Statistical anisotropy in CMB maps

Hanson & Lewis: 0908.0963
Hanson, Lewis & Challinor: 1003.0198

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The Vanilla Universe Assumptions

- Translation invariance - statistical homogeneity (observers see the same things on average after spatial translation)

- Rotational invariance - statistical isotropy (observations at a point the same under sky rotation on average)

- Primordial adiabatic nearly scale-invariant Gaussian fluctuations filling a flat universe

Statistically isotropic CMB with Gaussian fluctuations and smooth power spectrum
Gaussian statistical anisotropy

- CMB lensing
- Power asymmetries
- Anisotropic primordial power
- Spatially-modulated primordial power
- Non-Gaussianity

+ various systematics, anisotropic noise, beam-effects, …
Gaussian anisotropic models

\[-\mathcal{L}(\hat{\Theta}|h) = \frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \ln \det (C^{\hat{\Theta}\hat{\Theta}})\]

Or is it a statistically isotropic non-Gaussian model??
Anisotropy estimators

\[- \mathcal{L}(\hat{\Theta}|h) = \frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \ln \det(C^{\hat{\Theta}\hat{\Theta}})\]

Maximum likelihood:

\[\frac{\delta \mathcal{L}}{\delta h^\dagger} = -\frac{1}{2} \hat{\Theta}^\dagger (C^{\hat{\Theta}\hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta h^\dagger} (C^{\hat{\Theta}\hat{\Theta}})^{-1} \hat{\Theta} + \frac{1}{2} \Tr \left[ (C^{\hat{\Theta}\hat{\Theta}})^{-1} \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta h^\dagger} \right] = 0\]

First iteration solution: Quadratic Maximum Likelihood (QML)

\[\hat{h} = \mathcal{F}^{-1}[\tilde{h} - \langle \tilde{h} \rangle].\]

\[\tilde{h} = \mathcal{H}_0 = \frac{1}{2} \hat{\Theta}^\dagger \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta h^\dagger} \tilde{\Theta}\]

\[= \frac{1}{2} \sum_{lm, l'm'} \left[ \frac{\delta C^{\hat{\Theta}\hat{\Theta}}}{\delta h^\dagger} \right] \Theta^*_{lm} \Theta_{l'm'} ,\]

\[\tilde{\Theta} = (C^{\hat{\Theta}\hat{\Theta}})^{-1} |_0 \hat{\Theta} \]
Sky modulation?

Popular modulation model:

$$\Theta_f(\hat{n}) = [1 + f(\hat{n})] \Theta_f^i(\hat{n})$$

QML estimator for $f$:

$$\tilde{h}_{lm}^f = \int d\Omega Y_{lm}^* \left[ \sum_{l_1 m_1} \tilde{\Theta}_{l_1 m_1} Y_{l_1 m_1} \right] \left[ \sum_{l_2 m_2} C_{l_2} \tilde{\Theta}_{l_2 m_2} Y_{l_2 m_2} \right]$$

$\ell_{\text{max}} = 25$

WMAP power reconstruction
(V band, KQ85 mask, foreground cleaned; reconstruction smoothed to 10 degrees)

Following Eriksen et al, WMAP, etc..
$\ell_{\text{max}} = 64$

$\ell_{\text{max}} = 100$

+ peak of QML dipole
Only ~1% modulation allowed on small scales

Consistent with Hirata 2009
- Very small observed anisotropy in quasar distribution
Primordial power spectrum anisotropy

Look for direction-dependence in primordial power spectrum:

$$\langle \chi_0(k) \chi_0^*(k') \rangle = (2\pi)^3 \delta(k-k') P_\chi(k)$$

Simple case:

$$P_\chi(k) = P_\chi(k)[1 + a(k)g(\hat{k})]$$

e.g.

Ackerman et.al. astro-ph/0701357
Gumrukcuoglu et al 0707.4179

Anisotropic covariance:

$$C_{l_1 m_1 l_2 m_2} = \int_{-l_1 - l_2}^{\pi} d^3k P_\chi(k) \Delta_{l_1}(k) \Delta_{l_2}(k) Y_{l_1 m_1}^*(\hat{k}) Y_{l_2 m_2} (\hat{k})$$
Reconstruct $g(k)$

QML estimator:  
$$\tilde{h}_{l_m}^g = \frac{1}{2} \int d\Omega Y^{*}_{lm} \sum_{l_1l_2} i^{l_1-l_2} C_{l_1l_2}$$

$$\times \left[ \sum_{m_1} \Theta_{l_1m_1} Y_{l_1m_1} \right] \left[ \sum_{m_2} \Theta_{l_2m_2} Y_{l_2m_2} \right]$$

Many-sigma quadrupole primordial power anisotropy??

$C^g_l$ vs $l$

$\alpha(k) = 1$

WMAP5
Direction close to ecliptic! Also varies with frequency and detector.
Beam asymmetries?
Check with analytic model of scan strategy

\[ \tilde{\Theta}(\Omega_p) = \sum_s w(\Omega_p, -s) \left[ \sum_{lm} B_{ls} \Theta_{lm} Y_{lm}(\Omega_p) \right] \]

Scan strategy

Beam shape multipoles

\[ w(\Omega_p, -s) = \sum_{i \in p} e^{-i s \alpha_i} / H_p \]

\[ = u(\Omega_p, s) / u(\Omega_p, 0) \]

\[ u(\Omega_p, s) = \sum_{i \in p} e^{i s \alpha_i} \]

WMAP model

(1) a beam at an angle \( \theta_b \) to the satellite spin axis, which rotates with period \( \tau_s \);

(2) a precession at an angle \( \theta_p \) to the anti-solar direction, with period \( \tau_p \); and

(3) a continuous repointing of the anti-solar direction as the observer orbits the sun.

\[ [u(\Omega_p, s)]_{lm} = \delta_{m0} KP_l(0) P_l(\cos \theta_p) s Y_{l0}(\Omega_b, 0) \]

Monte Carlo with subtraction of mean field analytic model of beam asymmetries

\[ g_{20} = \frac{\left| g_{21} \right|^2 + \left| g_{22} \right|^2}{4} \]

No detection..

\[ |g_{2M}| < 0.07 \text{ at } 95\% \text{ confidence} \]

Consistent with Pullen et al 2010 constraint from large-scale structure 1003.0673

can be explained as correlated noise
Primordial spatial modulation

\[ \chi(x) = \chi_0(x)[1 + \phi(x)] \]

Gaussian and statistically homogeneous

\[ \tilde{h}_{lm}^\phi (r) = \int d\Omega Y_{lm}^* \left[ \sum_{l_1 m_1} \alpha_{l_1} (r) \bar{\Theta}_{l_1 m_1} Y_{l_1 m_1} \right] \times \left[ \sum_{l_2 m_2} \beta_{l_2} (r) \bar{\Theta}_{l_2 m_2} Y_{l_2 m_2} \right] \]

At recombination
Bispectrum non-Gaussianity

- Local model: small scale power correlated with large-scale temperature
- Considering large-scale modes to be fixed, expect power anisotropy

\[
\Psi = \Psi_0 + f_{NL} \Psi_0^2 \\
= \Psi_0 (1 + f_{NL} \Psi_0)
\]

Liguori et al 2007
Conclusions

• Can easily test for a variety of Gaussian anisotropic models using QML estimators

• Marginal evidence for $L < 64$ dipole power asymmetry in WMAP, small at high $L$

• Strong evidence for primordial power anisotropy model using uncorrected WMAP maps
  - varies between detectors, ecliptic alignment
  - appears to be fully explained by beam asymmetries

• Powerful method to test for a wide class anisotropic theoretical models AND instrumental systematics
Comparison with simulations

Symmetric beam and asymmetric beam simulations of Wehus et al, 0904.3998
<table>
<thead>
<tr>
<th>DA</th>
<th>$\langle g_{20} \rangle (\sigma)$</th>
<th>$\langle g_{40} \rangle (\sigma)$</th>
<th>$\langle g_{60} \rangle (\sigma)$</th>
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</thead>
<tbody>
<tr>
<td>Q1</td>
<td>-0.33 (12.1)</td>
<td>0.030 (0.83)</td>
<td>-0.003 (0.07)</td>
</tr>
<tr>
<td>Q2</td>
<td>-0.33 (12.3)</td>
<td>0.029 (0.81)</td>
<td>-0.003 (0.08)</td>
</tr>
<tr>
<td>V1</td>
<td>0.17 (6.51)</td>
<td>0.031 (0.86)</td>
<td>-0.003 (0.07)</td>
</tr>
<tr>
<td>V2</td>
<td>0.17 (6.74)</td>
<td>0.032 (0.92)</td>
<td>-0.002 (0.06)</td>
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<tr>
<td>W1</td>
<td>0.27 (9.10)</td>
<td>0.043 (1.07)</td>
<td>-0.002 (0.05)</td>
</tr>
<tr>
<td>W2</td>
<td>0.31 (9.79)</td>
<td>0.042 (0.97)</td>
<td>-0.003 (0.06)</td>
</tr>
<tr>
<td>W3</td>
<td>0.33 (9.99)</td>
<td>0.037 (0.85)</td>
<td>-0.002 (0.05)</td>
</tr>
<tr>
<td>W4</td>
<td>0.27 (8.63)</td>
<td>0.045 (0.95)</td>
<td>-0.003 (0.05)</td>
</tr>
</tbody>
</table>

TABLE I: Analytic predictions for the beam mean-field bias to the primordial power asymmetry estimator, with $l_{\text{max}} = 400$. The significance ($\sigma$) is given by the mean-field divided by the estimator noise.