $(P^{\psi}_{ijlm}(l))$ defined by:

$$\langle\tilde{\psi}_{ij}(\vec{l}) \quad \tilde{\psi}^{*}_{lm}(\vec{l'})\rangle = (2\pi)^2\delta^2(\vec{l}-\vec{l'})P^{\psi}_{ijlm}(l)$$

- Power spectra of ω_i and Π_i

$$\begin{aligned}
\langle\tilde{\omega}_i(\vec{k},\eta)\tilde{\omega}^{*}_j(\vec{k'},\eta')\rangle &= (2\pi)^3\delta^3(\vec{k}-\vec{k'})P_{ij}\frac{P_{\omega}(k\eta,k\eta')}{\sqrt{\eta\eta'}} \\
\langle\tilde{\Pi}_i(\vec{k},\eta)\tilde{\Pi}^{*}_j(\vec{k'},\eta')\rangle &= (2\pi)^3\delta^3(\vec{k}-\vec{k'})P_{ij}\frac{P_{\Pi}(k\eta,k\eta')}{\sqrt{\eta\eta'}} \\
\langle\tilde{\omega}_i(\vec{k},\eta)\tilde{\Pi}^{*}_j(\vec{k'},\eta')\rangle &= (2\pi)^3\delta^3(\vec{k}-\vec{k'})P_{ij}\frac{P_{\omega\Pi}(k\eta,k\eta')}{\sqrt{\eta\eta'}} \\
\langle\tilde{\omega}_i(\vec{k},\eta)\tilde{\Pi}^{*}_j(\vec{k'},\eta')\rangle &= \langle\tilde{\Pi}_i(\vec{k},\eta)\tilde{\omega}^{*}_j(\vec{k'},\eta')\rangle \\
P_{ij} &= \delta_{ij}-\hat{k}_i\hat{k}_j
\end{aligned}$$

plugging in and rearranging...

$$P^{\psi}_{ijlm}(l) = 256\pi^2 G^2 \int_0^{\chi_\infty} d\chi \frac{g^2(\chi)}{\chi^3} \hat{l}_j \hat{l}_m \times \Big\{ P_\omega(l) \hat{l}_i \hat{l}_l + \left(\delta_{il} - \hat{l}_i \hat{l}_l\right) \left(\frac{P_\Pi(l)}{4} + P_{\omega\Pi}(l) \frac{2\dot{a}\chi}{al} + \frac{4\dot{a}^2\chi^2}{a^2 l^2} P_\omega(l) \right) \Big\}$$

This is the power spectrum of components of the distortion matrix in terms of the powerspectra characterising a string network

Weaklensing surveys give results in terms of the power spectra of κ, ρ etc, we're interested in ρ, so need to go from P^{ψ}_{ijlm} to P_ρ

$$P_\rho(l) = \frac{1}{4}\left(P^{\psi}_{1212}(l) + P^{\psi}_{2121}(l) - 2P^{\psi}_{1221}(l)\right)$$

$$\begin{aligned} P_\rho(l) \; = \; & \int_0^{\chi_\infty} d\chi \frac{g^2(\chi)}{\chi^3} 64\pi^2 G^2 \times \\ & \left(\frac{4\dot{a}^2\chi^2}{a^2 l^2} P_\omega(l) + \frac{P_\Pi(l)}{4} + \frac{2\dot{a}\chi}{al} P_{\Pi\omega}(l)\right) \; , \end{aligned}$$

Result! The rotation power spectrum is not zero.

Why so important?

Scalars can't source rotations: $\psi_{ij} \sim \Phi_{,ij}$

Symmetric in i and j.

$$\Rightarrow P^{\psi}_{1212} = P^{\psi}_{2121} = P^{\psi}_{1221}$$

$$\Rightarrow P_{\rho} = 0$$

So a network of cosmic strings could generate a weak lensing signal that would not be contaminated by anything that we "know" exists.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Putting numbers in

Insert:

- Weight function $W(z) = z^2 e^{-5z/2}$
- $z_{\max} = 6$
- $P_{\omega/\Pi/\Pi\omega}(l) \sim (G\mu)^2 l^{-1}$
- Noise: background galaxy density$= 100$ galaxies/arcmin^2

  average ellipticity$= 0.3$
- $G\mu = 10^{-7}$

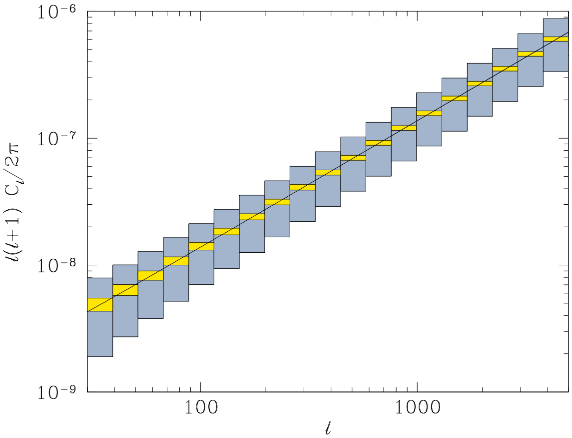

Figure: The angular power spectrum of rotation for a network of strings with $G\mu = 1 \times 10^{-7}$. The blue and yellow boxes show the forecasted error for two surveys with $f_{\rm sky} = 0.1$ and $f_{\rm sky} = 0.5$ respectively.

Conclusions

- We could observe signal with next generation surveys.
- Would be evidence of something exotic:

  cosmic strings, or something crazier...?