What do galaxy surveys really measure

Camille Bonvin
In collaboration with Ruth Durrer
PRD 84, 063505 (2011)

KICC and DAMTP, Cambridge

Rencontres de Moriond 2012
Galaxy surveys

Galaxy surveys measure the number of galaxies per unit of solid angle, in redshift bins measurement of the matter density fluctuation.

Two difficulties:

- The luminous matter does not necessarily trace directly dark matter ➔ bias.
- The number density is measured in redshift bins ➔ redshift space distortion.  

\[ \Delta(n, z) = b \cdot \delta(n, z) + \frac{1}{H} \partial_r (v \cdot n) \]

There are many corrections to this relation.
Corrections

- The volume of observation is distorted with respect to the physical volume at the source position.

- We observe the galaxy density as a function of redshift, rather than as a function of conformal time.

These effects modify the relation between

\[ \Delta(n, z) \text{ and } \delta(n, z) = \frac{\delta\rho(n, z)}{\bar{\rho}(\bar{z})} \]

These corrections will be relevant for future galaxy surveys that will observe galaxies at high redshift and over very large areas of the sky, like BOSS, DES, Euclid.
Outline

- Calculation of $\Delta(n, z)$ at first order in perturbation theory.

  → New contributions.

- Calculation of the angular power spectrum $C_\ell(z)$ of $\Delta(n, z)$

  → The effect of the corrections on $C_\ell$ becomes important for some configurations in the sky.

  → The corrections are useful to test cosmological models.
Derivation

Observers measure $N(n, z)$ and average it to obtain $\langle N \rangle(z)$

$$\Delta(n, z) = \frac{N(n, z) - \langle N \rangle(z)}{\langle N \rangle(z)}$$

We want to relate this to $\delta(n, z) = \frac{\delta\rho(n, z)}{\bar{\rho}(\bar{z})}$

♦ **Volume** perturbations enter in $\rho(n, z) = \frac{N(n, z)}{V(n, z)}$

♦ The background redshift $\bar{z}$ differs from the measured one $z$

$$\Delta(n, z) = \delta(n, z) - 3 \frac{\delta z}{1 + \bar{z}} + \frac{\delta V(n, z)}{V(z)}$$
\( \Delta(n, z_S) = D + \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n}) \)

\[
- \int_0^{r_S} d\lambda \frac{r_S - r}{r r_S} \Delta_\Omega (\Phi + \Psi) \rightarrow \text{lensing}
\]

\[
+ \left( \frac{\dot{H}}{\mathcal{H}^2} + \frac{2}{r_S \mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} \rightarrow \text{Doppler}
\]

\[
+ \frac{2}{r_S} \int_0^{r_S} d\lambda (\Phi + \Psi)
\]

\[
+ \left( \frac{\dot{H}}{\mathcal{H}^2} + \frac{2}{r_S \mathcal{H}} \right) \left( \Psi + \int_0^{r_S} d\lambda (\dot{\Phi} + \dot{\Psi}) \right)
\]

\[
- \frac{2a}{\Omega_m} \left( \frac{\mathcal{H}}{\mathcal{H}_0} \right)^2 \left( \Psi + \frac{\dot{\Phi}}{\mathcal{H}} \right)
\]
Angular power spectrum

\[ \Delta(n, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(n) \]

\[ \langle \Delta(n, z) \Delta(n', z') \rangle = \sum_{\ell} \frac{2\ell + 1}{4\pi} C_\ell(z, z') P_\ell(n \cdot n') \]

- We use Einstein’s equations to relate the density and velocity to the gravitational potential.
- We choose gaussian initial conditions, with a flat power spectrum.
- We compute the transfer function in a \( \Lambda \)CDM Universe with CAMB.
\[
\frac{\ell(\ell + 1)C_\ell(z)}{2\pi}
\]
Relative contributions

\[
\frac{C_\ell^{\text{new}}}{C_\ell^{\text{tot}}} = 20
\]

At most percent-level corrections
\frac{\ell(\ell + 1)C_\ell(z, z')}{2\pi} \quad \ell = 20

\begin{align*}
z &= 0.1 \\
0.105 &\quad 0.110 &\quad 0.115 &\quad 0.120 &\quad 0.125 &\quad 0.130
\end{align*}

\begin{align*}
z &= 0.5 \\
0.50 &\quad 0.55 &\quad 0.60 &\quad 0.65 &\quad 0.70 &\quad 0.75
\end{align*}

\begin{align*}
z &= 1 \\
1.0 &\quad 1.1 &\quad 1.2 &\quad 1.3 &\quad 1.4 &\quad 1.5
\end{align*}

\begin{align*}
z &= 3 \\
3.0 &\quad 3.2 &\quad 3.4 &\quad 3.6 &\quad 3.8
\end{align*}

\begin{align*}
\text{Doppler} &\quad \text{standard} &\quad \text{lensing} &\quad \text{potential}
\end{align*}
\[ \sigma_z = 0.1 \cdot z \]

\[ \frac{\ell(\ell + 1)C_\ell}{2\pi} \]

\[ z_m = 0.1, \quad z_m = 0.5, \quad z_m = 1, \quad z_m = 3 \]
Relative contributions

\[ \frac{C_{\text{new}}}{C_{\text{tot}}} \]

\[ \ell = 20 \]

\[ \sigma_z = 0.1 \cdot z \]

More than 50 percent corrections
Conclusion

- We computed the number density of galaxy taking into account volume and redshift perturbations.
- We found three types of corrections: lensing, Doppler and gravitational potential.
- We computed the angular power spectrum.
  - At the same redshift and without window function the corrections remain subdominant.
  - For different redshifts or with a window function the Doppler term and lensing term become important. Since these corrections have different dependence on dark energy or modified gravity they provide new cosmological tests.
Derivation

\[ \Delta(n, z) = \delta(n, z) - 3 \frac{\delta z}{1 + \bar{z}} + \frac{\delta V(n, z)}{V(z)} \]

In longitudinal gauge

- \[ \delta = D - 3 \frac{\mathcal{H}}{k}(1 + w)V \]
  - \( D \) energy density in comoving gauge

- \[ \frac{\delta z}{1 + \bar{z}} = - \left[ \Psi + \mathbf{n} \cdot \mathbf{V} + \int_0^{rS} dr (\dot{\Phi} + \dot{\Psi}) \right] \]

- \[ dV = \sqrt{-g} \epsilon_{abcd} u^a dx^b dx^c dx^d \]

We relate the photon direction at the source to the photon direction at the observer by solving null geodesic equations.
Gaussian window function $W(z)$ centred at $z_m$ of width $\sigma_z$

$$
\int_0^\infty dz \Delta(n, z) W(z)
$$

$$
\frac{\ell(\ell + 1)C_\ell}{2\pi} \quad z_m = 0.1
$$
Redshift dependence $\ell = 20$

\[
\frac{\ell(\ell + 1)C_\ell}{2\pi}
\]

No window function

10% window function

- density
- corr. dens-z
- Doppler
- z-distortion
- lensing
- potential
Total angular power spectrum

\[ \frac{\ell (\ell + 1)C_\ell(z)}{2\pi} \]

- No window function
- 10% window function

\( z = 0.1 \)
\( z = 0.5 \)
\( z = 1 \)
\( z = 3 \)