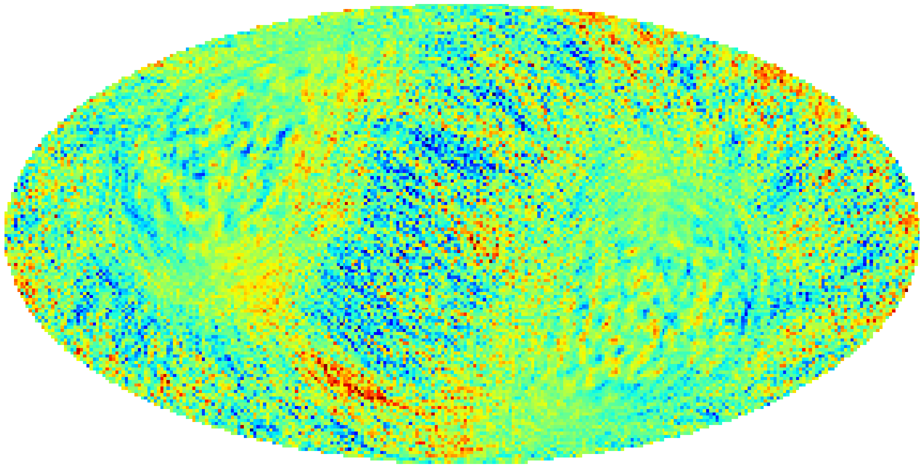
# Statistical anisotropy in CMB maps



**Hanson & Lewis: 0908.0963**
**Hanson, Lewis & Challinor: 1003.0198**

University of Sussex

**Antony Lewis**
http://cosmologist.info/

# The Vanilla Universe Assumptions

- Translation invariance - statistical homogeneity
  (observers see the same things on average after spatial translation)

- Rotational invariance - statistical isotropy
  (observations at a point the same under sky rotation on average)

- Primordial adiabatic nearly scale-invariant Gaussian fluctuations filling a flat universe

**Statistically isotropic CMB with Gaussian fluctuations and smooth power spectrum**

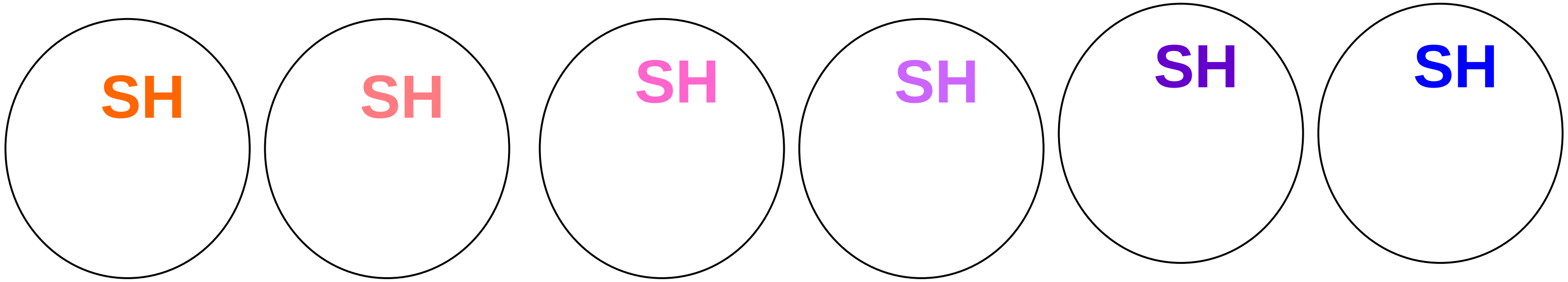
# Gaussian statistical anisotropy

- CMB lensing
- Power asymmetries
- Anisotropic primordial power
- Spatially-modulated primordial power
- Non-Gaussianity

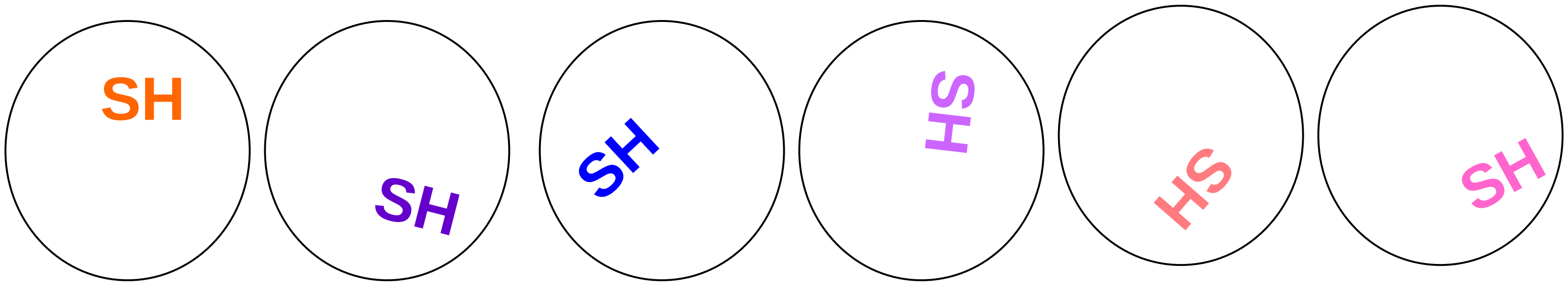
+ various systematics, anisotropic noise, beam-effects, …

# Gaussian anisotropic models

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^{\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$




Or is it a statistically isotropic non-Gaussian model??



# Anisotropy estimators

$$-\mathcal{L}(\hat{\Theta}|\mathrm{h}) = \frac{1}{2}\hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\ln\det(C^{\hat{\Theta}\hat{\Theta}})$$

Maximum likelihood:

$$\frac{\delta\mathcal{L}}{\delta\mathrm{h}^\dagger} = -\frac{1}{2}\hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}(C^{\hat{\Theta}\hat{\Theta}})^{-1}\hat{\Theta} + \frac{1}{2}\mathrm{Tr}\left[(C^{\hat{\Theta}\hat{\Theta}})^{-1}\frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}\right] = 0$$

First iteration solution: Quadratic Maximum Likelihood (QML)

$$\hat{\mathrm{h}} = \mathcal{F}^{-1}[\tilde{\mathrm{h}} - \langle\tilde{\mathrm{h}}\rangle].$$

$$\tilde{\mathrm{h}} = \mathcal{H}_0 = \frac{1}{2}\bar{\Theta}^\dagger \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta\mathrm{h}^\dagger}\bar{\Theta} = \frac{1}{2}\sum_{lm,l'm'}\left[\frac{\delta C^{\hat{\Theta}\hat{\Theta}}_{lm,\ l'm'}}{\delta\mathrm{h}^\dagger}\right]\Theta^*_{lm}\Theta_{l'm'},$$

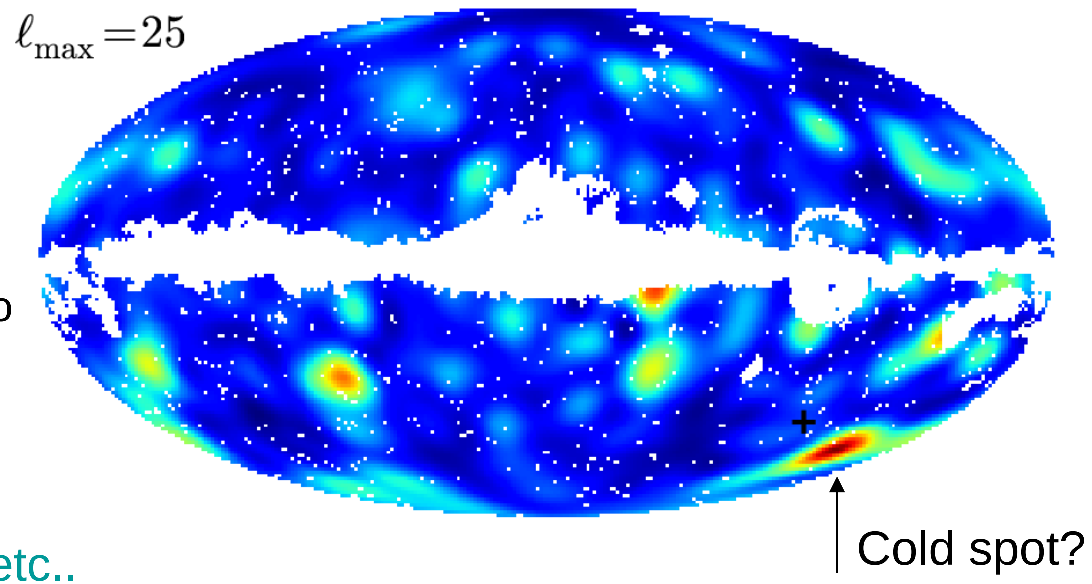
$$\bar{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1}|_0\hat{\Theta}$$

# Sky modulation?

Popular modulation model: $\Theta_f(\hat{\mathbf{n}}) = [1 + f(\hat{\mathbf{n}})]\Theta^i_f(\hat{\mathbf{n}})$

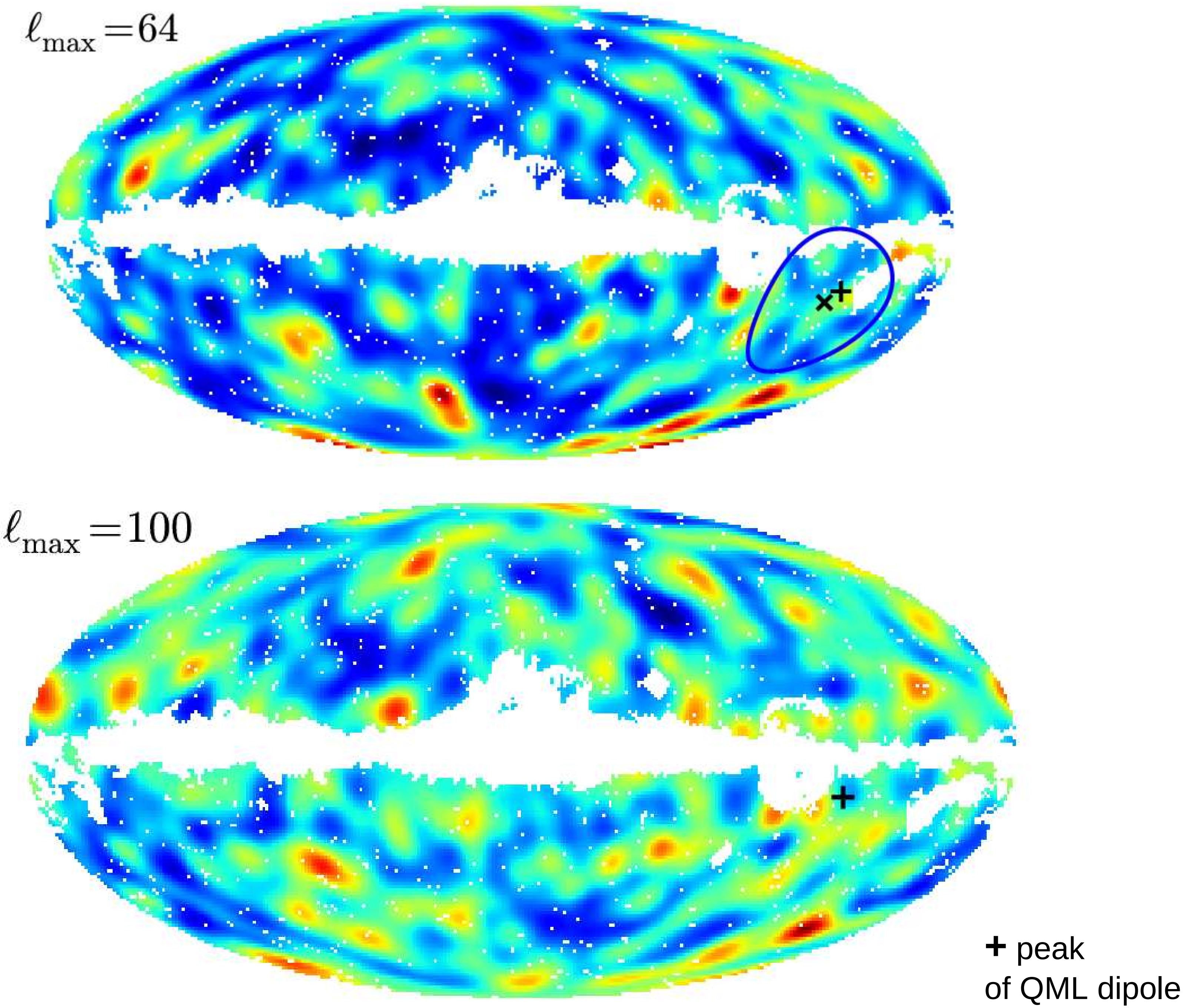
QML estimator for f:

$$\tilde{h}^f_{lm} = \int d\Omega Y^*_{lm} \left[\sum_{l_1 m_1}^{l_{\max}} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right] \left[\sum_{l_2 m_2}^{l_{\max}} C_{l_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$



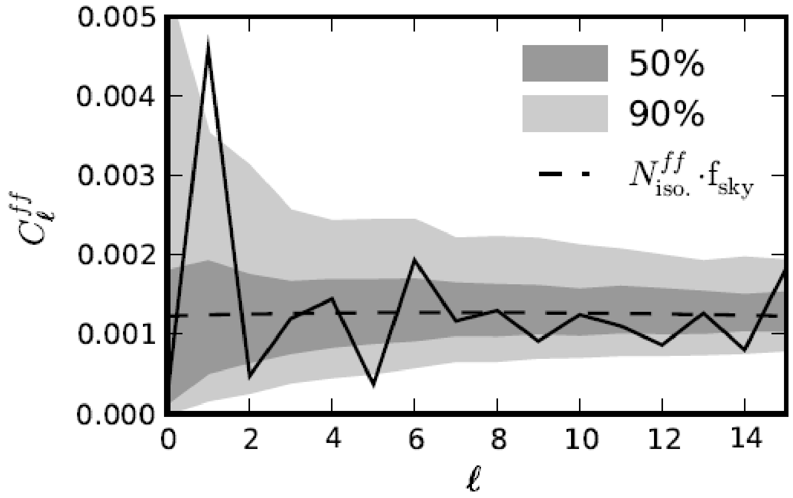

WMAP power reconstruction
(V band, KQ85 mask, foreground cleaned; reconstruction smoothed to 10 degrees)

Following Eriksen et al, WMAP, etc..

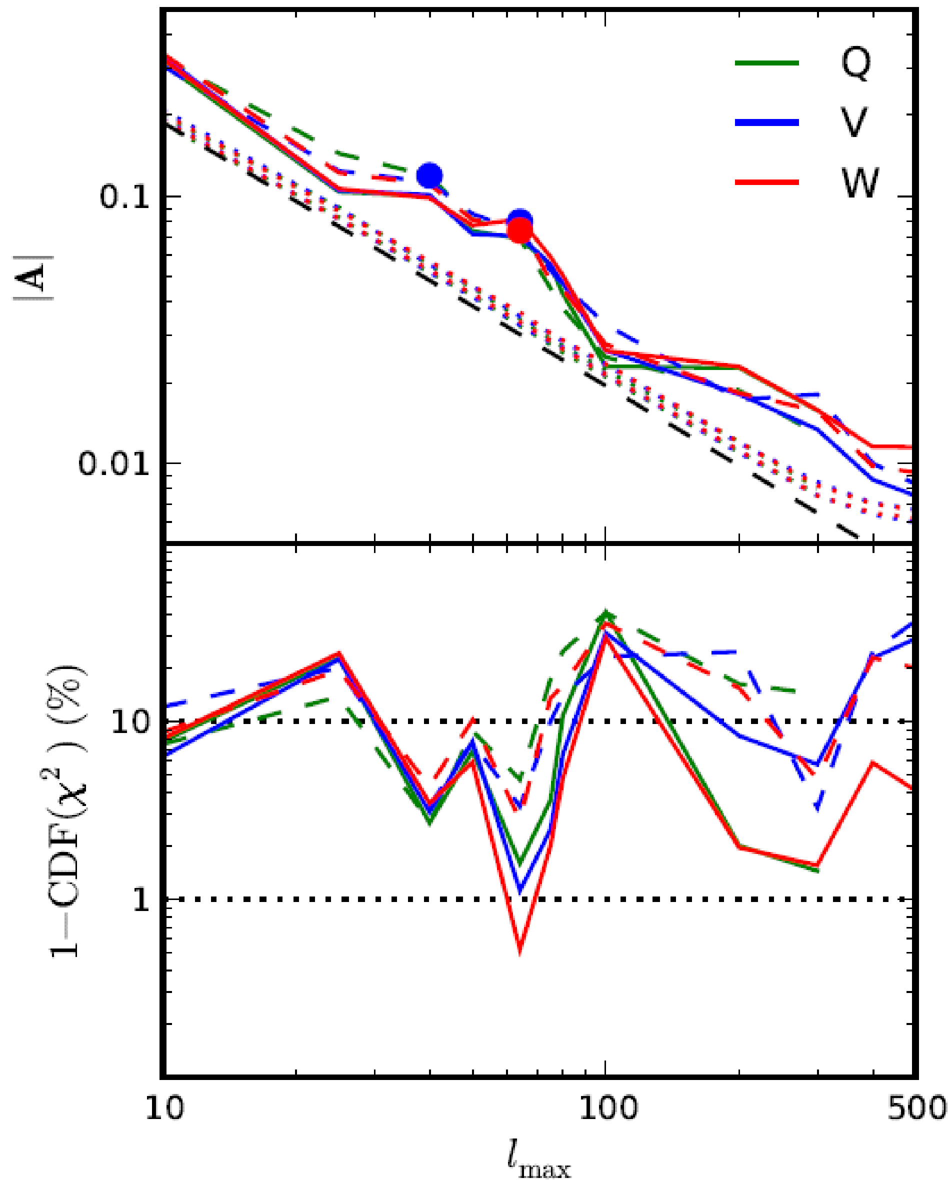


$\ell_{\rm max}=64$
$\ell_{\rm max}=100$
+ peak
of QML dipole

## Modulation power spectrum $l_{max}$=64




## Dipole amplitude as function of $l_{max}$




Only ~1% modulation allowed on small scales

Consistent with Hirata 2009
- Very small observed anisotropy in quasar distribution

# Primordial power spectrum anisotropy

Look for direction-dependence in primordial power spectrum:

$$\langle \chi_0(\mathbf{k}) \chi_0^*(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') P_\chi(\mathbf{k})$$

Simple case:

$$\mathcal{P}_\chi(\mathbf{k}) = \mathcal{P}_\chi(k)[1 + a(k) g(\hat{\mathbf{k}})]$$

e.g.
Ackerman et.al. astro-ph/0701357
Gumrukcuoglu et al 0707.4179

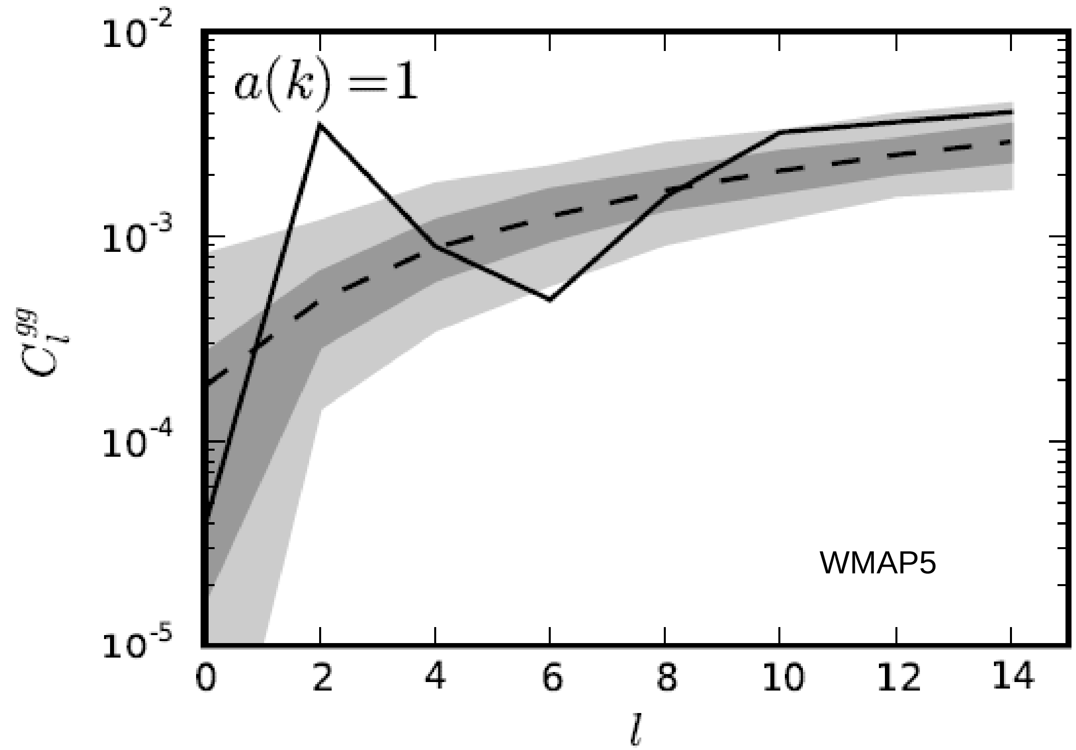
Anisotropic covariance:

$$C_{l_1 m_1 l_2 m_2} =$$
$$i^{l_1 - l_2} \frac{\pi}{2} \int \mathrm{d}^3\mathbf{k} P_\chi(\mathbf{k}) \Delta_{l_1}(k) \Delta_{l_2}(k) Y^*_{l_1 m_1}(\hat{\mathbf{k}}) Y_{l_2 m_2}(\hat{\mathbf{k}})$$

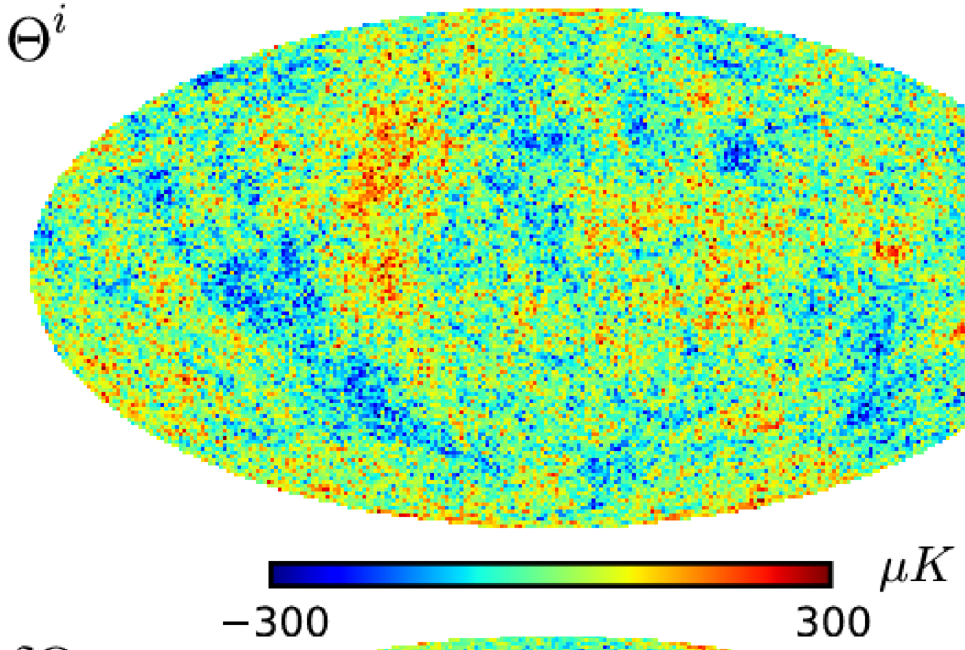
# Reconstruct g(k)

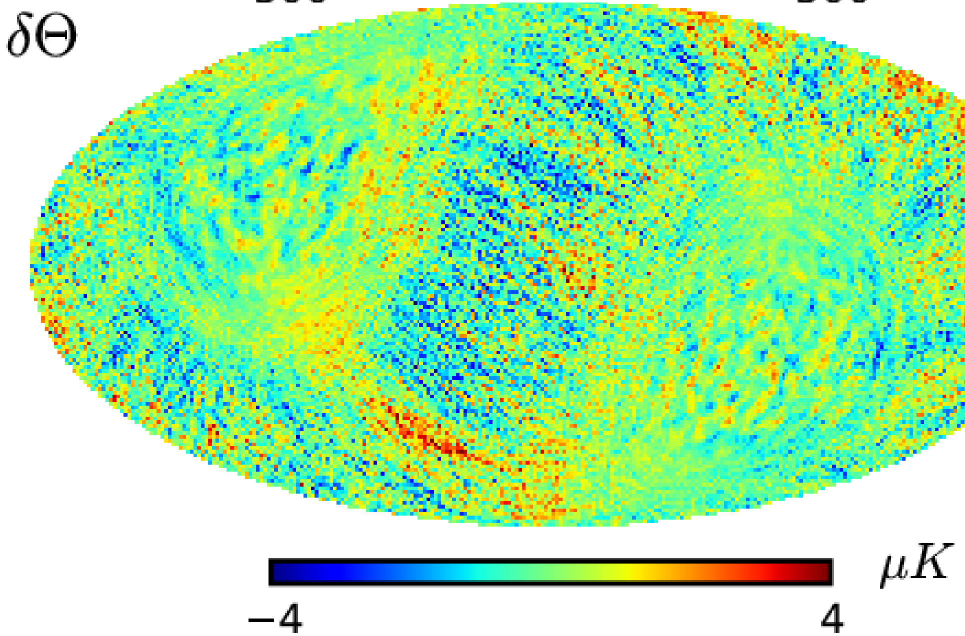
QML estimator:

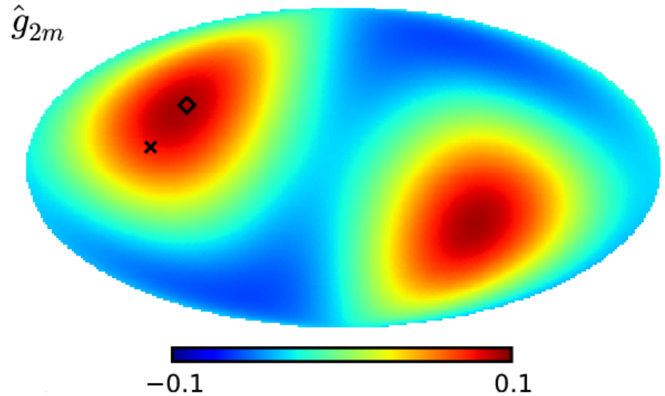
$$\tilde{h}^g_{lm} = \frac{1}{2}\int \mathrm{d}\Omega Y^*_{lm} \sum_{l_1 l_2} i^{l_1 - l_2} C_{l_1 l_2} \times \left[\sum_{m_1} \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1}\right]\left[\sum_{m_2} \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2}\right]$$




Many-sigma quadrupole primordial power anisotropy??




Direction close to ecliptic!
Also varies with frequency
and detector.

# Beam asymmetries?
# Check with analytic model of scan strategy

$$\tilde{\Theta}(\Omega_p) = \sum_s w(\Omega_p, -s) \left[ \sum_{lm} B_{ls} \Theta_{lm\; s} Y_{lm}(\Omega_p) \right]$$

Scan strategy

Beam shape multipoles

$$w(\Omega_p, -s) = \sum_{i \in p} e^{-is\alpha_i} / H_p$$
$$= v(\Omega_p, s) / v(\Omega_p, 0)$$

$$v(\Omega_p, s) = \sum_{i \in p} e^{is\alpha_i}$$

WMAP model

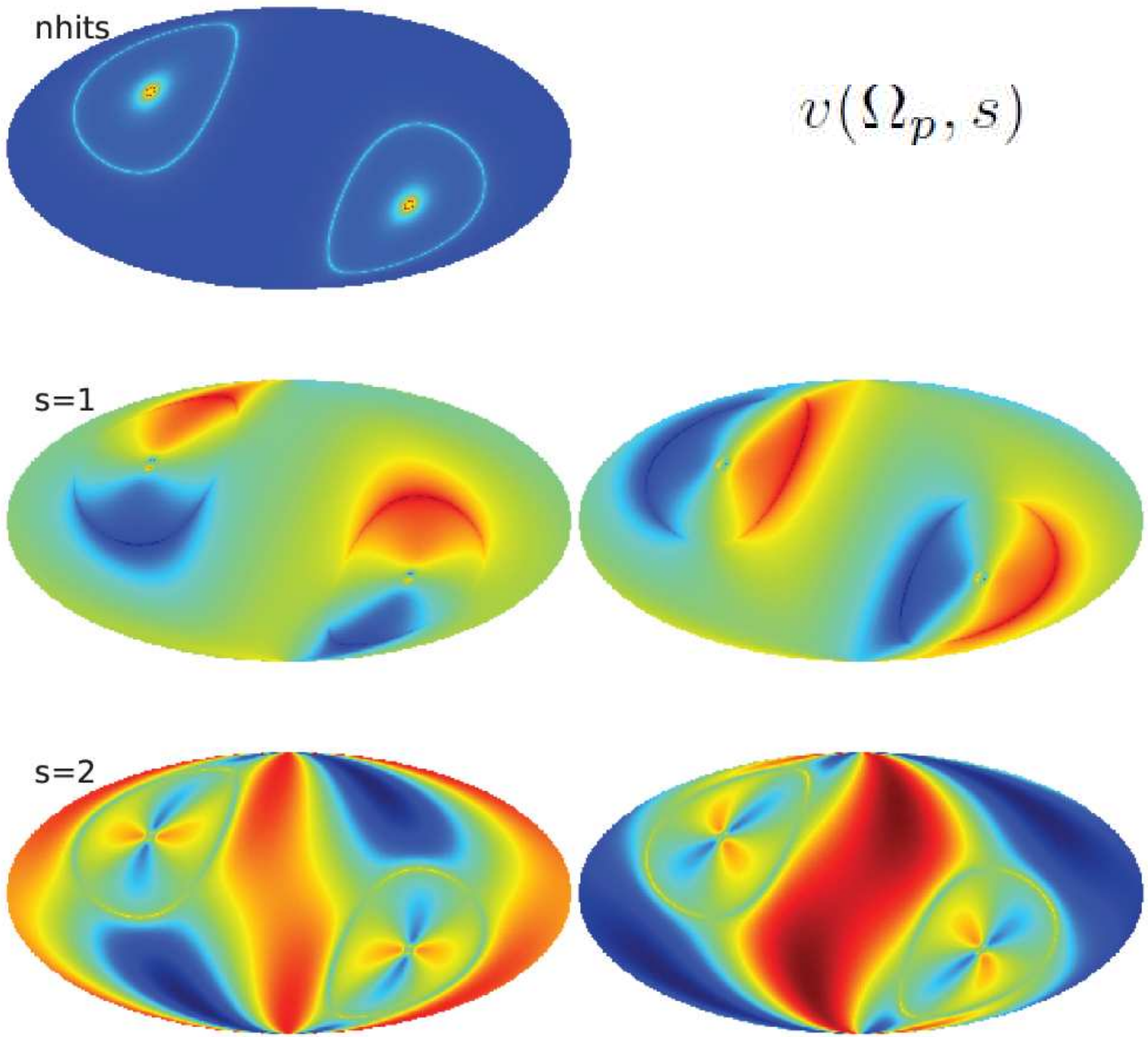
Hirata et al astro-ph/0406004.

(1) a beam at an angle $\theta_b$ to the satellite spin axis, which rotates with period $\tau_s$;

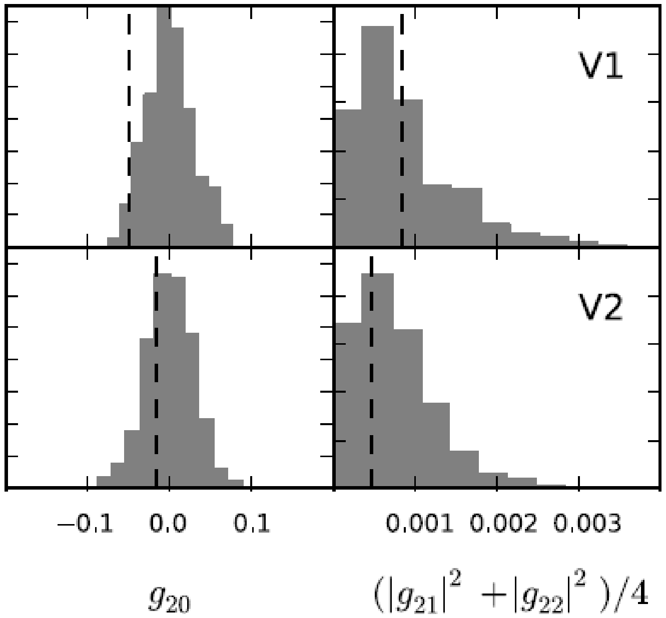
(2) a precession at an angle $\theta_p$ to the anti-solar direction, with period $\tau_p$; and

(3) a continuous repointing of the anti-solar direction as the observer orbits the sun.

$$\longrightarrow \quad [v(\Omega_p, s)]_{lm} = \delta_{m0} K P_l(0) P_l(\cos\theta_p)\, {}_sY_{l0}(\theta_b, 0)$$


nhits
$v(\Omega_p, s)$
s=1
s=2

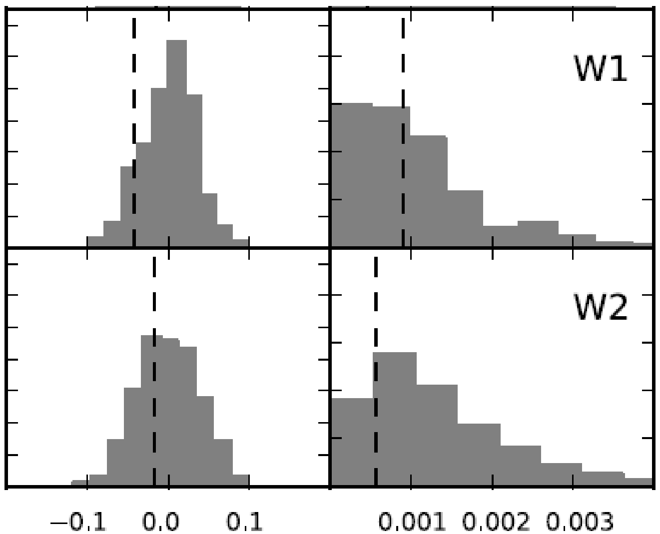
# Monte Carlo with subtraction of mean field analytic model of beam asymmetries

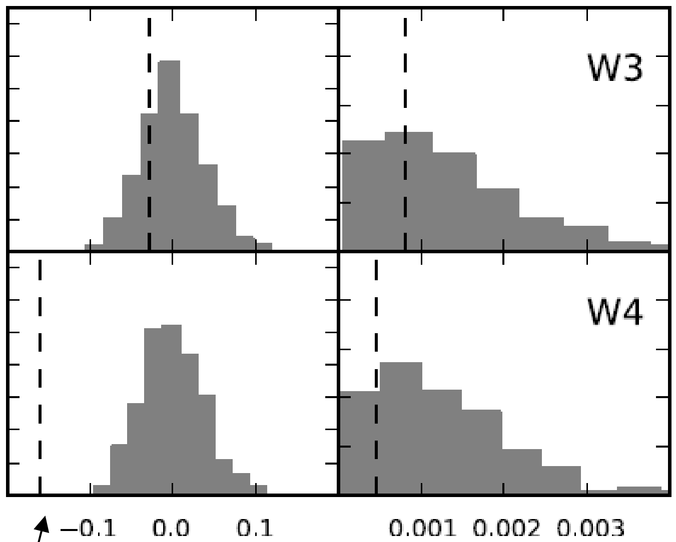



No detection..

$|g_{2M}| < 0.07$ at 95% confidence

Consistent with Pullen et al 2010
constraint from large-scale structure 1003.0673




can be explained as correlated noise

# Primordial spatial modulation

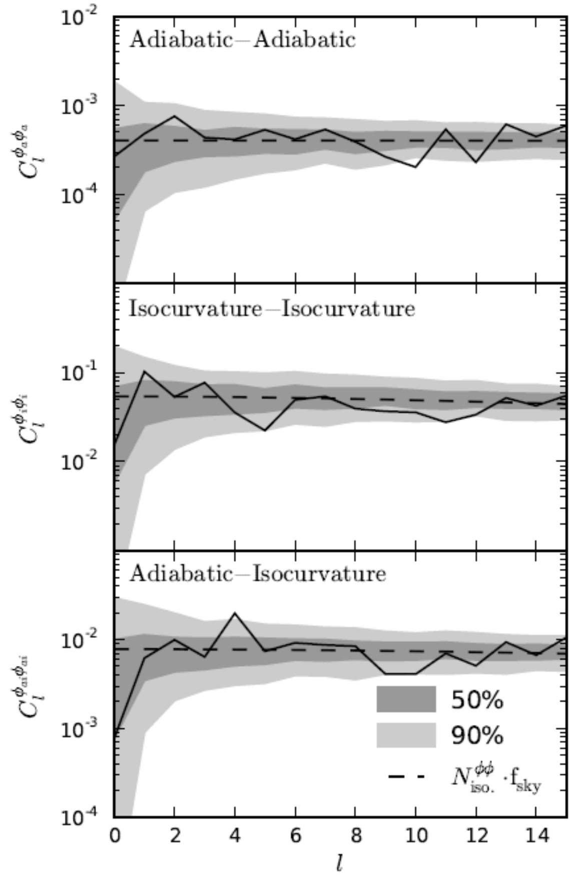
$$\chi(\mathbf{x}) = \chi_0(\mathbf{x})[1 + \phi(\mathbf{x})]$$

Gaussian and statistically homogeneous

Modulation field

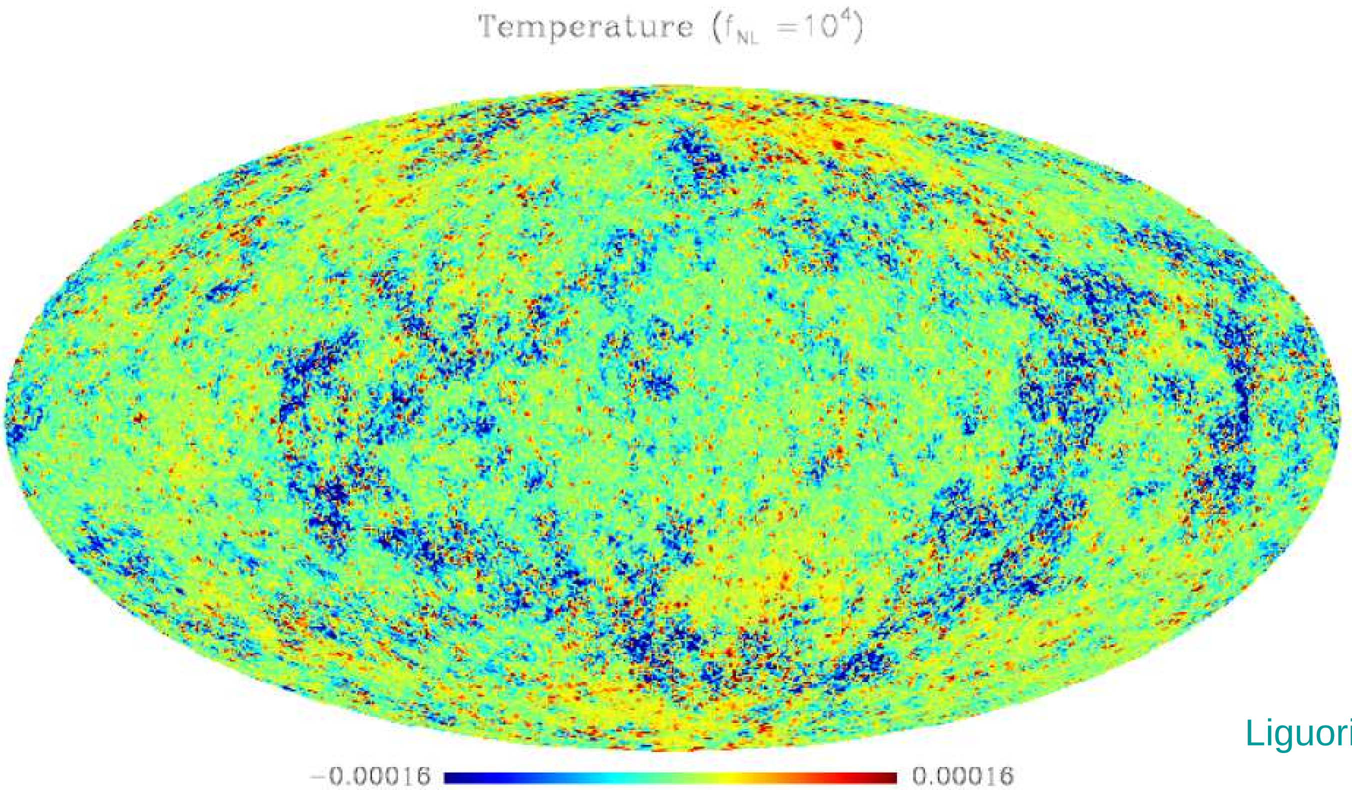
$$\tilde{h}^{\phi}_{lm}(r) = \int d\Omega Y^{*}_{lm} \left[ \sum_{l_1 m_1} \alpha_{l_1}(r) \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1} \right] \times \left[ \sum_{l_2 m_2} \beta_{l_2}(r) \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2} \right]$$

At recombination



# Bispectrum non-Gaussianity

- Local model: small scale power correlated with large-scale temperature
- Considering large-scale modes to be fixed, expect power anisotropy




Liguori et al 2007

Local primordial non-Gaussianity
$$\Psi = \Psi_0 + f_{\rm NL}\Psi_0^2$$
$$= \Psi_0(1 + f_{\rm NL}\Psi_0)$$

# Conclusions

- Can easily test for a variety of Gaussian anisotropic models using QML estimators
- Marginal evidence for L<~64 dipole power asymmetry in WMAP, small at high L
- Strong evidence for primordial power anisotropy model using uncorrected WMAP maps

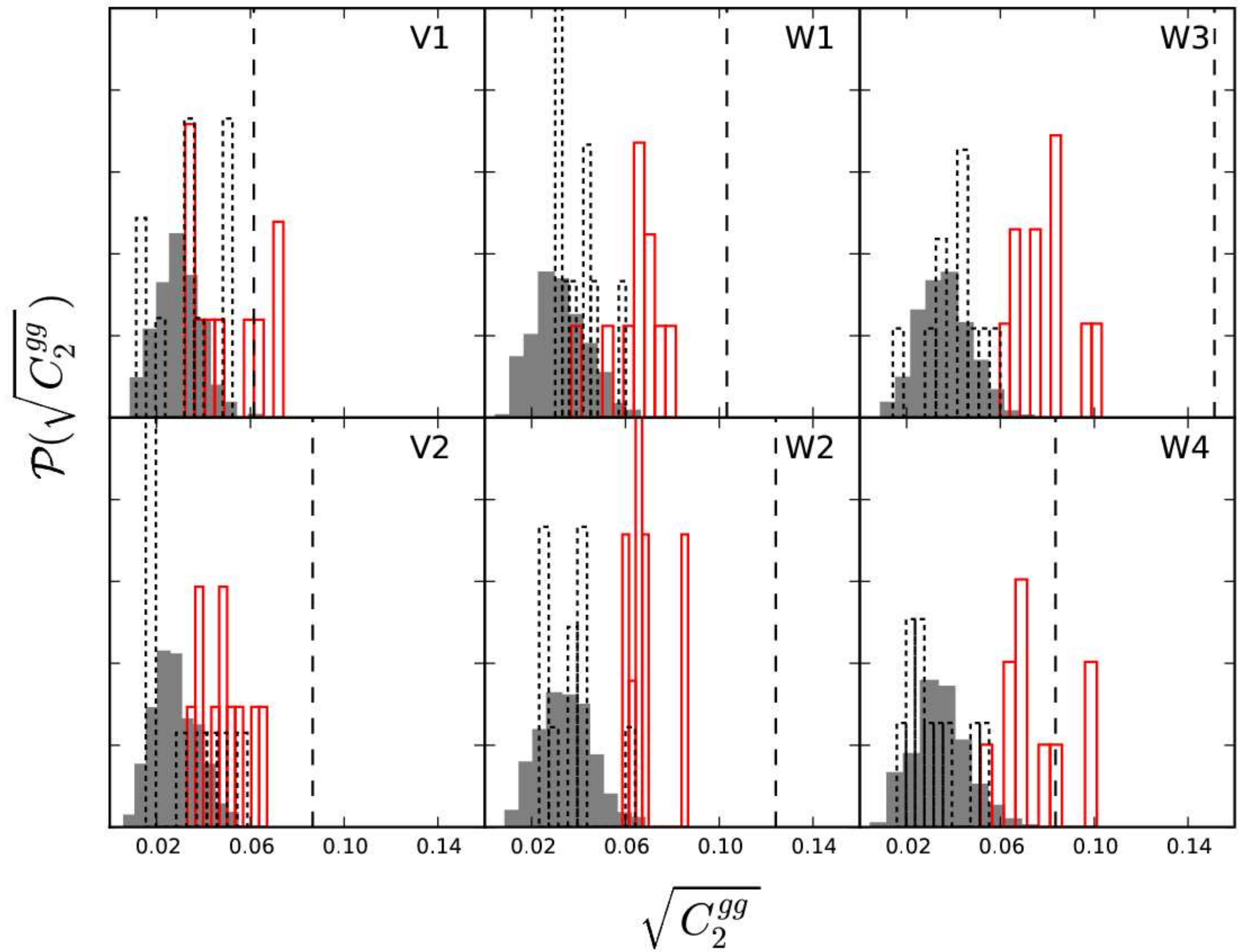
  - varies between detectors, ecliptic alignment

  - appears to be fully explained by beam asymmetries

- Powerful method to test for a wide class anisotropic theoretical models AND instrumental systematics

Comparison with simulations

Symmetric beam and asymmetric beam simulations of Wehus et al, 0904.3998



| DA | $\langle g_{20} \rangle$ ($\sigma$) | $\langle g_{40} \rangle$ ($\sigma$) | $\langle g_{60} \rangle$ ($\sigma$) |
|---|---|---|---|
| Q1 | -0.33 (12.1) | 0.030 (0.83) | -0.003 (0.07) |
| Q2 | -0.33 (12.3) | 0.029 (0.81) | -0.003 (0.08) |
| V1 | 0.17 (6.51) | 0.031 (0.86) | -0.003 (0.07) |
| V2 | 0.17 (6.74) | 0.032 (0.92) | -0.002 (0.06) |
| W1 | 0.27 (9.10) | 0.043 (1.07) | -0.002 (0.05) |
| W2 | 0.31 (9.79) | 0.042 (0.97) | -0.003 (0.06) |
| W3 | 0.33 (9.99) | 0.037 (0.85) | -0.002 (0.05) |
| W4 | 0.27 (8.63) | 0.045 (0.95) | -0.003 (0.05) |

TABLE I: Analytic predictions for the beam mean-field bias to the primordial power asymmetry estimator, with $l_{\rm max} = 400$. The significance ($\sigma$) is given by the mean-field divided by the estimator noise.