B-mode CMB spectrum estimation

pure pseudo cross-spectrum approach

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B-mode physics

- **large scale gaussian B-modes**
  - signature of primordial gravitational waves
  - Energy scale of inflation

- **small scale non gaussian B-modes**
  - leakage from E to B due to CMB lensing
  - GR tests

- **systematics**
  - foreground residuals
  - instrumental effects
  - ...
CMB anisotropies in polarization

- Spherical harmonic coefficients

\[
\begin{align*}
\Delta T(n) &= \sum_{\ell m} a_{\ell m}^T Y_{\ell m}(n) \\
(Q \pm iU)(n) &= \sum_{\ell m} a_{\ell m}^{\pm 2} Y_{\ell m}(n)
\end{align*}
\]

\[
\begin{align*}
a_{\ell m}^T &= \int_{4\pi} T \times Y_{\ell m} \\
a_{\ell m}^E &= \int_{4\pi} (Q, U) \times D^E Y_{\ell m} \\
a_{\ell m}^B &= \int_{4\pi} (Q, U) \times D^B Y_{\ell m}
\end{align*}
\]

- Angular power spectra

\[
C_{\ell}^{XY} = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}^X a_{\ell m}^{Y*}| \quad (X,Y) \in \{T,E,B\}
\]

contains all needed information if the field is gaussian
• **maximum likelihood**

- from the pixel-pixel correlation matrix \( M \)

\[
P(C_\ell | T) \propto \exp \left[ -\frac{1}{2} \left( T^T M^{-1} T + \text{Tr}(\ln M) \right) \right]
\]

- computationally expensive, requiring \( \mathcal{O}(N_{\text{pix}}^3) \) time and \( \mathcal{O}(N_{\text{pix}}^2) \) memory

- not adapted to large survey and high resolution maps

• **quadratic estimator (pseudo-Cl)**

- direct estimation of pseudo-spectra from data \( \tilde{C}_\ell \)

\[
\tilde{C}_\ell = \sum_{\ell'} M_{\ell \ell'} B_{\ell'}^2 C_{\ell'} + N_\ell
\]

- very fast (requiring \( \mathcal{O}(N_{\text{pix}}^{3/2}) \) time)

- frequently near-optimal in practice (in temperature !)
• Compute cross-correlation between two independent maps

• After pre-processing of data, noise could be considered as not correlated from one map to another
  ➡ no noise bias
  ➡ no noise Monte-Carlo needed

\[
\langle \hat{a}_\ell^X \hat{a}_{\ell m}^Y \rangle = \sum_{\ell'} M_{\ell \ell'} B_{\ell}^X B_{\ell}^Y F_{\ell'} \langle a_{\ell' m}^X a_{\ell' m}^Y \rangle + \langle n_{\ell m}^X n_{\ell m}^Y \rangle
\]

Kogut et al. 2003, Hinshaw et al. 2003, Tristram et al. 2005
E/B mixing

- CMB measurements are not full sky

\[
\begin{pmatrix}
\tilde{a}_{\ell m}^E \\
\tilde{a}_{\ell m}^B
\end{pmatrix}
= \begin{pmatrix}
W_+ & iW_- \\
-iW_- & W_+
\end{pmatrix}
\begin{pmatrix}
a_{\ell m}^E \\
a_{\ell m}^B
\end{pmatrix}
\]

- pseudo-Cl correct for mixing

\[
\begin{pmatrix}
\tilde{C}_{\ell}^{EE} \\
\tilde{C}_{\ell}^{BB}
\end{pmatrix}
= \begin{pmatrix}
M_{\ell\ell'}^+ & M_{\ell\ell'}^- \\
M_{\ell\ell'}^- & M_{\ell\ell'}^+
\end{pmatrix}
\begin{pmatrix}
C_{\ell'}^{EE} \\
C_{\ell'}^{BB}
\end{pmatrix}
\]

- can achieve E-B separation on ensemble average (mixing matrices can be computed analytically)
- BUT for any realization: B variance feels leakage

Lewis et al. 2002, Bunn et al. 2003

small-scale experiment simulation
why we need to separate?

B-mode

\[ \frac{l(l+1)}{2\pi} C_l \]

Model

Fisher estimate

r=0.05

multipole

l

1

10

100
why we need to separate?

B-mode power spectrum estimator

Model
Fisher estimate
pseudo-Cl estimator

$\frac{l(l+1)}{2\pi} C_l$
why we need to separate?

\[ \frac{l(l+1)}{2\pi} C_l \]

- Model
- Fisher estimate
- pseudo-Cl estimator

\[ r = 0.10 \]
\[ r = 0.05 \]
\[ r = 0.01 \]
pure estimator

- **standard approach**

\[ a_{\ell m}^B = \int_{4\pi} M \cdot (Q, U) \times D^B Y_{\ell m} \]

\[ = \int_{\Omega} D^B (Q, U) \times Y_{\ell m} + \oint_{\Omega} (Q, U) \partial Y_{\ell m} + \oint_{\Omega} \partial(Q, U) Y_{\ell m} \]

ambiguous modes (contains E and B)

- **change the base for harmonic decomposition to keep only B modes**

\[ a_{\ell m}^B = \int_{4\pi} (Q, U) \times D^B (WY_{\ell m}) \]

\[ = \int_{\Omega} W \cdot D^B (Q, U) \times Y_{\ell m} + \oint_{\Omega} (Q, U) \times \partial(WY_{\ell m}) + \oint_{\Omega} \partial(Q, U) \times WY_{\ell m} \]

choose W to zero contour integrals

- remove the leakage
- reduce the variance

Bunn et al. 2003
Smith 2005
Smith & Zaldarriaga 2007
Xpure implementation

- implementation of pure algorithm in two steps
  1. window function computation
  2. pure pseudo cross-spectrum estimator

- using Scalable Spherical HArmonic Transform package (S2HAT)
  - fully parallel (in CPU time and memory)
  - very fast: less than 30min for 1000 simulations on 1024 procs

\[
\begin{align*}
\alpha_{B,lm}^{\text{pure}} &= \alpha_{B,lm}^{(2)} + \lambda_{1,l} \alpha_{B,lm}^{(1)} + \lambda_{0,l} \alpha_{B,lm}^{(0)} \\
W(\vec{n}) &= W_1(\vec{n}) = \nabla W(\vec{n}) \\
W_2(\vec{n}) &= \nabla^2 W(\vec{n})
\end{align*}
\]
• need for an appropriate apodization for spin 0,1,2
  ➔ numerical derivatives (pixelization and edge issues)
• we propose and compare several type of apodization with any shape of the sky patch

Analytic weighting

Optimal weighting
• drop the ambiguous modes
  → remove the leakage
  → reduce the variance

\[
\begin{pmatrix}
\tilde{C}_E E \\
\tilde{C}_B B
\end{pmatrix} =
\begin{pmatrix}
M_+^{\ell\ell'} \\
M_-^{\ell\ell'}
\end{pmatrix}
\begin{pmatrix}
C_E E \\
C_B B
\end{pmatrix}
\]
leakage

- CMB simulations without B mode for a small-scale B-mode dedicated experiment
  - sky coverage 1%
  - illustrate the leakage from E to B in std and pure

- B mode signal (r=0.05)
- E to B leakage (standard estimator)
- E to B leakage (pure estimator)
variance

standard pseudo-CI
Xpol
SpicePol

pure algorithm
Xpure

fisher analysis
fisher

figure by H. Nishino (KEK)

\( \frac{l(l+1)C_{\ell}^{BB}/2\pi}{\mu K^2} \)

\( l \)

Tristram et al. 2005
Chon et al. 2004
Grain et al. 2009
conclusions

Grain, Tristram, Stompor PRD 2009

- B-mode studies require specific power spectrum algorithm

- pure pseudo cross-spectra methods are well adapted
  - fast: \((N_{\text{pix}})^{3/2}\) au lieu de \((N_{\text{pix}})^3\)
  - allow for large MonteCarlo
  - pure estimator: accurate enough for B-mode detection

- Xpure is used for B-mode dedicated experiments
  - EBEx: balloon-borne Oxley et al. 2004
  - PolarBear: ground based