The CMB bispectrum in the squeezed limit

Filippo Vernizzi - IPhT, CEA Saclay

JCAP 1111 - 025, 2011
with P. Creminelli and C. Pitrou

Moriond 2012, La Thuile (Valle d’Aosta), 13 March 2012
Bispectrum: angular 3-point function

\[
\left\langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \frac{\Delta T}{T}(\hat{n}_3) \right\rangle \quad \Rightarrow \\
\langle a_{\vec{l}_1} a_{\vec{l}_2} a_{\vec{l}_3} \rangle = (2\pi)^2 \delta(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) B_{l_1 l_2 l_3}
\]
Bispectrum: angular 3-point function

\[
\langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \frac{\Delta T}{T}(\hat{n}_3) \rangle \quad \Rightarrow \quad \langle a_{\vec{l}_1} a_{\vec{l}_2} a_{\vec{l}_3} \rangle = (2\pi)^2 \delta(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) B_{l_1 l_2 l_3}
\]

• Determining the CMB bispectrum in absence of primordial NG extremely important to interpret Planck data. Concentrate on recombination (for late-time effects see Mangilli’s talk on Thursday).

• Complete Boltzmann code at 2\textsuperscript{nd}-order extremely challenging: contains complicated non-linearities in baryon-photon fluid and from GR. Many recent developments:

Bartolo, Matarrese, Riotto ’06; Bernardeau, Pitrou, Uzan ’08; Pitrou ’08; Bartolo, Riotto ’08; Khatri, Wandelt ’08; Senatore, Tassev, Zaldarriaga ’09; Nitta et al. ’09, Beneke and Fidler ’10
Bispectrum: angular 3-point function

\[ \langle \frac{\Delta T}{T}(\hat{n}_1) \frac{\Delta T}{T}(\hat{n}_2) \frac{\Delta T}{T}(\hat{n}_3) \rangle \Rightarrow \langle a_{\vec{l}_1} a_{\vec{l}_2} a_{\vec{l}_3} \rangle = (2\pi)^2 \delta(\vec{l}_1 + \vec{l}_2 + \vec{l}_3) B_{l_1 l_2 l_3} \]

- Determining the CMB bispectrum in absence of primordial NG extremely important to interpret Planck data. Concentrate on recombination (for late-time effects see Mangilli’s talk on Thursday).

- Complete Boltzmann code at 2\textsuperscript{nd}-order extremely challenging: contains complicated non-linearities in baryon-photon fluid and from GR. Many recent developments:
  Bartolo, Matarrese, Riotto ’06; Bernardeau, Pitrou, Uzan ’08; Pitrou ’08; Bartolo, Riotto ’08; Khatri, Wandelt ’08; Senatore, Tassev, Zaldarriaga ’09; Nitta et al. ’09, Beneke and Fidler ’10

Can we find a consistency check in a particular physical limit?
For single-field model, Maldacena’s consistency relation holds:

\[
\langle \zeta_{\vec{k}_L} \zeta_{\vec{k}_S} \zeta_{-\vec{k}_S} \rangle = -(n_s - 1) P(k_L) P(k_S)
\]

Maldacena '02

\[
g_{ij} dx^i dx^j = a^2(t) e^{2\zeta} d\tau^2
\]
Squeezed limit

\[ k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3 \]

For single-field model, Maldacena's consistency relation holds:
\[ \langle \zeta_{k_L} \zeta_{k_S} \zeta_{-k_S} \rangle = -(n_s - 1)P(k_L)P(k_S) \quad \text{Maldacena '02} \]
\[ g_{ij} dx^i dx^j = a^2(t) e^{2\zeta} dx^2 \]

The effect of the long mode translates into a rescaling of the momenta:
\[ \Rightarrow \tilde{k} = ke^{-\zeta_L} \]

\[ \langle \zeta_{\tilde{k}_L} \langle \zeta_{\tilde{k}_S} \zeta_{-\tilde{k}_S} \rangle \rangle \approx \langle \zeta_{\tilde{k}_L} P(k_S e^{-\zeta_L}) \rangle \approx -\frac{d \ln(k_S^3 P_{k_S})}{d \ln k_S} P(k_S)P(k_L) \]

Flat spectrum \( n_s - 1 = 0 \)
Squeezed limit

\[ k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3 \]

For single-field model, Maldacena’s consistency relation holds:

\[ \langle \zeta_{k_L} \zeta_{k_S} \zeta_{-k_S} \rangle = -(n_s - 1)P(k_L)P(k_S) \quad \text{Maldacena '02} \]

The effect of the long mode translates into a rescaling of the momenta:

\[ \Rightarrow \tilde{k} = ke^{-\zeta_L} \]

\[ \langle \zeta_{\tilde{k}_L} \langle \zeta_{\tilde{k}_S} \zeta_{-\tilde{k}_S} \rangle \rangle \approx \langle \zeta_{\tilde{k}_L} P(k_S e^{-\zeta_{\tilde{k}_L}}) \rangle \approx - \frac{d \ln(k_S^3 P_{k_S})}{d \ln k_S} P(k_S)P(k_L) \]

Red spectrum \( n_s - 1 < 0 \)
Squeezed limit

\[ k_L \equiv k_1 \ll k_S \equiv k_2 \sim k_3 \]

For single-field model, Maldacena’s consistency relation holds:

\[
\langle \tilde{\zeta}_{k_L} \tilde{\zeta}_{k_S} \tilde{\zeta}_{-k_S} \rangle = - (n_s - 1) P(k_L) P(k_S) \quad \text{Maldacena '02}
\]

\[ g_{ij} dx^i dx^j = a^2(t) e^{2\zeta} dx^2 \]

The effect of the long mode translates into a rescaling of the momenta:

\[ \Rightarrow \tilde{k} = ke^{-\zeta_L} \]

\[
\langle \tilde{\zeta}_{\tilde{k}_L} \langle \tilde{\zeta}_{\tilde{k}_S} \tilde{\zeta}_{-\tilde{k}_S} \rangle \rangle \approx \langle \tilde{\zeta}_{\tilde{k}_L} P(k_S e^{-\zeta_{\tilde{k}_L}}) \rangle \approx - \frac{d \ln (k_S^3 P_{kS})}{d \ln k_S} P(k_S) P(k_L)
\]

Blue spectrum \( n_s - 1 > 0 \)

All single-field models predict negligible amount of local NG: a detection of local NG rules out all single-field models!
Particular squeezed limit

One of the angles must subtend a scale longer than Hubble radius at recombination (but smaller than Hubble radius today):

\[
\langle \frac{\Delta T}{T} (\hat{n}_1) \frac{\Delta T}{T} (\hat{n}_2) \frac{\Delta T}{T} (\hat{n}_3) \rangle
\]

This corresponds to:

\[ B_{l_1 l_2 l_3} , \quad l_1 \ll l_2 \sim l_3 \quad \& \quad l_1 \ll 200 \]
Two friends at different positions receive photons a bit after recombination at the same physical temperature or Hubble time. The long mode is out of their horizon: they will see the same CMB anisotropies.
The *long mode* is inside my horizon. Have to take into account the *modulation* due to it. This induces a *bispectrum*.

\[ C_{l_S} \rightarrow C_{l_S} + \Theta_L \frac{d}{d\Theta_L} C_{l_S}, \quad \Theta \equiv \Delta T/T \]
The long mode is inside my horizon. Have to take into account the modulation due to it. This induces a bispectrum.

\[ C_{l_S} \rightarrow C_{l_S} + \Theta_L \frac{d}{d\Theta_L} C_{l_S} \quad \Theta \equiv \Delta T/T \]

Rescaling of spatial coords ⇒ rescaling of angles:

\[ C_{l_S} \rightarrow C_{l_S} - \Theta_L (5\hat{n} \cdot \nabla \hat{n} C_{l_S}) \]

\[ B_{l_L l_S l_S} = \langle \Theta_L C_{l_S} \rangle = 5 \frac{C_{l_L}}{l_S^2} \frac{d(l_S^2 C_{l_S})}{d \ln l_S} \quad (\Theta_L = -\frac{1}{5} \zeta) \]
The long mode is inside my horizon. Have to take into account the modulation due to it. This induces a bispectrum.

\[ C_{l_S} \rightarrow C_{l_S} + \Theta_L \frac{d}{d\Theta_L} C_{l_S} \]

\[ \Theta \equiv \Delta T/T \]

Rescaling of spatial coords \(\Rightarrow\) rescaling of angles:

\[ C_{l_S} \rightarrow C_{l_S} - \Theta_L (5\hat{n} \cdot \nabla \hat{n} C_{l_S}) \]

\[ B_{l_LL_Sl_S} = \langle \Theta_L C_{l_S} \rangle = 5 \frac{C_{l_L}}{l^2_S} \frac{d(l^2_S C_{l_S})}{d \ln l_S} \]

\[ (\Theta_L = -\frac{1}{5} \zeta) \]

Long mode changes the local average temperature:

\[ B_{l_LL_Sl_S} = 2C_{l_L} C_{l_S} \]
The long mode is inside my horizon. Have to take into account the modulation due to it. This induces a bispectrum.

\[ C_{l_S} \rightarrow C_{l_S} + \Theta_L \frac{d}{d\Theta_L} C_{l_S} \quad \Theta \equiv \Delta T/T \]

\[ B_{l_L l_S l_S} = C_{l_L} C_{l_S} \left( 2 + 5 \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right) \]

Lensing close to last scattering displaces the 2-p function.
Boltzmann code: CMBquick

This relation can be used as consistency check of Boltzmann codes based on a physical limit. One of the most complete code is Pitrou’s CMBquick (http://www2.iap.fr/users/pitrou/).
One of the most complete code is Pitrou’s CMBquick (http://www2.iap.fr/users/pitrou/).
Boltzmann code: CMBquick

This relation can be used as consistency check of Boltzmann codes based on a physical limit.

One of the most complete code is Pitrou’s CMBquick (http://www2.iap.fr/users/pitrou/).

Suspense...

\[ B_{l_L l_S l_S} = C_{l_L} C_{l_S} \left( 2 + 5 \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right) \]

The check is nontrivial! Even though analytically the squeezed limit is easy, in the code all 2nd-order effects must conspire to reproduce the simple analytical formula.
Final result

Coordinate and average temperature redefinition

\[ B_{l_L l_S} = C_{l_L} C_{l_S} \left( 2 + 5 \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right) + 6 C_{l_L} C_{l_S} \left[ 2 \cos 2\theta - (1 + \cos 2\theta) \frac{d \ln (l_S^2 C_{l_S})}{d \ln l_S} \right] \]

\[ \left( \cos \theta = \hat{l}_L \cdot \hat{l}_S \right) \]

Boubekeur et al. '09

Lensing due to correlation between temperature at recombination and transverse displacement:

\[ \delta \vec{x}^\perp = -2 \int_{\eta_{rec}}^{\eta_0} \left( 1 - \frac{\eta_{rec}}{\eta} \right) \nabla \hat{n} \Phi(\vec{x}) d\eta \]
Final result

Coordinate and average temperature redefinition

\[ B_{LLS_lS} = C_{ll} C_{ls} \left( 2 + 5 \frac{d \ln(l^2_S C_{ls})}{d \ln l_S} \right) + 6 C_{ll} C_{ls} \left[ 2 \cos 2 \theta - (1 + \cos 2 \theta) \frac{d \ln(l^2_S C_{ls})}{d \ln l_S} \right] \]

Lensing

\[ (\cos \theta = \hat{l}_L \cdot \hat{l}_S) \]

Combining the two contributions:

\[ B_{LLS_lS} = C_{ll} C_{ls} (1 + 6 \cos 2 \theta) \left( 2 - \frac{d \ln(l^2_S C_{ls})}{d \ln l_S} \right) \]

Boubekeur et al. '09
Final result

Coordinate and average temperature redefinition

\[
B_{llslls} = C_{ll} C_{ls} \left( 2 + 5 \frac{d \ln (l^2_S C_{ls})}{d \ln l_S} \right) + 6 C_{ll} C_{ls} \left[ 2 \cos 2\theta - (1 + \cos 2\theta) \frac{d \ln (l^2_S C_{ls})}{d \ln l_S} \right]
\]  
\[
\cos \theta = \hat{l}_L \cdot \hat{l}_S
\]

Lensing

Combining the two contributions:

\[
B_{llslls} = C_{ll} C_{ls} (1 + 6 \cos 2\theta) \left( 2 - \frac{d \ln (l^2_S C_{ls})}{d \ln l_S} \right)
\]

\[
f_{\text{NL}}^\text{local} = -\left( \frac{1}{6} + \cos 2\theta \right) \left( 1 - \frac{1}{2} \frac{d \ln (l^2_S C_{ls})}{d \ln l_S} \right)
\]

Integrating only in the squeezed limit:

\[
f_{\text{NL}}^\text{loc} = -0.39, \quad l_{\text{max}} = 2000
\]
\[
f_{\text{NL}}^\text{loc} = -0.48, \quad l_{\text{max}} = 3000
\]

Negligible contamination to Planck.
Conclusions

Valid in the squeezed limit for adiabatic (single clock) perturbations. Includes already NG from single-field models. It is a consistency relation on the observable (CMB temperature).

Reasonable agreement with CMBquick code $\Rightarrow$ Code reliable in the regime that we studied

Planck will not be biased by $2^{\text{nd}}$-order effects at recombination. However, these may be detectable.
Conclusions

\[ B_{l_L l_S l_S} = C_{l_L} C_{l_S} (1 + 6 \cos 2\theta) \left( 2 - \frac{d \ln(l_S^2 C_{l_S})}{d \ln l_S} \right) \]

Valid in the squeezed limit for adiabatic (single clock) perturbations. Includes already NG from single-field models. It is a consistency relation on the observable (CMB temperature).

Reasonable agreement with CMBquick code ⇒ Code reliable in the regime that we studied

Planck will not be biased by 2\textsuperscript{nd}-order effects at recombination. However, these may be detectable.
The check is nontrivial! Even though analytically the squeezed limit is easy, in the code all 2nd-order effects must conspire to reproduce the simple analytical formula.