Evidence of Solar Influences on Nuclear Decay Rates

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Publications to date

Background
RADIATIONS
FROM
RADIOACTIVE SUBSTANCES

by

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Re-issue of the edition of 1930

CAMBRIDGE
AT THE UNIVERSITY PRESS
1951
§ 34 a. Emission of α particles and probability variations. The rate of disintegration of all radioactive substances is expressed by a simple law, namely, that the number of atoms \( n \) breaking up per second is proportional to the number \( N \) of atoms present. Consequently \( n = \lambda N \), where \( \lambda \) is a constant characteristic for a particular radioactive substance. The rate of transformation of an element has been found to be a constant under all conditions. It is unaltered by exposing the active matter to extremes of temperature or by change of its physical or chemical state. It is independent of the age of the active matter or its concentration. It is unaffected by exposure to strong magnetic fields. Hevesy has shown that the disintegration of the primary radioactive element uranium is unaltered by exposing it to the β and γ radiation from a strong source of radium, although these rays, of great individual energy, might be expected to penetrate the atomic nucleus.

Since the expulsion of an α or β particle results from an instability of the atomic nucleus, the failure to alter the rate of transformation shows that the stability of the atomic nucleus is not influenced to an appreciable extent by the forces at our command. This is not unexpected when we consider the enormous intensity of the forces, probably both electric and magnetic, which hold the charged parts of the nucleus together in such a minute volume.

E. v. Schweidler* showed that the exponential law of decay of the radioactive bodies could be deduced without any special hypotheses of the structure of the radioactive nuclei or of the mechanism of disintegration. He assumed only that the disintegration of an atom is subject to the laws of chance, and that the probability \( p \) that an atom of a certain type shall be transformed within a given interval of time \( \Delta \) is independent of the time which has elapsed since the formation of the atom and is a constant which is the same for all atoms of the same type or radioactive product.

For very small values of the time interval \( \Delta \), the chance \( p \) of transformation will be proportional to the length of the interval. There-

* Schweidler, Congrès International, Radiologie, Liège, 1905.
The decay of $^{116m}$In follows a linear relationship given by the equation $y = -0.000203x + 6.04443$ with a correlation coefficient $R^2 = 0.980358$. The plot shows the decay of counts over time (s) with a linear fit to the natural logarithm of counts.
$^{116m}\text{In Decay}$

$y = 421.762694e^{-0.000203x}$

$R^2 = 0.980358$

$T_{1/2} \approx 56 \text{ min}$
Motivation
A New Test of Randomness

PHYSICAL REVIEW E 67, 016113 (2003)

Geometric random inner products: A family of tests for random number generators

Shu-Ju Tu* and Ephraim Fischbach†
Department of Physics, Purdue University, West Lafayette, Indiana 47907-1396
(Received 4 October 2002; published 28 January 2003)

We present a computational scheme, GRIP (geometric random inner products), for testing the quality of random number generators. The GRIP formalism utilizes geometric probability techniques to calculate the average scalar products of random vectors distributed in geometric objects, such as circles and spheres. We show that these average scalar products define a family of geometric constants which can be used to evaluate the quality of random number generators. We explicitly apply the GRIP tests to several random number generators frequently used in Monte Carlo simulations, and demonstrate a statistical property for good random number generators.

DOI: 10.1103/PhysRevE.67.016113

PACS number(s): 02.50.Ng
Decay Rate Fluctuations
Brookhaven National Laboratory

Measurement of $^{32}\text{Si}$ Half-life

David Alburger
Garman Harbottle
Eleanor Norton
Half-life of $^{32}$Si

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Brookhaven National Laboratory, Upton, NY 11973 (U.S.A.)

Received August 15, 1985; revised version received March 3, 1986

Beta rays from a $^{32}$Si-$^{32}$P source, produced in 1968-69 via the $^{30}$Si(t,p)$^{32}$Si reaction using a Van de Graaff beam at $E_t = 3.4$ MeV, were counted with an end-window gas-flow proportional counter system including an automatic precision sample changer. Comparison counts were taken on the $\beta$ rays from a $^{36}$Cl source. Measurements beginning February, 1982 were made at approximately 4-week intervals, each consisting of a total of 40 hours of counting on each sample. The decay rate was determined from the $^{32}$Si/$^{36}$Cl ratio of counts. Small periodic annual deviations of the data points from an exponential decay curve were observed, but are of uncertain origin and had no significant effect on the result. Based on the analysis of 53 points taken in 48 months, the value $T_{1/2} = 172(4)$ yr is adopted for the half-life of $^{32}$Si. This result is substantially greater than two previously reported measurements of 108(18) yr and 101(18) yr but is lower than values based on geophysical evidence.
Fig. 3. Lower part, $^{32}\text{Si}$ singles counting rate measured over a period of 48 months; upper part, corresponding $^{36}\text{Cl}$ singles counting rate. Points are counts per 10 hr on each sample, averaged from 4 runs. Error bars are (arbitrarily) three times the statistical uncertainties. The solid curve shown for $^{32}\text{Si}$ is an exponential computer fit, although the ordinate is linear for convenience in plotting. The upper horizontal line is the average of all $^{36}\text{Cl}$ points. The results of the fit to the $^{32}\text{Si}$ data are $T_{1/2} = 173.8$ yr with an uncertainty of 4.8 yr and a standard deviation of 1.7 yr.

Fig. 2. Ratio of $^{32}$Si/$^{36}$Cl counts measured for 53 points over a period of 48 months. Points are averages of 4 runs, each with 10 hr on each sample. **Error bars are (arbitrarily) three times the statistical uncertainties** and the solid line is an exponential computer fit, although the ordinate is linear for convenience in plotting. The results of the fit are $T_{1/2} = 171.6$ yr with an uncertainty of 3.3 yr and a standard deviation of 3.2 yr.

Fig. 4. Points from Fig. 2 corrected for decay using $T_{1/2} = 172$ yr. normalized to 1,000 for the average of all points, and plotted with four 12-month groups superposed. Error bars are statistical uncertainties and $T = 0$ is January 1. A clear annual effect is evident and an arbitrary sine function fit gives an amplitude of 3.4 standard deviations, a maximum on February 9, and a minimum on August 6.

TABLE 1

Changes in counting rates with changes in various experimental parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Normal setting</th>
<th>Change in $^{32}\text{Si}$ singles rate</th>
<th>Change in $^{32}\text{Si}/^{36}\text{Cl}$ ratio of rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counter voltage</td>
<td>2150 V</td>
<td>3.1% per 100 V</td>
<td>0.086% per 100 V</td>
</tr>
<tr>
<td>Counter gas flow</td>
<td>0.042 gauge</td>
<td>0.16% per 0.01 gauge</td>
<td>0.025% per 0.01 gauge</td>
</tr>
<tr>
<td>Discrim. dial</td>
<td>0.40</td>
<td>0.8% per 0.1 dial</td>
<td>0.005% per 0.1 dial</td>
</tr>
<tr>
<td>Box pressure</td>
<td>29.38&quot;</td>
<td>0.1% per 0.1&quot;</td>
<td>0.016% per 0.1&quot;</td>
</tr>
</tbody>
</table>

...bined with average temperature variations over a range of at least $\pm 5^\circ\text{F}$, which is larger than the probable actual range. We therefore conclude that systematic periodic variations are present but that they cannot be fully accounted for by our tests or estimates.

Physikalisch-Technische Bundesanstalt (PTB)

Europium Half-lives and Long Term Detector Stability

Helmut Siegert
Heinrich Schrader
Ulrich Schoetzig
Half-life Measurements of Europium Radionuclides and the Long-term Stability of Detectors

HELMUT SIEGERT, HEINRICH SCHRADER* and ULRICH SCHÖTZIG

Physikalisch-Technische Bundesanstalt (PTB), Bundesallee 100, D-38116 Braunschweig, Germany

Fig. 1. Residuals of the ionization chamber measurement data of $^{226}$Ra as a function of time from a fit with an exponential decay function. A datum point is an average value and contains about 30 individual measurements of current taken over about 3 days and corrected for background. The vertical dotted lines are positioned at 1st January at an interval of 10 years.

Purdue Experiments
PHARM Mn-54 Consecutive 4 hr Counts

\[ y = -0.000355x + 30.958742 \]
\[ R^2 = 0.213746 \]

\[ T_{1/2} = 1953 \text{d} \]

\[ T_{1/2} = 313.5 \text{d} \]

\[ y = -0.002211x + 103.393949 \]
\[ R^2 = 0.998237 \]
Early December 2006

![Graph showing the data for ln(4 hr count) over dates from December 2nd to December 11th, 2006. The graph includes data points and lines for measured, published decay, and fit to measured values.]
Solar Flare Forces Shuttle Astronauts to Seek Shelter From Radiation

Wednesday, December 13, 2006

FOX NEWS

Astronauts scammed to shielded areas of the international space station and space shuttle Discovery Tuesday night to protect themselves from possibly being exposed to high levels of radiation from an unusually large solar flare, NASA said.

Activity aboard Discovery and the space station was interrupted when the solar flare erupted late Tuesday, as two astronauts were finishing the first spacewalk of the current shuttle mission.

Space.com categorized it as an X-3 flare, in the most dangerous category. Such storms are fairly common when the Sun is at its most active, but they are rare during the current low point in the 11-year cycle of solar activity.

- Click here to visit FOXNews.com's Space Center

NASA spokesman Bill Jeffs told FOXNews.com that crew members slept overnight in "heavily shielded areas" of their respective craft - such as airlocks and the Destiny science lab aboard the space station - as a precautionary measure.

"That move was made to avoid having to walk the crew during their sleep period," NASA spokesman John Ira Petry told Space.com. "It was never a danger to the crew."

Severe Geomagnetic Storm Expected From Tuesday's Solar Flare

Thursday, December 14, 2006

By Robert Roy Britt

Space weather forecasters revised their predictions for storminess Wednesday after a major flare erupted on the Sun overnight, threatening damage to communication systems and power grids while offering up the wonder of the Northern Lights.

"We're looking for very strong, severe geomagnetic storms" to begin probably around mid-day Thursday, Joe Kunches, lead forecaster at the NOAA Space Environment Center, told SPACE.com.

The storm is expected to generate aurora or Northern Lights as far south as the northern United States Thursday night.

- Click here to visit FOXNews.com's Space Center

Astronauts aboard the International Space Station are not expected to be put at additional risk, Kunches said.
ESA mission controllers react to solar flare

15 December 2006

An energetic storm on the Sun has forced ESA mission controllers to react to anomalies or take action to avoid damage to spacecraft. Several missions, including Integral, Cluster and Envisat, felt the storm's effects, highlighting the need for ESA's ongoing development of space weather forecasting tools.

The joint ESA/NASA spacecraft SOHO (Solar & Heliospheric Observatory) imaged a large solar flare on 13 December that led to an energetic solar radiation storm.

The LASCO (Large Angle and Spectrometric Coronagraph Experiment) instrument on board SOHO detected a powerful coronal mass ejection (CME) generated by the storm; the CME – a stream of fast-moving atomic particles – was directed towards Earth. The flare also generated X-rays.

The ejection arrived at Earth 14 December between 13:00-19:00 CET (12:00-18:00 UTC), where it gave rise to a strong geomagnetic storm; initial edges of the ejection were detected as early as 04:00 CET (03:00 UTC) on 13 December.

The coronal mass ejection came during a week of intense solar activity that is not yet over. An additional peak event occurred during the night of 14 December, and ground controllers on several ESA missions have reported varying effects on their spacecraft.
Mn-54 4 Hr Counts, Published, Fit and Actual Data

(7.51 1.07) x 10^5
Events missing
Mn-54 4 Hr Counts, Published, Fit and Actual Data

Date (2006, EST)

Measured  Published Decay  Fit to Actual Decayed

X3/4B Flare
13 Dec 06 02:37 UT
(21:37 EST)
Physics 167 Mn-54 Consecutive 4 hr Counts Normalized with Linear GEOS11 x-ray Data
December $^{54}$Mn Decay Data with Integral GOES-11 X-rays

- Normalized Counts/4hrs
- Sum X-ray

40 hours precursor
Logarithmic Decay of $^{54}$Mn with Integral X-ray Flux

- Representative Error $3\sigma$
- X3/4B Flare
- X1 Flare & CME Class C Long duration flare
- Region 930 drops over West Limb
- Severe Solar Storm

<table>
<thead>
<tr>
<th>In(CR)</th>
<th>X-ray Flux (W/m²)</th>
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<tr>
<td>17.06</td>
<td>1.0E-02</td>
</tr>
<tr>
<td>17.04</td>
<td>1.0E-03</td>
</tr>
<tr>
<td>17.02</td>
<td>1.0E-04</td>
</tr>
<tr>
<td>17.00</td>
<td>1.0E-05</td>
</tr>
<tr>
<td>17.04</td>
<td>1.0E-06</td>
</tr>
<tr>
<td>17.06</td>
<td>1.0E-07</td>
</tr>
</tbody>
</table>

Time (EST=UT-5, 2006)

- In($^{54}$Mn Count Rate)
- Representative Error
- Measured X-Ray Data
- Linear (In($^{54}$Mn Count Rate))

Fischbach, Jenkins, and Sturrock et al.; Rencontres de Moriond; La Thuile, Italy 2011
5 Dec 2006: X-9 class flare
13 Dec 2006: X3/4B Flare
Fischbach, Jenkins, and Sturrock et al.; Rencontres de Moriond; La Thuile, Italy 2011

West Lafayette
40°26' N, 86°54' W
21:37:00 EST

\[ \nu_{\text{solar}} \]

SUN

\[ d = 9270 \text{ km} \]

\[ \theta = 23.5^\circ \]

\[ R = 6378 \text{ km} \]

4°16’ N, 174°42’ E

\sim \text{ Butaritari}
Logarithmic Decay of $^{54}$Mn for December 2008

- $\ln(CR)$
- Linear ($\ln(\text{gross})$)
Logarithmic Decay of $^{54}$Mn for December 2008

ln(CR)

Date (2008)

- ln(gross)
- Linear (ln(gross))

Fischbach, Jenkins, and Sturrock et al.; Rencontres de Moriond; La Thuile, Italy 2011
**SOLAR ACTIVITY:** Hours ago, **something on the far side of the sun exploded** and hurled a massive cloud of debris (a CME) over the eastern limb. Using a coronagraph to block the sun's glare, the Solar and Heliospheric Observatory (SOHO) photographed the cloud expanding into space:

NASA's Stereo-B spacecraft is stationed over the sun's eastern limb, but it was not taking pictures at the probable time of the eruption, so details of the blast are unknown. The CME could herald an active region (e.g., a sunspot or perhaps an unstable magnetic filament) turning to face Earth in the days ahead.
Logarithmic Decay of $^{54}$Mn for December 2008

Non Earth-directed large CME recorded by SOHO at 0942 16 Dec 08 UT

$T_{1/2} = 776 \text{d}$

$T_{1/2} = 307 \text{d}$

$y = -0.002251x + 105.8637406$

$R^2 = 0.9977750$

$y = -0.0008937x + 51.8287553$

$R^2 = 0.5886092$

$\ln(CR)$

Date (2008)


Fischbach, Jenkins, and Sturrock et al.; Rencontres de Moriond; La Thuile, Italy 2011
Solar Neutrinos?

Eccentricity of Earth’s Orbit
Back to BNL and PTB
Pearson Correlation Coefficient $r=0.52$, $N=239$, Prob=$4.17\times10^{-18}$

Pearson Correlation Coefficient \( r=0.66 \), \( N=233 \), Prob=1.0\times10^{-31}

Pearson Correlation Coefficient $r=0.62$, $N=1974$, Prob=$5.13 \times 10^{-210}$

Data from Siegert, et al., Appl. Radiat. Isot. 49, 1397 (1998) Fig. 1
Pearson Correlation Coefficient $r=0.65$, $N=1968$, Prob=$3.12\times10^{-246}$

Data from Siegert, et al., Appl. Radiat. Isot. 49, 1397 (1998) Fig. 1
BNL/PTB Correlation
Pearson Correlation Coefficient $r=0.66$, $N=39$, Prob=$5.8 \times 10^{-6}$
Pearson Correlation Coefficient $r=0.87$, $N=35$, Prob=$3.78\times10^{-12}$
Data from other institutions
Data from other institutions

ELLIS: CNRC
The effective half-life of a broad beam $^{238}\text{PuBe}$ total body neutron irradiator

Kenneth J Ellis

USDA/ARS Children’s Nutrition Research Center, Department of Pediatrics, Baylor College of Medicine, Houston, TX 77030, USA

Received 12 January 1990, in final form 2 April 1990

Abstract. A broad-beam $^{238}\text{PuBe}$ neutron irradiator has been previously developed for exclusive use in in vivo neutron activation analysis in humans. The initial calibrations of the facility provided only a fixed value for the thermal neutron flux. Adjustment of this flux value for decay of the neutron source was later introduced and was based on the physical half-life of $^{238}\text{Pu}$. The current findings would suggest, however, that a more appropriate value for the effective half-life for the total body irradiator is $141.7 \pm 2.5$ y. In addition, variations in the induced counts for a Mn standard on a yearly basis indicate that seasonal differences of approximately 0.5% can be present between the winter and summer months.
Normalized Decay Data

Data from other institutions

OSURR
OSU Power Spectrum

\[ \nu = 1.06 \text{ year}^{-1} \]
\[ S = 35.8 \]
\[ \phi = -48.8^\circ \]

\[ \nu = 1.06 \text{ year}^{-1} \]
\[ S = 35.8 \]
\[ \phi = -48.8^\circ \]
Data from other institutions

PARKHOMOV
Fig. 1. Count rate of the $^{60}\text{Co}$ and $^{90}\text{Sr-}^{90}\text{Y}$ β sources, measured by G-M counters, adjusted for a drop of activity with half-lives 5.27 and 28.6 years, and count rate of the $^{239}\text{Pu}$ α source, measured by the silicon detector [3, 5].

Data from other institutions

SIEGERT, PTB (EUROPIUM)
Fig. 2. Efficiency values of a Ge(Li) detector at the 1408 keV full-energy peak from a $^{152}$Eu point source at a distance of about 16 cm from the detector. The plotted cosine function has a period of 1 year. Vertical dotted lines are positioned at intervals of 5 years, each located at 1st January of the corresponding year.

# Data Summary

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Detector</th>
<th>Type</th>
<th>Decay Type</th>
<th>Measured Radiation Type</th>
<th>Observed Variations</th>
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</thead>
<tbody>
<tr>
<td>PTB ($^{226}$Ra)</td>
<td>Ion Chamber</td>
<td>Gas</td>
<td>$\alpha, \beta, \gamma$</td>
<td>$\gamma$</td>
<td>Annual</td>
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<td>PTB ($^{152}$Eu)</td>
<td>GeLi</td>
<td>Solid State</td>
<td>$\varepsilon, \gamma$</td>
<td>$\gamma$</td>
<td>Annual</td>
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<td>BNL ($^{32}$Si/$^{36}$Cl)</td>
<td>Prop. Counter</td>
<td>Gas</td>
<td>$\beta$</td>
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<tr>
<td>Purdue ($^{54}$Mn)</td>
<td>NaI(Tl)</td>
<td>Scintillator</td>
<td>$\varepsilon, \gamma$</td>
<td>$\gamma$</td>
<td>Flare</td>
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<td>NaI(Tl)</td>
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<td>$\beta, \gamma$</td>
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<td>Annual</td>
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<td>G-M</td>
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<td>$\beta$</td>
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<tr>
<td>Parkhomov ($^{90}$Sr/$^{90}$Y-I)</td>
<td>G-M</td>
<td>Gas</td>
<td>$\beta$</td>
<td>$\beta$</td>
<td>Annual</td>
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<tr>
<td>Parkhomov ($^{90}$Sr/$^{90}$Y-II)</td>
<td>G-M</td>
<td>Gas</td>
<td>$\beta$</td>
<td>$\beta$</td>
<td>Annual</td>
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<td>Parkhomov ($^{60}$Co)</td>
<td>G-M</td>
<td>Gas</td>
<td>$\beta, \gamma$</td>
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<td>Weak Annual</td>
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<tr>
<td>Parkhomov ($^{239}$Pu)</td>
<td>Si</td>
<td>Solid State</td>
<td>$\alpha$</td>
<td>$\alpha$</td>
<td>None</td>
</tr>
</tbody>
</table>
Possible Mechanisms
Possible Mechanisms and New Long Range Forces

• The Sun can influence nuclear decays either through new long-range fields or through particles such as neutrinos, axions, or other non-standard objects.

• If the effect arises from a new long-range field, then any such field must have a range of ~1 a.u. Such spin-independent fields are highly constrained.

• If the sun influences nuclear decays through emission of particles, then these can influence nuclear decays through relatively short-range fields which tend to be less highly constrained.

• Spin-dependent long-range forces are relatively poorly constrained, and may provide an appropriate mechanism to explain time-varying decay rates.
Possible Mechanisms-I

• Spatial Variation of the Fine Structure Constant $\alpha$
  – This is an example of a mechanism whose existence depends on a field whose range is $\sim 1$ a.u.
  – Such a field is strongly constrained by both 5th force type experiments, and atomic physics experiments.
Spatial Variation of the Fine Structure Constant $\alpha$

For alpha decay (e.g., $^{226}\text{Ra} \rightarrow ^{222}\text{Rn} + ^4\text{He}$)

$$\frac{\delta \alpha}{\alpha} \approx \frac{\delta \Gamma}{\Gamma} \left( \frac{1}{4\pi Z\alpha c} \right) \rightarrow 6.3 \times 10^{-3} \left( \frac{\delta \Gamma}{\Gamma} \right)$$

From our $^{226}\text{Ra}$ data,

$$\frac{\delta \Gamma}{\Gamma} \approx 3 \times 10^{-3} \quad \Rightarrow \quad \frac{\delta \alpha}{\alpha} \approx 2 \times 10^{-5}$$

This *may* be incompatible with existing WEP and 5th force constraints.

Possible Mechanisms-II

- Spin-dependent long range force coupling to neutrinos
  - This is an example of where a spin-dependent force of relatively short range could provide an explanation of the decay data.
Spin-dependent long range force coupling to neutrinos

\[ p \rightarrow n \quad W^- \quad \bar{\nu}_e \quad \nu_s \]

\[ e^- \quad \nu_s \]

\[ \Delta \]
Variation in Solar Neutrino Flux

1. For beta-decay,\[
\frac{d\Gamma}{dE} \propto E \sqrt{E^2 - m_e^2} (E_0 - E)^2
\]
where Gamma is extremely sensitive to small shifts in $E_0$.

2. Assume $E_0 \rightarrow E_0 + \Delta$, where Delta arises from solar neutrinos, then
\[
(E_0 - E)^2 \rightarrow (E_0 - E)^2 + 2\Delta(E_0 - E) + \Delta^2
\]

3. Next, assume
\[
\Delta = \left\langle \frac{A}{r^3} [3(\hat{\sigma}_1 \cdot \hat{r})(\hat{\sigma}_2 \cdot \hat{r}) - \hat{\sigma}_1 \cdot \hat{\sigma}_2] \right\rangle
\]
where
\[
1 = \nu_e, \quad 2 = p, e, n, \nu_e
\]

4. For an un-polarized sample,
\[
(E_0 - E)^2 \rightarrow (E_0 - E)^2 + \Delta^2
\]
Variation in Solar Neutrino Flux (cont’d)

5. Compare this to the change induced by $m^2_v \neq 0$

$$(E_0 - E)^2 \to (E_0 - E)\sqrt{(E_0 - E)^2 - m^2_v}$$

For $(E_0 - E)^2 \gg m^2_v$, $\Delta^2 \Rightarrow \Delta^2 \approx -\frac{1}{2}m^2_v$

$$m^2_v = -100 \text{ eV}^2 \text{ to } -10 \text{ eV}^2.$$  

$\Rightarrow \Delta^2 = 50 \text{ eV}^2 \text{ to } 5 \text{ eV}^2$

This *may* be compatible with current limits on neutrino magnetic dipole moments.
Given troubling systematics which result in improbably negative estimators of $m_{\nu_{e}}^{2(\text{eff})} \equiv \sum_i \left| U_{ei} \right|^2 m_{\nu_i}^2$, in many experiments, we use only KRAUS 05 and LOBASHEV 99 for our average.

<table>
<thead>
<tr>
<th>VALUE (eV^2)</th>
<th>CL%</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
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<tbody>
<tr>
<td>- 1.1 ± 2.4</td>
<td>OUR AVERAGE</td>
<td>15 KRAUS 05</td>
<td>SPEC</td>
<td>^3H β decay</td>
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<td>- 1.9 ± 3.4</td>
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<td></td>
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<td>- 3.7 ± 5.3</td>
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<td>- 39 ± 34</td>
<td>15</td>
<td>22 WEINHEIMER 93</td>
<td>SPEC</td>
<td>^3H β decay</td>
</tr>
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<td>- 24 ± 48</td>
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<td>- 147 ± 68</td>
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<td>25 ROBERTSON 91</td>
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* * * We do not use the following data for averages, fits, limits, etc. * * *

HTTP://PDG.LBL.GOV