Modified non-local gravity: exact solutions and applications

Alexey Koshelev

*Vrije Universiteit Brussel,*
*Rencontre de Moriond, March 15, 2012*
The problem

• The initial singularity problem is the problem with most cosmological solutions because they hit a singularity approaching the time when Universe had begun

• Reformulation of gravity in terms of other models makes it looking like a strong coupling regime still not providing a reasonable resolution for the singular behaviour

• Standard ways to avoid a singularity meets another may be even more serious problem of ghosts

• These ghosts are related to higher derivatives in for example $F(R)$ models which feature non-singular bouncing solutions

• Is there a way around of ghosts?
Model

Biswas, Koivisto, Mazumdar, 2010
Biswas, A.K., Mazumdar, Vernov to appear

This type of Lagrangian follows almost immediately from open SFT for a scalar field

\[ S = \int d^4x \left( \phi \mathcal{F}(\Box) \phi - v(\phi) + \ldots \right) \]

This describes the modification of gravity expected from the closed String Field Theory. There are deep reasons to have operators like those in the above mentioned model in SFT.

\[ S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G_N} + \frac{c}{2} \mathcal{F}(\Box) R + \ldots \right) \]
Model (continued)

To give some feeling of what we are dealing with the trace of Einstein equations with an additional cosmological constant is

\[
\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left( \partial_\mu \square R \partial_\mu \square^{n-1-l} R + 2 \square^l R \square^{n-l} R \right) + 6 \square \mathcal{F}(\square) R = \frac{R}{8\pi G_N c} + \frac{4}{c} \Lambda
\]

and it (really not obviously) has the following exact solution (we need some radiation for it and put \( \mathcal{F}'(r_1) = 0 \))

\[
a = a_0 \cosh(\lambda t) \quad \Rightarrow \quad H = \lambda \tanh(\lambda t)
\]

Here \( a \) is the scale factor of the spatially flat FRW metric. And yes, we always discuss 4 dimensions.

The latter exact solution is the manifestly non-singular bouncing solution moreover with a de Sitter late time asymptotic

Other solutions which can be easily constructed are \( H = H_0 t \) and \( H = H_0/t \).
Model (continued)

The previous action is equivalent to the following one

\[ S = \int d^4x \sqrt{-g} \left( R \left( \frac{1}{16\pi G_N} + \Psi \right) - \Psi \frac{1}{\mathcal{F}(\Box)} \Psi + \ldots \right) \]

The important property here is the non-minimal coupling of a scalar field to gravity.

Model without the non-minimal coupling was intensively studied by A.K., Vernov (2009 and 2010) and Galli, A.K (2010)

The principal point here is the operator \( \mathcal{F}(\Box) \) which may contain any (infinite) degrees of the box operator making the action non-local but at the same time giving a way to evade Ostrogradski ghosts in the spectrum.

The main objective of our work in progress is to analyse the cosmological perturbations and find out what is new compared to the Einstein gravity.
What is good about $\mathcal{F}(\Box)$?

$1/\mathcal{F}(\Box)$ determines the propagator. The physical spectrum is read from the poles of this expression, i.e. we must know zeros of the operator $\mathcal{F}(\Box)$.

For example, $\mathcal{F}(\Box) = e^{\Box}(1 - \Box)$ has only one zero and is absolutely equivalent to just $(1 - \Box)$ from the point of view of physics.

More than one zero indicates the presence of ghosts.

For the above mentioned modified gravity model the ghost-free condition is

$$\frac{1}{8\pi G_N} - 6\Box \mathcal{F}(\Box)$$

has at most one zero.

Things become more complicated when we deal with perturbations since every $\Box$ depends on the metric and will contribute to the story.
Perturbations

\[-(\Phi + \Psi) = \lambda \Box \zeta + 8\pi G_N \frac{a^2}{k^2} \pi^{(s)}\]

\[\Psi = \sigma H \dot{\zeta} + 4\pi G_N \rho \frac{a^2}{k^2} \varepsilon\]

Here $\zeta$ is a combination of $\Phi$ and $\Psi$ (i.e. it is gauge invariant) acted by a non-local operator

Two things are to mention here:

even if the matter has no anisotropic stresses one cannot avoid the effects of “anisotropic” perturbations

in this particular setup we can re-express $\Phi = G(\Box) \varepsilon$ making us possible to write down the single equation for the behavior of $\varepsilon$

It is a non-local equation again.

One simple condition $\mathcal{F}''(r_1) = 0$ guarantees there are no growing modes. (Recall that the only condition imposed so far to build the solution was $\mathcal{F}'(r_1) = 0$).
Summary and Outlook

• Non-local generalization of gravity is presented and several exact solutions are constructed

• There are solutions which manifestly have a non-singular bouncing behavior

• Perturbations are studied and in the given setup a significant progress is achieved.

• One can guarantee the good behavior of the model imposing very weak restrictions on the non-local operator $\mathcal{F}(\Box)$

• We are working on to generalize this analysis beyond the described modification
Thank you for listening!