Clustering of dark matter haloes: insights from the peak model

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outline

• Clustering with Gaussian initial conditions
• Clustering with primordial non-Gaussianity
Galaxies are now routinely measured up to high redshift
Biased galaxy formation

Galaxies trace high overdensity regions
= dark matter haloes

Courtesy of Ilian Iliev
Science with large galaxy surveys

A plethora of ongoing and planned galaxy surveys, e.g. BOSS, DES, Euclid, HETDEX, JDEM, Pan-STARR, WiggleZ ..., to measure

- matter power spectrum: matter content, neutrino masses etc.
- baryon acoustic oscillation (BAO): dark energy equation of state
- redshift space distortions: growth rate of structures
- primordial non-Gaussianity

... requires solid understanding of galaxy clustering!
The local bias model

Kaiser (1984), Fry & Gaztanaga (1993), ...

- Essentially all models of galaxy biasing are based on the local bias model

\[ \delta_g(x) = \sum_N \frac{b_N}{N!} \delta(x)^N \]

- The bias parameters are derived using the peak-background split argument:

\[ b_N = \left( -\frac{1}{\sigma} \right)^N \bar{n}^{-1} \frac{\partial^N [\bar{n}(\nu)]}{\partial \nu^N}, \quad \nu \equiv \delta_c(z)/\sigma(M) \]
The peak model


• DM haloes indeed are local density maxima of the evolved mass distribution
• Too difficult to work out the properties of density peaks of a highly non-Gaussian field -> consider instead the density maxima of the initial Gaussian density field
• good approximation for massive haloes (M>M*)
Peak correlation functions

Desjacques (2008), Desjacques, Scoccimarro & Sheth (2010)

• compute the ensemble averages

$$\langle n_{pk}(x_1) \cdots n_{pk}(x_n) \rangle$$

• (connected) 2-point correlation

$$\xi_{pk}(r) = (b_I - \bar{b}_I \Delta)^2 \xi_{\delta_S}(r) + \frac{1}{2} \left[ b_{II} - \bar{b}_{II} \Delta \right]^2 (\xi_{\delta_S}(r))^2 + \text{other terms}$$

• A calculation shows that the peak-background split holds (at least) up to 3rd order

$$b_N = \left( -\frac{1}{\sigma} \right)^N \bar{n}_{pk} \frac{\partial^N \bar{n}_{pk}(\nu)}{\partial \nu^N}$$
Mo & White (1996):

\[ b_I \approx 1 + \frac{\nu^2 - 1}{\delta_{sc}} \]

Peaks:

\[ b_I \approx 1 + \frac{\nu^2 - 3}{\delta_{sc}} \]

Sheth & Tormen (1999):

\[ b_I \approx 1 + \frac{a\nu^2 - 1}{\delta_{sc}}, \quad a = 0.75 \]
**Peak bias at first order**

- The leading order contribution to the 2-point correlation of density peaks is

\[ \xi_{pk}(r) = b_I^2 \xi_{\delta S}(r) - 2b_I \tilde{b}_I \Delta \xi_{\delta S}(r) + \tilde{b}_I^2 \Delta^2 \xi_{\delta S}(r) \]

- Can be thought of as arising from the bias relation

\[ \delta n_{pk}(x) = b_I \delta S(x) - \tilde{b}_I \Delta \delta S(x) \]

- In Fourier space,

\[ \delta n_{pk}(k) = b_{pk}(k) \delta(k), \quad b_{pk}(k) \equiv \left( b_I + \tilde{b}_I k^2 \right) W(k, R_S) \]
In a Cold Dark Matter cosmology:

Baryon acoustic oscillation (BAO)
Scale-dependence across the BAO

\[ M_s = 5 \times 10^{13} M_\odot / h \]

\[ z = 0.3 \]
\[ \nu = 2.3 \]
\[ b_\nu = 1.5 \]
\[ b_\xi = 31.7 \]

Local bias: \( b_I^2 \xi_\delta \)

Density peaks: \( b_I^2 \xi_\delta - 2b_I \tilde{b}_I \Delta \xi_\delta + \tilde{b}_I^2 \Delta^2 \xi_\delta \)
**Velocity bias**

Bardeen et al. (1986), Desjacques & Sheth (2010)

Assumption: galaxies/DM haloes locally move with the dark matter flows. This implies

- **local bias model:**
  \[ \sigma_{vg}^2 = \sigma_{vdm}^2 \]
  \[ \theta_g(k) = \theta(k) \quad (\theta = \nabla \cdot v) \]

- **peak model:**
  \[ \sigma_{vpk}^2 = \sigma_{vdm}^2 (1 - \gamma_0^2), \quad 0 < \gamma_0 < 1 \]
  \[ \theta_{pk}(k) = \left(1 - \frac{\sigma_0^2}{\sigma_1^2} k^2\right) \theta(k) \equiv b_{vel}(k)\theta(k) \]
Gravitational evolution

In a first approximation, we can assume that the initial density peaks move along straight lines (Zeldovich 1970)

\[ x_{pk}(t) = q_{pk} - D(a) \nabla \Phi(q_{pk}) \]

The peak 2-point correlation as a function of \( z \) is

\[ \xi_{pk}(r, z) = \int \frac{d^3k}{(2\pi)^3} G^2(k, z) \left[ b_{vel}(k) + b_{pk}(k, z) \right]^2 P_{\delta \delta}(k, z_0) e^{i k \cdot r} \]

\[ G^2(k, z) = \left( \frac{D(z)}{D(z_0)} \right)^2 \exp \left( -\frac{1}{3} k^2 \sigma_{vpk}^2(z) \right) \]
Peaks

Linear tracers

Work in progress: mode-coupling term still missing
The KIAS Horizon run: $>10^7$ mock LRGs at $z=0$ ($V=300\text{Gpc}^3/h^3$)

(Kim et al. 2009)
Are primordial fluctuations (non-)Gaussian?

- Single field slow roll inflation, which gives the best fit to CMB and LSS data, predicts a negligible level of primordial non-Gaussianity (NG)
- Any evidence for or against the detection of primordial NG would strongly constrain inflationary scenarios
Primordial NG of the local type

Local mapping: \( \Phi(x) = \phi(x) + f_{NL} \phi(x)^2 + g_{NL} \phi(x)^3 + \cdots \)
\( \phi \) Gaussian, \( |\phi| \sim 10^{-5} \), \( P_\phi(k) \sim k^{n_s - 4} \)

Three-point function (bispectrum):
\( B_\Phi(k_1, k_2, k_3) \approx 2f_{NL} [P_\phi(k_1)P_\phi(k_2) + \text{cyclic}] \)

Four-point function (trispectrum):
\( T_\Phi(k_1, k_2, k_3, k_4) \approx 6g_{NL} [P_\phi(k_1)P_\phi(k_2)P_\phi(k_3) + \text{cyclic}] \)
Scale-dependent bias in the local $f_{NL}$ model

Dalal et al. (2008), Matarrese & Verde (2008), McDonald (2008)

$$\Delta b_\kappa(k, f_{NL}) = 3f_{NL} [b(M) - 1] \delta_c \frac{\Omega_m H_0^2}{k^2 T(k) D(z)} = \frac{b_\phi}{k^2}$$

At large scales:

$$P_{hh}(k) = \left( b(M) + \frac{b_\phi}{k^2} \right)^2 P_{mm}(k)$$
The coupling $f_{\text{NL}} \phi^2$ also affects:

\[
\frac{P_{\text{mm}}(k, f_{\text{NL}})}{P_{\text{mm}}(k, 0)} \equiv 1 + \beta_m(k, f_{\text{NL}})
\]
At the first order in $f_{\text{NL}}$, the correction to the halo bias thus is

$$\Delta b(k, f_{\text{NL}}) = \Delta b_\kappa(k, f_{\text{NL}}) + \Delta b_\text{I}(f_{\text{NL}}) + b(M)\beta_m(k, f_{\text{NL}})$$

Peak-background split: $\Delta b_\text{I}$ is negative for $f_{\text{NL}}>0$!
The scale-dependent bias shift can be derived from the statistics of highly overdense regions

\[ \xi_{hh} (|x_2 - x_1|) = -1 + \exp \left\{ \sum_{n=2}^{\infty} \sum_{j=1}^{n-1} \frac{v^n \sigma^n}{j!(n-j)!} \xi^{(n)} \left( \begin{array}{c} x_1, \cdots, x_1 \\ j \text{ times} \end{array} \begin{array}{c} x_2, \cdots, x_2 \\ n-j \text{ times} \end{array} \right) \right\} \]

Matarrese, Lucchin, Bonometto (1986)

We find

\[ \Delta b(k, g_{NL}) = \Delta b_\kappa(k, g_{NL}) + \Delta b_I(g_{NL}) \]

\[ \Delta b_\kappa(k, g_{NL}) = \frac{3}{4} g_{NL} [b(M) - 1] \delta_c^2 \frac{D(0)}{D(z)} S_3^{(1)}(M) \frac{\Omega_m H_0^2}{k^2 T(k)} \]

skewness \( \sim 3 \times 10^{-4} \)
To match the simulations, we consider \( \Delta b = \epsilon_\kappa \Delta b_\kappa + [\Delta b_I + \epsilon_I] \)
Limits on $g_{NL}$ from large scale structure

Using the quasar sample analyzed by Slosar et al. (2008):

$$-3 \times 10^5 < g_{NL} < 8.2 \times 10^5 \quad (95\% \text{ C.L.})$$

For a single tracer with $b(M)=2$, the $1\sigma$ detection limit is

$$\sim 4 \times 10^5 \quad (\text{BOSS})$$

$$\sim 2 \times 10^4 \quad (\text{EUCLID})$$
Conclusion

- In the peak model, the spatial bias parameters and the (first order) velocity bias are scale-dependent.
- This has nontrivial effect on the BAO and redshift space correlations.
- Some discrepancy remains between the theory and simulations of the non-Gaussian halo bias.
- Future galaxy surveys should test realistic models of cubic non-Gaussianity.
- Measure galaxy bispectrum to improve sensitivity to the primordial bispectrum shape.