Testing isotropy of light propagation with CMB polarization data

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based on
G. Gubitosi, et al., JCAP (2011)
Motivation - Lorentz Invariance Violations

• a fundamental length/energy scale can be built combining the three main constants of Nature

\[ L_P \equiv \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35} m \quad \quad E_P \equiv \sqrt{\frac{\hbar c^5}{G}} \sim 10^{28} eV \]

• at this scale quantum and gravitational effects are expected to be comparable:

  heuristic argument: equality between Schwarzschild radius and Compton length

\[ r_S = \frac{2Gm}{c^2} \quad \quad r_C = \frac{\hbar}{mc} \quad \quad \rightarrow \quad \quad r_S \sim r_C \sim L_P \quad \text{if} \quad m \sim \frac{E_P}{c^2} \]
Motivation - Lorentz Invariance Violations

- existence of a fundamental length scale is incompatible with invariance under Lorentz transformations

  under boosts $L_P$ should contract, but then what happens to its building constants?

→ Quantum Gravity might violate Lorentz Invariance

and actually Lorentz Invariance Violations emerge in many theoretical frameworks for Quantum Gravity:

- string theory tensor VEV
- spin network calculations in Loop Quantum Gravity
- Spacetime Foam
- Non-commutative Geometry
- Horava-Lifshitz Gravity
Effective Field Theory approach

• use EFT to describe low energy features of Quantum Gravity theory
  is well defined, provides a dynamical framework, is insensitive to details of underlying theory, we know how to deal with it

• construct most general (non-renormalizable) extension of Standard Model Lagrangian compatible with fundamental symmetries
  for QED it is possible to build only one operator with mass-dimension 5

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2E_P} n^\alpha F_{\alpha\delta} n^\sigma \partial_\sigma \left( n_\beta \epsilon^{\beta\delta\gamma\lambda} F_{\gamma\lambda} \right)
\]

(n^\alpha is an external four-vector)

most general effective Lagrangian under a few conditions:
  quadratic in the fields, gauge invariant, formally Lorentz invariant, not reducible to lower-dimension operators, nor to a total derivative

R.C. Myers, M. Pospelov, PRL 2003
Effective Field Theory approach

- usually assume no space components for $n^\alpha$:
  (preserved rotation invariance in some reference frame)

$$n_\alpha = (n_0, 0, 0, 0)$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2 E_P} \varepsilon^{jkl} F_{0j} \partial_0 F_{kl}$$

$$[\xi \equiv n_0^3]$$

$\xi$ should be roughly of order one according to Quantum Gravity intuition

L. Maccione, S. Liberati, JCAP 2008
Photons propagation

• equation of motion from effective Lagrangian:

\[
\left( -\omega^2 + p^2 \pm \frac{2\xi}{E_P} \omega^2 p \right) E_\pm = 0
\]

for right- and left- circular polarizations

• The two helicity states of electromagnetic waves propagate with different phase velocities:

→ linearly polarized radiation rotates polarization direction during propagation

amount of rotation \( \alpha(T) = \delta \omega T = 2 \frac{\xi}{E_p} p^2 T \)
CMB constraints

• CMB radiation ~10% linearly polarized

  describe polarization field with Stokes parameters:

  \[ Q \equiv E_x^2 - E_y^2 \]

  \[ U \equiv 2 \Re \{ E_x E_y^* \} \]

• polarization rotation mixes Stokes parameters

  \[ Q' = Q \cos(2\alpha) + U \sin(2\alpha) \]

  \[ U' = U \cos(2\alpha) - Q \sin(2\alpha) \]

\[ (Q \pm iU) = \sum_{\ell m} \pm 2a_{\ell m} \pm 2Y_{\ell m} \]

\[ E_{\ell m} \equiv \frac{1}{2} \left( +2a_{\ell m} + -2a_{\ell m} \right) \]

\[ B_{\ell m} \equiv \frac{1}{2i} \left( +2a_{\ell m} - -2a_{\ell m} \right) \]

\[ E = \sum_{\ell m} E_{\ell m} Y_{\ell m} \]

\[ B = \sum_{\ell m} B_{\ell m} Y_{\ell m} \]

\[ \rightarrow \text{mixing of E and B modes} \]
Birefringence effects on CMB

- rotation of power spectra due to mixing of E and B modes:

\[
\begin{align*}
C^{EE}_l' &= C^{EE}_l \cos^2(2\alpha) + C^{BB}_l \sin^2(2\alpha) \\
C^{BB}_l' &= C^{EE}_l \sin^2(2\alpha) + C^{BB}_l \cos^2(2\alpha) \\
C^{EB}_l' &= \frac{1}{2} (C^{EE}_l - C^{BB}_l) \sin(4\alpha) \\
C^{TE}_l' &= C^{TE}_l \cos(2\alpha) \\
C^{TB}_l' &= C^{TE}_l \sin(2\alpha)
\end{align*}
\]

EB and TB spectra acquire power from TE and EE

black: standard power spectra
red: spectra after rotation of 20°
Exploiting energy dependence to improve constraints and disentangle from other effects

Present constraints and forecasts

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\alpha \pm \sigma(\alpha)$</th>
<th>$\xi \pm \sigma(\xi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP (94 GHz)</td>
<td>$-1.6 \pm 2.1$</td>
<td>$-0.09 \pm 0.12$</td>
</tr>
<tr>
<td>BOOMERanG (145 GHz)</td>
<td>$-5.2 \pm 4.0$</td>
<td>$-0.123 \pm 0.096$</td>
</tr>
<tr>
<td>WMAP+BOOMERanG</td>
<td>--</td>
<td>$-0.110 \pm 0.075$</td>
</tr>
<tr>
<td>PLANCK 70 GHz</td>
<td>0.64</td>
<td>$6.0 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>PLANCK 100 GHz</td>
<td>0.14</td>
<td>$6.5 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>PLANCK 143 GHz</td>
<td>0.073</td>
<td>$1.7 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>PLANCK 217 GHz</td>
<td>0.10</td>
<td>$1.0 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>PLANCK all</td>
<td>--</td>
<td>$8.5 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

GG, L. Pagano, G. Amelino-Camelia, A. Melchiorri, A. Cooray, JCAP 2009

• constraints from other CMB experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$\alpha \pm \sigma(\alpha)$</th>
<th>only constraints on the rotation angle at the moment, mixing different energy channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>BICEP (100+150 GHz)</td>
<td>$-2.60 \pm 1.02$</td>
<td></td>
</tr>
<tr>
<td>QUAD (100 GHz)</td>
<td>$-1.89 \pm 2.24$</td>
<td></td>
</tr>
<tr>
<td>QUAD (150 GHz)</td>
<td>$0.83 \pm 0.94$</td>
<td></td>
</tr>
</tbody>
</table>

J. Q. Xia, H. Li, X. Zhang, PLB 2010
E. Y. S. Wu et al. [QUAD collaboration] PRL 2009
Comparison with astrophysical limits

- strong limits on isotropic birefringence from astrophysical observations
  (GRBs: $|\xi| \lesssim 10^{-14}$, Crab Nebula observations: $|\xi| \lesssim 10^{-9}$)


but $\xi \equiv n_0^3$ related to the time-component of a 4-vector and astrophysical sources
have a rest frame different from CMB

one may have $n_\alpha \sim (0, 1, 1, 1)$ in one reference frame and $n_\alpha \sim (10^{-3}, 1, 1, 1)$ in
another with velocity $\beta = 10^{-3}$, and this is the typical boost factor between CMB
and extragalactic source

→ can’t constrain the general model without information on
the space components of the vector
Generalization to non-isotropic case

- general form of the effective field theory Lagrangian

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2 E_P} n^\alpha F_{\alpha\delta n^\sigma \partial_\sigma} (n_\beta \varepsilon^{\beta\delta\gamma\lambda} F_{\gamma\lambda}) \]

- implies anisotropic behavior of the field:

*direction-dependent* birefringent dispersion law

\[ \omega_\pm = p \pm \frac{p^2}{E_P} (n_0 + \vec{n} \cdot \hat{p})^3 \]

*non-transverse* field eigenstates

\[ \vec{E}_\pm = \begin{pmatrix} 0 \\ \pm \frac{\hat{n}}{\sqrt{2}} \end{pmatrix} + \frac{1}{E_P} \begin{pmatrix} \sqrt{2} \frac{|\vec{n}|}{|\vec{k}|^2} (\vec{k} \cdot \vec{n} + |\vec{k}| n_0)^2 |\vec{k} - (\vec{k} \cdot \vec{n}) \hat{n}| \\ \frac{\hat{n}}{\sqrt{2}} \frac{1}{|\vec{k}|^2} (\vec{k} \cdot \vec{n} + |\vec{k}| n_0) |\vec{k} - (\vec{k} \cdot \vec{n}) \hat{n}|^2 \\ \frac{1}{\sqrt{2}} \frac{1}{|\vec{k}|^2} (\vec{k} \cdot \vec{n} + |\vec{k}| n_0) |\vec{k} - (\vec{k} \cdot \vec{n}) \hat{n}|^2 \end{pmatrix} \]
**Non-isotropic Lorentz breaking**

- Large attenuation of anomalous effects in a significant portion of the sky (at least $10^{-4}$ in ~10% of the sky)

\[
\theta \equiv \cos (\vec{k} \cdot \vec{n}), \quad \frac{n_0}{|\vec{n}|} = 0.9
\]

- Crowns of partial blindness to anomalous effects

→ can’t use a few astrophysical point sources, need full-sky analysis (CMB)

**GG, G. Genovese, G. Amelino-Camelia, A. Melchiorri, PRD 2010**
Non-isotropic birefringence

• Look for direction-dependent rotation of linear polarization

\[ \alpha(\theta, \phi, t) = -\frac{1}{E_P} |\vec{p}|^2 |\vec{n}|^3 \left( \sin \theta \sin \theta_n \cos(\phi - \phi_n) + \cos \theta \cos \theta_n \right)^3 t \]

\{\theta, \phi\} : observation direction

\{\theta_n, \phi_n\} : preferential direction pointed out by \( \vec{n} \)

\[ \alpha(\theta, \phi) = \sum_{LM} \alpha_{LM} Y_{LM}(\theta, \phi) \]

\[ \rightarrow \quad L=1,3 \]

Map of the expected amount of rotation effect for \( \theta_n = 45^\circ, \phi_n = 90^\circ \)

Effects on CMB

- anisotropic birefringence induces correlations between different multipoles

\[
\langle E'_{lm} T * l' m' \rangle = C^{ET}_l \delta_{l'l} \delta_{mm'} + i2 \sum_{LM \text{ (odd)}} \alpha_{LM} C^{ET}_{l'} H^L_{l'l} \xi^{LM}_{lm l' m'}
\]

\[
\langle B'_{lm} T * l' m' \rangle = 2 \sum_{LM \text{ (even)}} \alpha_{LM} C^{ET}_{l'} H^L_{l'l} \xi^{LM}_{lm l' m'}
\]

- but no effect on full-sky power spectra:

\[
C^{BE'}_l = -\frac{2}{2l+1} \sum_m \sum_{LM}^{2l+L=\text{even}} \alpha_{LM} (C^{BB}_l - C^{EE}_l) H^L_{ll} \xi^{LM}_{lm lm}
\]

\[
C^{EE'}_l = C^{EE}_l
\]

\[
C^{BT'}_l = \frac{2}{2l+1} \sum_m \sum_{LM}^{2l+L=\text{even}} \alpha_{LM} C^{ET}_l H^L_{ll} \xi^{LM}_{lm lm}
\]

\[
C^{ET'}_l = C^{ET}_l
\]

\[
C^{BB'}_l = C^{BB}_l
\]

→ need to use “reduced” power spectra on patches of sky
Method of analysis

• consider circular regions of the sky of diameter 20°, uniformly distributed

• in each circular patch approximate rotation effect as constant

if the disk is centered on the direction of maximum rotation then the spectra are rotated as in the isotropic case, but with a rescaled angle given by the average of the function $\alpha(\theta, \phi)$ on the disk:

$$\alpha_0 = A \alpha_{max}$$

$$A = \langle \left( \sin \theta \sin \theta_n \cos(\phi - \phi_n) + \cos \theta \cos \theta_n \right)^3 \rangle$$

$$\alpha_{max} = \frac{1}{E_P} \frac{1}{|\vec{p}|^2 |\vec{n}|^3 t}$$

• correlate information from disks in opposite directions

find best fit for $\alpha_0$ for each couple of disks
Method of analysis

• the couple of opposite disks with maximum best-fit value for $\alpha_0$ will identify the symmetry-breaking vector direction

• through Monte Carlo simulations estimate uncertainty on $\alpha_0$

• draw the distribution of the estimated polarization rotation angle for each patch of the sky

Uncertainties on the symmetry-breaking vector direction are obtained slicing the map along lines of $1\sigma$ and $2\sigma$ errors on $\alpha_0$.
**Planck simulation**

- results from analysis of simulated map

  input values \( \alpha_{max} = 1.98 \) \( \{\theta_n, \phi_n\} = \{45^\circ, 90^\circ\} \)

map of effective measured rotation angle:

<table>
<thead>
<tr>
<th>( \bar{\alpha}<em>{max} \pm \Delta \bar{\alpha}</em>{max} )</th>
<th>( \Delta \bar{\theta}_n )</th>
<th>( \Delta \bar{\phi}_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.97 ( \pm ) 0.20</td>
<td>[23.9, 63.0]</td>
<td>[49.5, 114.7]</td>
</tr>
</tbody>
</table>

(fore direction angles we report 2\( \sigma \) intervals)

Contour plot of the confidence intervals on \( \{\theta_n, \phi_n\} \), obtained by slicing the distribution of \( \alpha_0 \) in correspondence to the 1\( \sigma \) and 2\( \sigma \) errors.

The black dot marks the input value.
Conclusions

• Quantum Gravity research gives motivations to look for Lorentz symmetry breaking

• the most common prediction is anomalous light propagation, which might depend on observation direction

• availability of (almost) full sky coverage from CMB data allows to test non-isotropic effects

• need to compare CMB observations in different directions, separately analyzing small portions of CMB maps

• Planck data will be good enough to perform this kind of tests, and will allow to constrain Quantum Gravity effects

• the method presented here can be easily generalized to test other kind of direction-dependent effects