Planck-scale induced birefringence and the CMB

Giulia Gubitosi

University of Rome “Sapienza”

45th Rencontres de Moriond March 2010

G. Gubitosi, L. Pagano, G. Amelino-Camelia, A. Melchiorri and A. Cooray,
Modified dispersion relations

quantum spacetime $\leadsto$ “quantum” spacetime symmetries


$\leadsto$ Dispersion relation modifications governed by the Planck scale $[E_p \approx 10^{28} \text{eV}]$

$$E^2 = p^2 + m^2 + \Delta_{QG}(p, m, E_p)$$

(with $\Delta_{QG} \xrightarrow{E_p \to \infty} 0$ and $\Delta_{QG} \xrightarrow{p \to 0} 0$)

First-order correction to massless particles dispersion relation

$$E = p + \eta \frac{p^2}{E_P}$$

($\eta$ can be either positive or negative)
Orthogonal polarization states of photons can behave differently:

\[ E_\pm \approx p + \eta_\pm \frac{p^2}{E_P} \]

If the polarization states are left/right circular and \( \eta_+ = -\eta_- \)

→ The two helicity states of electromagnetic waves have different phase velocities

\[ \vec{E}_\pm = \text{Re} \left( (\hat{e}_1 \pm i\hat{e}_2) e^{i(\omega_\pm t - \vec{k} \cdot \vec{x})} \right) \]

→ linearly polarized radiation rotates its polarization vector during propagation

⇒ \textit{in-vacuo} birefringence

(as for light propagation through materials with chiral molecules, as sugar and quartz)

- R.C. Myers, M.Pospelov, PRL 90 (2003) 211601
Effective field theory for in-vacuo birefringence

Modifications of photons dispersion relation of the form $\frac{p^2}{E_{Pl}}$ can be generated by an effective field theory with mass-dimension five corrections to the Lagrangian of electrodynamics

Myers-Pospelov model: [R.C. Myers, M.Pospelov, PRL 90 (2003) 211601]

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2 E_{Pl}} n^\alpha F_{\alpha\delta} n^\sigma \partial_\sigma (n_\beta \varepsilon^{\beta\delta\gamma\lambda} F_{\gamma\lambda})$$

where $n^\alpha$ is an external four-vector (Lorentz invariance violated)
This is the only admissible correction term under a few simple assumptions
(quadratic, gauge invariant, one more derivative than kinetic term, Lorentz invariant (except for $n^\alpha$), not reducible to lower-dimension operators, nor to a total derivative)
Myers-Pospelov model

MP assumed pure-time vector $n_\alpha = (n_0, 0, 0, 0)$:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2E_p} \varepsilon^{ijkl} F_{0j} \partial_0 F_{kl} \quad \left[ \xi \equiv n_0^3 \right]$$

→ violated invariance under boost transformation
preserved space isotropy

Quantum gravity intuition: $\xi$ roughly of order one

Several astrophysical analyses on observable features of this model


→ are CMBP data sensitive to the Planck scale?
Myers-Pospelov model

MP assumed pure-time vector $n_\alpha = (n_0, 0, 0, 0)$:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2E_p} \varepsilon^{jkl} F_{0j} \partial_0 F_{kl} \quad [\xi \equiv n_0^3]$$

→ violated invariance under boost transformation
preserved space isotropy

Quantum gravity intuition: $\xi$ roughly of order one

Several astrophysical analyses on observable features of this model


→ are CMBP data sensitive to the Planck scale?
Birefringence emerging from MP Lagrangian

Modified Maxwell equations

\[ \nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{2\xi}{E_p} \frac{\partial^2 \vec{B}}{\partial t^2} \quad \nabla \cdot \vec{E} = 0 \]

Dispersion law:

\[ \omega_{\pm} = p \left( 1 \pm \frac{\xi}{E_p} p \right) \]

→ Polarization rotation angle (if radiation initially linearly polarized):

\[ \alpha(T) = 2 \frac{\xi}{E_p} p^2 T \]
CMB photons are known to be $\sim 10\%$ linearly polarized

- free streaming toward us since when the universe was $\sim 350000$ yr old
  - large accumulation of the polarization rotation effect
- $\alpha$ depends on energy $\rightarrow$ must consider energy redshift
  \[ \omega(z) = (1 + z)\omega(0) \]

\[
\alpha(z) = 2 \frac{\xi}{E_p} \frac{p_0^2}{H_0} \int_0^z \frac{(1+z')}{\sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}} \, dz'
\]

[assuming standard $\Lambda$-CDM cosmological model]
Measuring rotation

Polarization can be decomposed into “Electric” and “Magnetic” modes:

\[
E(\hat{n}) = \sum_{lm} E_{lm} Y_{lm}(\hat{n}) \quad \text{even under parity}
\]

\[
B(\hat{n}) = \sum_{lm} B_{lm} Y_{lm}(\hat{n}) \quad \text{odd under parity}
\]

Density perturbations are scalars → can produce only E-modes of polarization

Rotation of original polarization generates B-modes
Cross-Correlation Power Spectra

→ tool of analysis: cross-correlation power spectra

\[ C_{l}^{XY} = \frac{1}{2l+1} \sum_{m} < X_{lm}^{*} Y_{lm} > , \quad X, Y = T, E, B \]

if a rotation occurs:

\[
\begin{align*}
C_{l}^{EE} &= \tilde{C}_{l}^{EE} \cos^2(2\alpha) + \tilde{C}_{l}^{BB} \sin^2(2\alpha) \\
C_{l}^{BB} &= \tilde{C}_{l}^{EE} \sin^2(2\alpha) + \tilde{C}_{l}^{BB} \cos^2(2\alpha) \\
C_{l}^{EB} &= \frac{1}{2} \left( \tilde{C}_{l}^{EE} - \tilde{C}_{l}^{BB} \right) \sin(4\alpha) \\
C_{l}^{TE} &= \tilde{C}_{l}^{TE} \cos(2\alpha) \\
C_{l}^{TB} &= \tilde{C}_{l}^{TE} \sin(2\alpha)
\end{align*}
\]
Cross-Correlation Power Spectra

→ tool of analysis:
cross-correlation power spectra

\[ C_{l}^{XY} = \frac{1}{2l + 1} \sum_{m} < X_{lm}^* Y_{lm} >, \quad X, Y = T, E, B \]

if a rotation occurs:

\[ C_{l}^{EE} = \tilde{C}_{l}^{EE} \cos^2 (2\alpha) + \tilde{C}_{l}^{BB} \sin^2 (2\alpha) \]
\[ C_{l}^{BB} = \tilde{C}_{l}^{EE} \sin^2 (2\alpha) + \tilde{C}_{l}^{BB} \cos^2 (2\alpha) \]
\[ C_{l}^{EB} = \frac{1}{2} \left( \tilde{C}_{l}^{EE} - \tilde{C}_{l}^{BB} \right) \sin (4\alpha) \]
\[ C_{l}^{TE} = \tilde{C}_{l}^{TE} \cos (2\alpha) \]
\[ C_{l}^{TB} = \tilde{C}_{l}^{TE} \sin (2\alpha) \]
Effects of rotation on power spectra

BLACK: standard spectra
RED: spectra resulting from a total rotation of 30°

Figure: EE, TE, EB and TB spectra (from top to bottom and left to right)
Analysis on recent experiments data

Data from WMAP5 (94 GHz) and Boomerang (145 GHz)
· E. Komatsu et al. [WMAP Collaboration], arXiv:astro-ph/0803.0547
· C. J. MacTavish et al., arXiv:astro-ph/0507503

- $\alpha$ is energy-dependent, we cannot give a WMAP+BOOMERANG joint estimation on it, but a joint estimation is useful for $\xi$, which does not depend on energy.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\alpha \pm \sigma(\alpha)$</th>
<th>$\xi \pm \sigma(\xi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP (94 GHz)</td>
<td>-1.6 $\pm$ 2.1</td>
<td>-0.09 $\pm$ 0.12</td>
</tr>
<tr>
<td>BOOMERanG (145 GHz)</td>
<td>-5.2 $\pm$ 4.0</td>
<td>-0.123 $\pm$ 0.096</td>
</tr>
<tr>
<td>WMAP+BOOMERanG</td>
<td>-</td>
<td>-0.110 $\pm$ 0.075</td>
</tr>
</tbody>
</table>

$[\alpha \propto \frac{\xi}{E^p_r}]$


At 95% confidence level:

$-0.260 < \xi < 0.040$

First limit on $\xi$ in CMB reference frame
Analysis on recent experiments data

Data from WMAP5 (94 GHz) and Boomerang (145 GHz)
- C. J. MacTavish et al., arXiv:astro-ph/0507503

- \( \alpha \) is energy-dependent, we cannot give a WMAP+BOOMERANG joint estimation on it, but a joint estimation is useful for \( \xi \), which does not depend on energy.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>( \alpha \pm \sigma(\alpha) )</th>
<th>( \xi \pm \sigma(\xi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP (94 GHz)</td>
<td>-1.6 ( \pm ) 2.1</td>
<td>-0.09 ( \pm ) 0.12</td>
</tr>
<tr>
<td>BOOMERanG (145 GHz)</td>
<td>-5.2 ( \pm ) 4.0</td>
<td>-0.123 ( \pm ) 0.096</td>
</tr>
<tr>
<td>WMAP+BOOMERanG</td>
<td>-</td>
<td>-0.110 ( \pm ) 0.075</td>
</tr>
</tbody>
</table>

\[ \alpha \propto \frac{\xi}{E_p^2} p_{\Omega}^2 \]


At 95% confidence level:

\(-0.260 < \xi < 0.040\)

First limit on \( \xi \) in CMB reference frame
Check on systematic effects

- Misleading detection of rotation can be due to a miscalibration of polarimeters (bad orientation)
  ⇒ check how much a miscalibration could have influenced our result
- we considered a realistic miscalibration of BOOMERANG polarimeters of $(0.9 \pm 0.7)^\circ$
  ⇒ new estimate on $\alpha$ from BOOMERANG:

\[ \alpha = (-4.3 \pm 4.1)^\circ \]

⇒ new estimate on $\xi$ (including WMAP)

\[ \xi = 0.097 \pm 0.075 \]
Forecasts on future experiments

Future experiments will be able to significantly improve limits on $\xi$, thanks also to multi-frequency data availability (disentanglement from other effects)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Channel</th>
<th>$\sigma(\alpha)$</th>
<th>$\sigma(\xi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLANCK</td>
<td>70</td>
<td>0.64</td>
<td>$6.0 \cdot 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.14</td>
<td>$6.5 \cdot 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>143</td>
<td>0.073</td>
<td>$1.7 \cdot 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>217</td>
<td>0.10</td>
<td>$1.0 \cdot 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>100+143+217</td>
<td>-</td>
<td>$8.5 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Spider</td>
<td>145</td>
<td>0.27</td>
<td>$6.1 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>EPIC</td>
<td>70</td>
<td>$2.1 \cdot 10^{-3}$</td>
<td>$1.9 \cdot 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>$1.8 \cdot 10^{-3}$</td>
<td>$7.8 \cdot 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>150</td>
<td>$1.5 \cdot 10^{-3}$</td>
<td>$2.9 \cdot 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>220</td>
<td>$1.2 \cdot 10^{-3}$</td>
<td>$1.1 \cdot 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>70+100+150+220</td>
<td>-</td>
<td>$1.0 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>CVL</td>
<td>150</td>
<td>$6.1 \cdot 10^{-4}$</td>
<td>$1.3 \cdot 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>217</td>
<td>$6.1 \cdot 10^{-4}$</td>
<td>$6.1 \cdot 10^{-6}$</td>
</tr>
</tbody>
</table>

Table: Expected $1\sigma$ error for PLANCK 70, 100, 143, 217 GHz, Spider 145 GHz, EPIC 70, 100, 150, 220 GHz and two ideal CVL experiment at 150 GHz and 217 GHz on $\alpha$ (in degrees) and $\xi$.

very strong limits on $\xi$ coming from astrophysical analyses
(GRBs: $|\xi| \lesssim 2 \times 10^{-7}$, Crab Nebula observations: $|\xi| \lesssim 10^{-9}$).
L. Maccione, S. Liberati, A. Celotti, J. G. Kirk, P. Ubertini

but $\xi = n_{0}^{3}$ → related to the time-component of a 4-vector
→ can’t give an absolute limit on the parameter
(one may have $n_{\alpha} \sim (0, 1, 1, 1)$ in one reference frame and
$n_{\alpha} \sim (10^{-3}, 1, 1, 1)$ in another moving with velocity $\beta = 10^{-3}$)

→ need informations on the other components of the vector to compare results

(see GG, G. Genovese, G. Amelino-Camelia, A. Melchiorri [arXiv: :1003.0878])

but $\xi = n_0^3$ → related to the time-component of a 4-vector → can’t give an absolute limit on the parameter (one may have $n_\alpha \sim (0, 1, 1, 1)$ in one reference frame and $n_\alpha \sim (10^{-3}, 1, 1, 1)$ in another moving with velocity $\beta = 10^{-3}$) → need informations on the other components of the vector to compare results

(see GG, G. Genovese, G. Amelino-Camelia, A. Melchiorri [arXiv: :1003.0878])
Conclusions and Outlook

- CMB data do gain sensitivity to Planck scale
- Peculiar energy dependence of Planck-scale birefringence
- CMB covers entire sky \(\rightarrow\) test anisotropic LIV effects
  (work in progress)