Inflation, or What?

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Recontres de Moriond, La Thuile
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The CMB Angular Power Spectrum (2012)

Angular scale

\( \ell (\ell + 1) C_\ell / 2\pi \) (\( \mu K^2 \))

- WMAP 7-year
- ACBAR
- QUaD
- ACT
- SPT

(Shirokoff, et al., arXiv:1012.4788)
CMB: Basic Properties

- Adiabatic density perturbations
- Superhorizon correlations
- Gaussian statistics
- Scale Invariance

Q: What does this really tell us?
Generating Superhorizon Perturbations

Punch line:

In an *expanding universe*, to generate perturbations consistent with observation, must have one of:

1. Accelerated Expansion
2. Superluminal Sound Speed
3. Super-Planckian Energy Density

(Geshnizjani, WHK, Moradinezhad Dizgah, arXiv:1107.1241)
Generation of Perturbations

Horizon

Mode freezing

$\frac{u_k}{a}$ vs. $(aH/k)$
Key ingredient for generation of perturbations: Shrinking comoving Hubble length.
The Horizon in Inflation

Decelerating

Accelerating

\[ ds^2 = a^2(\tau)(d\tau^2 - dx^2) \]
Mode Exit and Reentry

classical

quantum

$\lambda$

re-entry

exit

$\tau$
Shorter Wavelength Modes Exit Later
Longer Wavelength Modes Exit Earlier
We See The Last 60 E-folds
Initial Conditions: Inaccessible
Superhorizon Perturbations
The Canonical Case

Mukhanov-Sasaki variable for curvature perturbations with constant sound speed

\[ v \equiv z\zeta \quad z = a\sqrt{2\epsilon} \]

\[ v''_k + \left( k^2 - \frac{z''}{z} \right) v_k = 0 \]
The Canonical Case

Mukhanov-Sasaki variable for curvature perturbations with constant sound speed

\[ v \equiv z \zeta \quad z = a \sqrt{2\epsilon} \]

\[ v_k'' + \left( k^2 - \frac{z''}{z} \right) v_k = 0 \]

\[ \epsilon = -\frac{\dot{H}}{H^2} = \frac{3(\rho + p)}{2\rho} \]

Inflation: \( \epsilon < 1 \)
The Freezeout Horizon

Scale invariance:

\[ v'' + \left( k^2 - \frac{z''}{z} \right) v_k = 0 \]
\[ \frac{z''}{z} = \frac{2}{\tau^2} \]
The Freeezeout Horizon

Scale invariance:

\[ v_k'' + \left( k^2 - \frac{z''}{z} \right) v_k = 0 \]

\[ \frac{z''}{z} = \frac{2}{\tau^2} \equiv R_\zeta^{-2} \]

Freezeout Horizon

Perturbations are generated when *Freezeout Horizon* shrinks.

\[ R_\zeta \propto -\tau \]
Two Horizons

Hubble Length: \[ R_H = \frac{1}{aH} = \frac{a}{a'} \]

Freezeout Horizon: \[ R_\xi = \sqrt{\frac{a \sqrt{\epsilon}}{(a \sqrt{\epsilon})''}} \]
Two Horizons

Hubble Length: \[ R_H = \frac{1}{aH} = \frac{a}{a'} \propto -\tau \]

Freezeout Horizon: \[ R_\zeta = \sqrt{\frac{a\sqrt{\epsilon}}{(a\sqrt{\epsilon})''}} \propto -\tau \]

Slow Roll Inflation: \( \epsilon \sim \text{const.} \ll 1 \)
Two Horizons

Hubble Length: \[ R_H = \frac{1}{aH} = \frac{a}{a'} \]

Freezeout Horizon: \[ R_\zeta = \sqrt{\frac{a\sqrt{\epsilon}}{(a\sqrt{\epsilon})''}} \]

General Case: \( R_\zeta \neq R_H \)

What conditions allow us to have a shrinking freezeout horizon without inflation?

(Khoury & Miller, arXiv:1012.0846)
Power spectrum is approximately scale-invariant over \textit{at least} a factor of 1000 in wavelength.
Long Wavelength Modes
Short Wavelength Modes
Horizon Crossing and Scale

Assume decelerating expansion: \( \epsilon > 1 \Rightarrow \dot{R}_H > 0 \)

Horizon Crossing:

\[
\lambda_i(\tau_i) = R_\zeta(\tau_i) = |\tau_i|
\]

\[
\lambda_f(\tau_f) = R_\zeta(\tau_f) = |\tau_f| > R_H(\tau_f)
\]
Horizon Crossing and Scale

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Horizon Crossing: $\lambda_i(\tau_i) = R_\zeta(\tau_i) = |\tau_i|$

$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |\tau_f| > R_H(\tau_f)$

CMB / LSS: $\lambda_i \geq 1000\lambda_f$

$$\frac{\tau_f - \tau_i}{R_H(\tau_f)} > 1000 \quad (\tau < 0)$$
Assume decelerating expansion: \( \epsilon > 1 \Rightarrow \dot{R}_H > 0 \)

Continuity:

\[
\frac{\dot{\rho}}{\rho} = -2\epsilon H
\]

\[
\ln \frac{\rho_i}{\rho_f} = 2 \int_{t_i}^{t_f} \epsilon H \, dt = 2 \int_{\tau_i}^{\tau_f} \epsilon R_H^{-1} \, d\tau
\]
Continuity

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Continuity: $\frac{\dot{\rho}}{\rho} = -2\epsilon H$

$$\ln \frac{\rho_i}{\rho_f} = 2 \int_{t_i}^{t_f} \epsilon H \, dt = 2 \int_{\tau_i}^{\tau_f} \epsilon R_H^{-1} \, d\tau$$

$$> 2R_H^{-1}(\tau_f) \int_{\tau_i}^{\tau_f} \epsilon \, d\tau \quad (\dot{R}_H > 0)$$
Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Continuity:

$$\frac{\dot{\rho}}{\rho} = -2\epsilon H$$

$$\ln \frac{\rho_i}{\rho_f} = 2 \int_{t_i}^{t_f} \epsilon H \, dt = 2 \int_{\tau_i}^{\tau_f} \epsilon R_H^{-1} \, d\tau$$

$$> 2R_H^{-1}(\tau_f) \int_{\tau_i}^{\tau_f} \epsilon \, d\tau \quad (\dot{R}_H > 0)$$

$$> 2R_H^{-1}(\tau_f) (\tau_f - \tau_i) \quad (\epsilon > 1)$$
Density

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

CMB/LSS: $\frac{\tau_f - \tau_i}{R_H(\tau_f)} > 1000$

Continuity: $\ln \frac{\rho_i}{\rho_f} > 2 \frac{\tau_f - \tau_i}{R_H(\tau_f)}$
Density

Assume decelerating expansion: \( \epsilon > 1 \Rightarrow \dot{R}_H > 0 \)

CMB/LSS: \( \frac{\tau_f - \tau_i}{R_H(\tau_f)} > 1000 \)

Continuity: \( \ln \frac{\rho_i}{\rho_f} > 2 \frac{\tau_f - \tau_i}{R_H(\tau_f)} > 2000 \)

\[ \rho_i > 10^{868} \rho_f! \]

\( \rho_f \geq (100 \text{ MeV})^4 \Rightarrow \rho_i \gg M_P^4 \)

(Linde, Mukhanov, Vikman arXiv:0912.0944)
Mukhanov-Sasaki variable for curvature perturbations with variable sound speed

\[ v \equiv q \zeta \quad q = \frac{a \sqrt{2 \epsilon}}{\sqrt{c_S}} \]

\[ v_k'' + \left( k^2 - \frac{q''}{q} \right) v_k = 0 \]

Time variable: \( dy \equiv c_S d\tau \)

(Khoury & Piazza, arXiv:0811.3633)
The Non-Canonical Case

Mukhanov-Sasaki variable for curvature perturbations with variable sound speed

\[ v \equiv q \zeta \quad q = \frac{a \sqrt{2\epsilon}}{\sqrt{c_s}} \]

Time variable: \[ dy \equiv c_s d\tau \]

Scale Invariance: \[ R_\zeta = \sqrt{\frac{q}{q''}} \propto -y \]
Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

Horizon Crossing: $\lambda_i(\tau_i) = R_\zeta(\tau_i) = |y_i|$

$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |y_f| > R_H(\tau_f)$
Assume decelerating expansion: $\epsilon > 1 \Rightarrow \ddot{R}_H > 0$

Horizon Crossing: $\lambda_i(\tau_i) = R_\zeta(\tau_i) = |y_i|$

$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |y_f| > R_H(\tau_f)$

$$y_f - y_i = \int_{\tau_i}^{\tau_f} c_S d\tau = \bar{c}_S (\tau_f - \tau_i)$$

$$\frac{\bar{c}_S(\tau_f - \tau_i)}{R_H(\tau_f)} > 1000$$
Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

CMB/LSS: $\frac{\bar{c}_S (\tau_f - \tau_i)}{R_H(\tau_f)} > 1000$

Continuity: $\ln \frac{\rho_i}{\rho_f} > 2 \frac{\tau_f - \tau_i}{R_H(\tau_f)} > \frac{2000}{\bar{c}_S}$
Density

Assume decelerating expansion: $\epsilon > 1 \Rightarrow \dot{R}_H > 0$

CMB/LSS: $\frac{\overline{c}_S (\tau_f - \tau_i)}{R_H(\tau_f)} > 1000$

Continuity: $\ln \frac{\rho_i}{\rho_f} > 2 \frac{\tau_f - \tau_i}{R_H(\tau_f)} > \frac{2000}{\overline{c}_S}$

\[
\begin{align*}
\rho_i & \leq M_P^4 \\
\rho_f & \geq (100 \text{MeV})^4 \\
\Rightarrow \quad \overline{c}_S & > 10
\end{align*}
\]
In an *expanding universe*, to generate perturbations consistent with observation, must have one of:

(1) Accelerated Expansion

(2) Superluminal Sound Speed

(3) Super-Planckian Energy Density

(Geshnizjani, WHK, Moradinezhad Dizgah, arXiv:1107.1241)
Broken Scale Invariance

What if scale invariance is broken?

\[ P(k) \propto k^{n_s-1}, \quad n_s \neq 1 \]

\[
R_\zeta^{-2} = \frac{q''}{q} = \frac{2 + (3/2)(1 - n_s)}{y^2}
\]

Limit still holds!

\[
q = \frac{a\sqrt{2\epsilon}}{\sqrt{c_S}} \quad dy \equiv c_s d\tau
\]