Modified gravity: a theorist’s perspective

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General Relativity is ________ theory

Is there any strong motivation to try to modify GR at large distances?
General Relativity is _________ theory

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Dark matter

Present accelerated expansion
What is GR?

It is the only consistent Lorentz invariant theory of a massless spin 2 field at low energies

Weinberg '65
What is GR?

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Modify GR in the infrared

There must be extra light degrees of freedom (Brans-Dicke, f(R), Pauli-Fierz massive gravity, DGP, ...)

Weinberg ’65
What is GR?

It is the only consistent Lorentz invariant theory of a massless spin 2 field at low energies

Modify GR in the infrared

There must be extra light degrees of freedom

one extra scalar $\phi$

Effect comes from the coupling of $\phi$ to $T_{\mu\nu}$

scalar contribution to the stress energy tensor small

self-acceleration vs acceleration by dark energy

Weinberg ’65
\[ M_{\text{Pl}}^2 R - (\partial \varphi)^2 + \frac{1}{M_{\text{Pl}}} h_{\mu\nu} T^{\mu\nu} + \frac{1}{M_{\text{Pl}}} \varphi T \]

No microscopic violation of EP
\[ M_{\text{Pl}}^2 R - (\partial \varphi)^2 + \frac{1}{M_{\text{Pl}}} h_{\mu \nu} T^{\mu \nu} + \frac{1}{M_{\text{Pl}}} \varphi T \]
Almost GR: \( \leq O(10^{-3}) \)

Scalar-tensor: \( O(1) \)

- \( r_{\text{pluto}} \): 10^{14} \text{ cm}
- \( r_{\text{IR}} \): \( \sim H_0^{-1} \), 10^{28} \text{ cm}
almost GR \[ \leq O(10^{-3}) \]

scalar-tensor \[ O(1) \]

screening mechanism

\[ r_{\text{pluto}} \sim 10^{14} \text{ cm} \]

\[ r_{\text{IR}} \sim H_0^{-1} \sim 10^{28} \text{ cm} \]
$\lesssim O(10^{-3})$
Chameleon mechanism
Khoury, Weltman ‘03

the mass of the scalar depends on the local matter density

\[ M_{\text{Pl}}^2 R - (\partial \varphi)^2 + V(\varphi) + \frac{1}{M_{\text{Pl}}} h_{\mu\nu} T^{\mu\nu} + \frac{1}{M_{\text{Pl}}} \varphi T \]
Chameleion mechanism

\[ M_{P_1}^2 R - (\partial \varphi)^2 + V(\varphi) + \frac{1}{M_{P_1}} h_{\mu\nu} T^{\mu\nu} + \frac{\beta}{M_{P_1}} \varphi T \]
Chameleion mechanism

\[ M_{Pl}^2 R - (\partial \varphi)^2 + V(\varphi) + \frac{1}{M_{Pl}} h_{\mu\nu} T^{\mu\nu} + \frac{\beta}{M_{Pl}} \varphi T \]

\( f(R) \) is just a specific subgroup of this class

\[ M_{Pl}^2 f(\tilde{R}) + S_{\text{matter}}[\tilde{g}_{\mu\nu}, \Psi_i] \]

when rewritten in terms of \( \varphi, g_{\mu\nu} \)

\[ f'(\tilde{R}) = \exp \left(-\frac{2\beta \varphi}{M_{Pl}} \right) \]

\[ g_{\mu\nu} = e^{-\frac{2\beta \varphi}{M_{Pl}}} \tilde{g}_{\mu\nu} \]

\[ V(\varphi) = \frac{M_{Pl}^2 (\tilde{R} f'(\tilde{R}) - f(\tilde{R}))}{2 f'(\tilde{R})^2} \]

\( \beta = \sqrt{1/6} \)
Chameleon mechanism

\[ M_{\text{Pl}}^2 R - (\partial \varphi)^2 + V(\varphi) + \frac{1}{M_{\text{Pl}}} h_{\mu\nu} T^{\mu\nu} + \frac{\beta}{M_{\text{Pl}}} \varphi T \]

\[ \Box \varphi = V'(\varphi) - \frac{\beta}{M_{\text{Pl}}} \rho \]

Scalar has a large mass inside the object the exterior scalar profile sourced only by a thin shell of mass at the boundary.
Chameleon mechanism

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Constraint on cosmological value of \( \varphi \)

\[ \frac{\beta \varphi}{M_{Pl}} < 10^{-5} \]

Brax, van de Bruck, Davies, Shaw ’08
Chameleon mechanism

\[ M_{\text{Pl}}^2 R - (\partial \varphi)^2 + V(\varphi) + \frac{1}{M_{\text{Pl}}} h_{\mu\nu} T^{\mu\nu} + \frac{\beta}{M_{\text{Pl}}} \varphi T \]

\[ \frac{\beta \varphi}{M_{\text{Pl}}} < 10^{-5} \quad \text{Origin of acceleration} \]

Jordan frame ("physical") metric

\[ \tilde{g}_{\mu\nu} = e^{\frac{2\beta \varphi}{M_{\text{Pl}}}} g_{\mu\nu} \]

Einstein metric flat, not dark energy effect
Caused by the non-minimal scalar coupling instead

Chameleon theories cannot support genuine self-acceleration
Vainshtein screening

higher derivative self-interactions suppress the scalar at short scales

\[ M_{P1}^2 R - (\partial \pi)^2 + \mathcal{L}(\partial \pi, \partial^2 \pi, \ldots) + \frac{1}{M_{P1}} h_{\mu \nu} T^{\mu \nu} + \frac{1}{M_{P1}} \pi T \]
Vainshtein screening

Robustly implemented in an Effective Field Theory

\[-(\partial \pi)^2 + \mathcal{L}_\pi + \frac{1}{M_{Pl}} \pi T\]

The Galileon  \[\pi(x) \rightarrow \pi(x) + c + b_\mu x^\mu\]

\[\mathcal{L}^{(2)} = (\partial \pi)^2\]
\[\mathcal{L}^{(3)} = (\partial \pi)^2 \Box \pi\]
\[\mathcal{L}^{(4)} = (\partial \pi)^2 [(\Box \pi)^2 - \partial_\mu \partial_\nu \partial^\mu \partial^\nu \pi]\]
\[\mathcal{L}^{(5)} = (\partial \pi)^2 [(\Box \pi)^3 - 3 \Box \partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi + 2 \partial_\mu \partial_\nu \pi \partial^\nu \partial^\alpha \pi \partial^\mu \partial^\alpha \pi]\]

2 derivatives EOM  \[\rightarrow\]  No extra ghost-like dof

Nicolis, Rattazzi, ET ’08
$M_{Pl}^2 \mathcal{R}$

$\left(\partial h_c\right)^2 + \frac{h_c}{M_{Pl}} \left(\partial h_c\right)^2 + \frac{h_c^2}{M_{Pl}^2} \left(\partial h_c\right)^2 + \ldots + \frac{1}{M_{Pl}^2} \left(\partial^2 h_c\right)^2 + \frac{h_c}{M_{Pl}^3} \left(\partial^2 h_c\right)^2 + \ldots + \frac{1}{M_{Pl}^4} h_c T$

$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}^c}{M_{Pl}}$
General Relativity

$M_{Pl}^2 \mathcal{R}$

$\ell = M^3 / (r h c) \ll M_{Pl}^2 r$

$\rho = M \delta^3(r)$

$h_c \sim \frac{M}{r M_{Pl}}$

Non-linearities become important at a scale $r_s$ where

$\frac{h_c}{M_{Pl}} \sim 1 \Rightarrow r_s \sim \frac{M}{M_{Pl}^2}$
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$$\frac{h_c}{M_{\text{Pl}}} \sim 1 \Rightarrow r_s \sim \frac{M}{M_{\text{Pl}}^2}$$

All the other terms are suppressed by extra-powers of

$$\frac{\partial}{\Lambda} \sim \frac{1}{r M_{\text{Pl}}} \ll 1$$

We can compute classical non-linearities without knowing the UV compl
The Galileon

Non renormalization theorem

Loops of quantum fields with interactions $\mathcal{L}^{(3)}, \mathcal{L}^{(4)}, \mathcal{L}^{(5)}$ generate terms involving at least 2 derivatives on the external legs. In particular galilean terms are not renormalized.

\begin{align*}
(\partial \pi)^2 &+ \frac{c_3}{\Lambda^3} (\partial \pi)^2 \Box \pi + \frac{c_4}{\Lambda^6} (\partial \pi)^2 (\partial^2 \pi)^2 + \frac{c_5}{\Lambda^9} (\partial \pi)^2 (\partial^2 \pi)^3 \\
+ \frac{d_2}{\Lambda^2} (\partial^2 \pi)^2 &+ \frac{d_3}{\Lambda^5} (\partial^2 \pi)^3 + \ldots + \frac{1}{M_{Pl}} \pi T
\end{align*}

Luty, Porrati, Rattazzi '03
The Galileon

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$$+ \frac{d_2}{\Lambda^2} (\partial^2 \pi)^2 + \frac{d_3}{\Lambda^5} (\partial^2 \pi)^3 + \ldots + \frac{1}{M_{Pl}} \pi T$$

$$\pi \sim \frac{M}{M_{Pl}} \frac{1}{r}$$

Classical non-linearities important

$$\frac{\partial^2 \pi}{\Lambda^3} \sim 1 \Rightarrow r_V \sim \left( \frac{M}{M_{Pl} \Lambda^3} \right)^{\frac{1}{3}}$$

Luty, Porrati, Rattazzi '03
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Non renormalization theorem Luty, Porrati, Rattazzi ’03
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All the other operators are suppressed by extra powers of $\frac{\partial}{\Lambda}$.
Results

Stable self-accelerating dS solutions

Stable spherically symmetric Vainshtein-like solutions around compact objects

\[ \Lambda = \left( H_0^2 M_{\text{Pl}} \right)^{1/3} \]

Nicolis, Rattazzi, ET ’08
Results

**Stable** self-accelerating dS solutions

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**Stable** spherically symmetric Vainshtein-like solutions around compact objects

Nicolis, Rattazzi, ET ‘08

Superluminal propagation around non trivial background

Nicolis, Rattazzi, ET ‘09

The UV completion **cannot** be a Lorentz-invariant local QFT

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi ‘06
Conclusions

When one comes up with a model that tries to explain the accelerated expansion by a modification of gravity

Order zero questions:
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i) Is it a modification of gravity in the IR?

Self-acceleration vs dark energy
Conclusions

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Order zero questions:

i) Is it a modification of gravity in the IR? vs dark energy

ii) Is the self-accelerating solution stable?
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iii) How are the extra DOFs screened at shorter scales?
Conclusions

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iv) Is the lagrangian of the extra DOFs robust (stable under quantum corrections)?
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Self-acceleration vs dark energy