Rencontres de Moriond

GRAVITATIONAL THEORY AND DARK MATTER

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ASTROPHYSICAL MOTIVATION
The cosmological concordance model $\Lambda$-CDM

This model brilliantly accounts for

1. the mass discrepancy between the dynamical and luminous masses of clusters of galaxies
2. the precise measurements of the anisotropies of the cosmic microwave background (CMB)
3. the formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
4. the fainting of the light curves of distant supernovae
The problem of the fundamental constituents of the Universe begins with the assumption that General Relativity is the correct theory of gravity, as in the Λ-CDM model. However, this poses the problem of the unknown constituents of the Universe:

1. No known particle in the standard model of particle physics could be the particle of dark matter.
2. Extensions of the standard model of particle physics provide well-motivated but yet to be discovered candidates.
3. The cosmological constant Λ does not have the right value when interpreted as a vacuum energy associated with the fluctuations of the gravitational field [Zel’dovich 1967].
Problems of CDM with galactic halos

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004; Famaey 2007]

1. Prediction of numerous but unseen satellites of large galaxies

2. Generic formation of cusps of DM in central regions of galaxies while the rotation curves seem to favor a constant density profile in the core

3. Evidence that tidal dwarf galaxies are dominated by DM contrary to CDM predictions [Bournaud et al. 2007; Gentile et al. 2007]

4. Failure to explain in a natural way Milgrom’s law, that DM arises only in regions where gravity falls below some universal acceleration scale $a_0$

5. Difficulty at explaining in a natural way the flat rotation curves of galaxies and the baryonic Tully-Fisher relation
The relation between the asymptotic flat velocity and the mass of spirals is

$$v_f = \left( G M_b a_0 \right)^{1/4}$$

where $a_0 = 1.2 \times 10^{-10} \text{ m/s}^2$
Problems of MOND in galaxy clusters [Gerbal, Durret et al 1992]

The bullet cluster [Clowe et al 2006] and more generally X-ray emitting galaxy clusters can be fitted with MOND and a component of baryonic dark matter and hot/warm neutrinos [Angus, Famaey & Buote 2008]
Different approaches to the DM problem

Faced with the *unreasonable effectiveness* of MOND, three solutions are possible:

1. **Standard**: MOND could be explained within the CDM paradigm

2. **Modified Gravity**: There is a fundamental modification of the law of gravity in a regime of weak gravity (this is the traditional approach of MOND and its relativistic extensions like TeVeS)

3. **Modified Dark Matter**: The law of gravity is not modified but DM is endowed with special properties which make it able to explain the phenomenology of MOND
The MOND equation can be written as the modified Poisson equation

\[ \nabla \cdot \left[ \mu \left( \frac{g}{a_0} \right) g \right] = -4\pi G \rho_b \]

where \( g = \nabla U \) is the gravitational field and \( \rho_b \) the density of ordinary matter.

In the MOND regime \( g \ll a_0 \) we have \( \mu = g/a_0 + \mathcal{O}(g^2) \)
In electrostratics the **Gauss equation** is modified by the polarization of the dielectric material

\[
\nabla \cdot \left[ (1 + \chi_e) E \right] = \frac{\rho_e}{\varepsilon_0} \quad \iff \quad \nabla \cdot E = \frac{\rho_e + \rho_{e}^{\text{polar}}}{\varepsilon_0}
\]

Similarly MOND can be viewed as a modification of the **Poisson equation** by the polarization of some “digravitational” medium

\[
\nabla \cdot \left[ \mu \left( \frac{g}{a_0} \right) g \right] = -4\pi G \rho_b \quad \iff \quad \nabla \cdot g = -4\pi G \left( \rho_b + \rho_{b}^{\text{polar}} \right)
\]

We may write \( \mu = 1 + \chi \) where \( \chi \) appears to be the gravitational susceptibility of the DM medium.
The DM action in standard general relativity is of the type

\[ S = \int d^4x \sqrt{-g} L \left[ J^\mu, \xi^\mu, \dot{\xi}^\mu, g_{\mu\nu} \right] \]

where the current density \( J^\mu \) and the dipole moment \( \xi^\mu \) are two independent dynamical variables.

- The current density \( J^\mu = \sigma u^\mu \) is conserved
  \[ \nabla_\mu J^\mu = 0 \]
- The covariant time derivative is denoted
  \[ \dot{\xi}^\mu \equiv \frac{D\xi^\mu}{d\tau} = u^\nu \nabla_\nu \xi^\mu \]
We propose a modification of the CDM Lagrangian

\[ L = -\sigma + J^\mu \dot{\xi}_\mu - \mathcal{W}(\Pi_\perp) \]

1. A mass term \( \sigma \) in an ordinary sense (like for ordinary CDM)
2. An interaction term between the fluid’s mass current \( J^\mu = \sigma u^\mu \) and the dipole moment \( \xi_\mu \)
3. A potential term \( \mathcal{W} \) describing an internal force and depending on the norm of the polarization field \( \Pi_\perp = \sigma \xi_\perp \)
The potential $\mathcal{W}$ is phenomenologically determined through third order

$$\mathcal{W} = \frac{\Lambda}{8\pi} + 2\pi \Pi_\perp^2 + \frac{16\pi^2}{3a_0} \Pi_\perp^3 + \mathcal{O}(\Pi_\perp^4)$$

1. The minimum of that potential is the cosmological constant $\Lambda$ and the third-order deviation from the minimum contains the MOND scale $a_0$
2. In this unification scheme the natural order of magnitude of the cosmological constant is comparable with $a_0$ namely $\Lambda \sim a_0^2$
Agreement with $\Lambda$-CDM at large scales

In a cosmological perturbation around a FLRW background, the dipole moment, which is space-like, will break the spatial isotropy of the background, and must belong to the first-order perturbation

$$\xi^\mu_\perp = O(1)$$

The stress-energy tensor reduces to $T^{\mu\nu} = T^{\mu\nu}_{\text{de}} + T^{\mu\nu}_{\text{dm}}$ where

1. the DE is given by the cosmological constant $\Lambda$
2. the DM takes the form of a perfect fluid with zero pressure

$$T^{\mu\nu}_{\text{dm}} = \rho U^\mu U^\nu + O(2)$$

The dipolar fluid is undistinguishable from standard DE (a cosmological constant) standard CDM (a pressureless perfect fluid) at the level of first-order cosmological perturbations.
TEST OF MOND IN THE SOLAR SYSTEM

Luc Blanchet (GRECO)
Open star clusters in our Galaxy do not show evidence for dark matter despite their typical low internal gravity $g_i \ll a_0$.

In the presence of the external Galactic field $g_e$ the MOND equation which is non-linear can be approximated by

$$\mu \left( \frac{|g_i + g_e|}{a_0} \right) g_i = g_i^{\text{Newtonian}}$$

- When $a_0 \lesssim g_e$ the sub-system exhibits Newtonian behavior.
- When $g_i \lesssim g_e \lesssim a_0$ the system is still Newtonian but with an effective Newton’s constant $G/\mu_e$.

The EFE results from a violation of the strong version of the equivalence principle.

The gravitational dynamics of a system is influenced by the external gravitational field in which the system is embedded.
Deformation of the field of the Sun by the Galactic field

The external field effect is a prediction of the non-linear Poisson equation

\[ \nabla \cdot \left[ \mu \left( \frac{g}{a_0} \right) \nabla U \right] = -4\pi G \rho_b \]

The MOND field of the Sun, in the presence of the external field of the Galaxy, is deformed along the direction of the Galactic center

\[ U = g_e \cdot x + \frac{GM_\odot / \mu_e}{r \sqrt{1 + \lambda_e \sin^2 \theta}} + O \left( \frac{1}{r^2} \right) \]

This will influence the motion of inner planets of the Solar System [Milgrom 2009]
The Newtonian physicist measures from the motion of planets the internal gravitational potential \( u = U - g_e \cdot x \) and detects the anomaly

\[
\delta u = u - u_N = G \int \frac{d^3x'}{|x - x'|} \rho_{\text{pdm}}(x', t)
\]

Since the phantom dark matter vanishes in the strong-field regime near the Sun \( \delta u \) is an harmonic function and admits the multipole expansion

\[
\delta u = \sum_{l=0}^{+\infty} \frac{(-)^l}{l!} x^L Q_L
\]

where \( Q_L \) are trace-free multipolar coefficients.

This expansion is valid in all the region inside the MOND transition radius of the Sun

\[
r_0 = \sqrt{\frac{GM_\odot}{a_0}} \approx 7100 \text{ AU}
\]
The dominant effect is quadrupolar and grows with the distance squared:

\[ u = \frac{GM_\odot}{r} + \frac{1}{2} x^i x^j Q_{ij} \]

The quadrupole moment is aligned in the direction of the Galactic center:

\[ Q_{ij} = Q_2 \left( e_i e_j - \frac{1}{3} \delta_{ij} \right) \]

The quadrupole moment is computed by solving numerically the MOND equation in the presence of the external galactic field. We find:

\[ 2.1 \times 10^{-27} \text{ s}^{-2} \lesssim Q_2 \lesssim 4.1 \times 10^{-26} \text{ s}^{-2} \]

depending on the MOND function in use.
Quadrupole moment as a function of distance
Quadrupole moment as a function of distance

\[ Q_{22}(r) \] [s^{-2}]

Radius \( r \) [AU]

Values:
- \( 3.8 \times 10^{-26} \)
- \( 3.82 \times 10^{-26} \)
- \( 3.84 \times 10^{-26} \)

\[
\langle \frac{d\tilde{\omega}}{dt} \rangle = \frac{Q_2 \sqrt{1 - e^2}}{4n} \left[ 1 + 5 \cos(2\tilde{\omega}) \right]
\]

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<thead>
<tr>
<th></th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Saturn</th>
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<tbody>
<tr>
<td>(\mu_1)</td>
<td>0.04</td>
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<td>(\mu_5)</td>
<td>(7 \times 10^{-3})</td>
<td>(3 \times 10^{-3})</td>
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<td>-0.22</td>
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<td>(\mu_{20})</td>
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<td>(10^{-3})</td>
<td>(9 \times 10^{-3})</td>
<td>(-9 \times 10^{-3})</td>
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<td>(\mu_{\text{exp}})</td>
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<td>(\mu_{\text{TeVeS}})</td>
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<td>-0.17</td>
<td>-1.21</td>
<td>5.81</td>
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The predicted values are close to the best published residuals for precession

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<td>[Pitjeva 2005]</td>
<td>-3.6 ± 5</td>
<td>-0.4 ± 0.5</td>
<td>-0.2 ± 0.4</td>
<td>0.1 ± 0.5</td>
<td>-</td>
<td>-6 ± 2</td>
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<tr>
<td>[Fienga et al. 2009]</td>
<td>-10 ± 30</td>
<td>-4 ± 6</td>
<td>0 ± 0.016</td>
<td>0 ± 0.2</td>
<td>142 ± 156</td>
<td>-10 ± 8</td>
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Conclusions

1. $\Lambda$-CDM is an extremely powerful model in cosmology but poses the problem of the fundamental constituents of the Universe.

2. MOND is a successful alternative for interpreting the galactic rotation curves and the baryonic Tully-Fisher relation without dark matter.

3. Reconciling $\Lambda$-CDM at cosmological scale and MOND at galactic scale into a single unified theory would constitute a great challenge.

4. A non-standard form of dark matter might exist, while keeping the standard law of gravity unchanged, able to successfully address both cosmological and galactic scales.

5. The external field effect in MOND could be checked in the Solar System dynamics by looking at precession anomalies in the motion of inner planets.