Probing Lorentz invariance in Cosmology

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Modified dispersion relations

• main observable consequence of LIV is modification of particle
dispersion relations:

\[ E^2 = p^2 + m^2 + \Delta_{QG}(p, m, E_P) \]

ultra-relativistic limit (p/m \rightarrow \infty), first order in 1/E_P, for massive particles

\[ E^2 \simeq p^2 + m^2 + \frac{1}{E_P}(\eta_1 p^3 + \eta_2 m p^2 + \eta_3 m^2 p) \]

R. Gambini, J. Pullin, PRD 1999

(see also previous talk by Vlasios Vasileiou)
Modified dispersion relations

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ultra-relativistic limit for massless particles:

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• orthogonal polarization states can behave differently:

(more in the next slides)

\[ E_\pm \simeq p + \eta_\pm \frac{p^2}{E_P} \]

if \( \eta_+ = -\eta_- \) for right/left helicity states

\[
\rightarrow \text{in-vacuo birefringence}
\]
In vacuo birefringence

- The two helicity states of electromagnetic waves propagate with different phase velocities
decomposing a linearly polarized field into circularly polarized modes:

\[
\vec{E}_\pm = \Re \left( (\hat{e}_1 \pm i \hat{e}_2) e^{i(\omega_\pm t - \vec{k} \cdot \vec{x})} \right)
\]

\[
\omega_\pm \equiv \omega_0 \pm \delta \omega = \omega_0 \pm \eta \frac{p^2}{E_P}
\]

→ linearly polarized radiation rotates polarization direction during propagation (birefringence)

amount of rotation:

\[
\alpha(T) = \delta \omega T
\]

(analogous to propagation in media with chiral molecules, like sugar and quartz)
In vacuo birefringence

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Effective Field Theory model

- use EFT to describe low energy features of Quantum Gravity theory
  is well defined, provides a dynamical framework, is insensitive to details of underlying theory, we know how to deal with it

- construct most general (non-renormalizable) extension of Standard Model Lagrangian compatible with fundamental symmetries
  for QED it is possible to build only one operator with mass-dimension 5

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2E_P} n^\alpha F_{\alpha\delta} n^\sigma \partial_\sigma \left( n_\beta \epsilon^{\beta\delta\gamma\lambda} F_{\gamma\lambda} \right) \]

(\( n^\alpha \) is an external four-vector)

most general effective Lagrangian under a few conditions:
quadradic in the fields, gauge invariant, formally Lorentz invariant, not reducible to lower-dimension operators, nor to a total derivative

R.C. Myers, M. Pospelov, PRL 2003
Effective Field Theory model

- for convenience no space components for $n^\alpha$ are usually assumed:
  (preserved rotation invariance in some reference frame, possibly the CMB frame)

\[ n_\alpha = (n_0, 0, 0, 0) \]

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2 E_P} \varepsilon^{jkl} F_{0j} \partial_0 F_{kl} \]

\[ [\xi \equiv n_0^3] \]

$\xi$ should be roughly of order one according to Quantum Gravity intuition
(the coupling constant sets the scale where new physics is relevant)
Photons propagation

• equation of motion from effective Lagrangian:

\[
\left(-\omega^2 + p^2 \pm \frac{2\xi}{E_P} \omega^2 p\right) E_\pm = 0
\]

for right- and left- circular polarizations

→ Plank-scale modified dispersion relation, dependent on the helicity of photons

\[
\omega_\pm = p \pm \frac{\xi}{E_P} p^2
\]

• birefringent propagation of light, with polarization rotation angle:

\[
\alpha(T) = \delta \omega T = 2 \frac{\xi}{E_P} p^2 T
\]

suppressed by Planck scale but:

• linear in \textit{propagation time} \(T\)
• quadratic in \textit{photon energy} \(p\)
Astrophysical constraints

- observation of the (linear) polarization direction in different energy ranges for GRB sources
  
  (exploit energy dependence of the effect)

\[ \Delta \alpha \equiv |\alpha(p_1) - \alpha(p_2)| = \xi\frac{|p_1^2 - p_2^2| T}{E_P} \]

- for GRB041219A (INTEGRAL observation):

  \[ E_1 \sim 2 \times 10^2\text{keV} \quad T \sim 10^8\text{years} \quad (z = 0.02) \]

  \[ E_2 \sim 3 \times 10^2\text{keV} \quad \Delta \alpha < 47^\circ \]

  \[ \xi < 1.1 \times 10^{-14} \]
Cosmic Microwave Background (CMB)

- Black-body spectrum, peaked at $T = (2.725 \pm 0.001) K$
  (equivalent to $\sim 6.6 \times 10^{-4} eV$)

- Radiation is $\sim 10\%$ linearly polarized
  $\rightarrow$ we can look for birefringence

- free streaming since the Universe was 350,000 years old ($z \sim 1100$)
  $\rightarrow$ large accumulation of effect

- because of energy redshift CMB photons started $\sim 10^3$ times more energetic
  $\rightarrow$ extra amplification

Anisotropies: $\Delta T / T \lesssim 10^{-5}$
Detecting birefringence with CMB polarization

• decompose polarization field in two components

  E modes are curl free
  B modes are divergence free

• B modes are parity-odd,
  E modes are parity-even

  standard physics predicts E and B modes should not correlate

• E and B modes mix under rotation

  birefringence correlates these two modes
**Present constraints**

- constraints from single experiments

<table>
<thead>
<tr>
<th>Experiment (channels in GHz)</th>
<th>$\alpha \pm \sigma_{\text{stat}}(\pm \sigma_{\text{syst}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP (41+61+94)</td>
<td>$-0.9 \pm 1.4 \pm 1.5$</td>
</tr>
<tr>
<td>BOOMERanG (145)</td>
<td>$-4.3 \pm 4.1$</td>
</tr>
<tr>
<td>QUAD (100)</td>
<td>$-1.89 \pm 2.24 \pm 0.5$</td>
</tr>
<tr>
<td>QUAD (150)</td>
<td>$0.83 \pm 0.94 \pm 0.5$</td>
</tr>
<tr>
<td>BICEP (100+150)</td>
<td>$-2.60 \pm 1.02 \pm 0.7$</td>
</tr>
</tbody>
</table>

can not trivially combine datasets to get $\xi$ because of energy dependence

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E. Komatsu et al. (WMAP coll.) Astrophys.J.Suppl. 2011
GG, L. Pagano, G. Amelino-Camelia, A. Melchiorri, A. Cooray, JCAP 2009
E. Wu et al. (QUAD coll) PRL102 2009
J.Q. Xia, H. Li, X. Zhang, PLB 687 2010
Present constraints

- combined constraint from all available experiments

\[ \xi = -0.87 \pm 0.34 \]

- one order of magnitude improvement expected with Planck satellite data
- possibility to discriminate among different energy dependence laws (e.g. systematic effects mimicking birefringence would be energy independent)
Comparison with astrophysical limits

• apparently much stronger limits on birefringence coming from astrophysical observations ($\xi \lesssim 10^{-14}$)

• but $\xi \equiv n_0^3$ related to the time-component of a 4-vector and astrophysical sources have a rest frame different from CMB

one may have $n_\alpha \sim (0, 1, 1, 1)$ in one reference frame and $n_\alpha \sim (10^{-3}, 1, 1, 1)$ in another with velocity $\beta = 10^{-3}$, and this is the typical boost factor between CMB and extragalactic sources

→ to compare constraints we need information on all the components of the Lorentz violating vector in some reference frame
Generalization to non-isotropic model

- general form of the effective field theory Lagrangian

\[ L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2E_P} n^\alpha F_{\alpha \delta} n^\sigma \partial_\sigma (n_\beta \varepsilon^{\beta \delta \gamma \lambda} F_{\gamma \lambda}) \]

- implies anisotropic behavior of the field:

direction-dependent birefringent dispersion law

\[ \omega_{\pm} = p \pm \frac{p^2}{E_P} (n_0 + \vec{n} \cdot \hat{p})^3 \]

non-transverse field eigenstates

\[ \vec{E}_{\pm} = \begin{pmatrix} 0 \\ \pm \frac{i}{\sqrt{2}} \end{pmatrix} + \frac{1}{E_P} \begin{pmatrix} \sqrt{2} \frac{|\vec{n}|}{|k|^2} (\vec{k} \cdot \vec{n} + |\vec{k}| n_0)^2 |\vec{k} - (\vec{k} \cdot \vec{n})\hat{n}| \\ \frac{i}{\sqrt{2}} \frac{|\vec{n}|^2}{|k|^2} (\vec{k} \cdot \vec{n} + |\vec{k}| n_0) |\vec{k} - (\vec{k} \cdot \vec{n})\hat{n}|^2 \\ \frac{1}{\sqrt{2}} \frac{|\vec{n}|^2}{|k|^2} (\vec{k} \cdot \vec{n} + |\vec{k}| n_0) |\vec{k} - (\vec{k} \cdot \vec{n})\hat{n}|^2 \end{pmatrix} \]
Non-isotropic Lorentz breaking

- Large attenuation of anomalous effects in a significant portion of the sky

\[ (\theta \equiv \cos(\vec{k} \cdot \vec{n}), \frac{n_0}{|\vec{n}|} = 0.9) \]

- Crowns of partial blindness to anomalous effects

\[ \to \text{would need data from many astrophysical point sources or full-sky analysis (CMB)} \]

GG, G. Genovese, G. Amelino-Camelia, A. Melchiorri, PRD 2010
Non-isotropic birefringence

• Look for direction-dependent rotation of linear polarization

\[
\alpha(\theta, \phi, t) = -\frac{1}{E_P} |\vec{p}|^2 |\vec{n}|^3 \left( \sin \theta \sin \theta_n \cos(\phi - \phi_n) + \cos \theta \cos \theta_n \right)^3 t
\]

\{\theta, \phi\} : observation direction
\{\theta_n, \phi_n\} : preferential direction pointed out by \(\vec{n}\)

Map of the expected amount of rotation effect for \(\theta_n = 45^\circ, \phi_n = 90^\circ\)

• no effect on full-sky correlations between E and B modes

→ need to study correlations on patches of the sky
   (correlate information from disks in opposite directions)
Planck data simulation

• results from analysis of simulated map

input values $\alpha_{max} = 1.98 \ \ \ \ \{\theta_n, \phi_n\} = \{45^\circ, 90^\circ\}$

map of effective measured rotation angle:

<table>
<thead>
<tr>
<th>constraints: $\bar{\alpha}<em>{max} \pm \Delta \bar{\alpha}</em>{max} \Delta \bar{\theta}_n \Delta \bar{\phi}_n$</th>
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<tbody>
<tr>
<td>$1.97 \pm 0.20 \ [23.9, 63.0] \ [49.5, 114.7]$</td>
</tr>
</tbody>
</table>

(for direction angles $2\sigma$ intervals are reported)

Contour plot of the confidence intervals on $\{\theta_n, \phi_n\}$ obtained.
The black dot marks the input

Conclusions

• Quantum Gravity research gives motivations to look for Lorentz symmetry breaking

• the most common prediction is anomalous light propagation, and one possibility is birefringence, which in general depends on observation direction

• birefringence can be tested with CMB observations and with astrophysical ones, but hard to compare results

• astrophysics observations are in principle much more sensitive, but need to collect many data from different observation directions

• availability of (almost) full sky coverage from CMB data allows to test non-isotropic effects

• Planck satellite data will be good enough to perform this kind of tests for the first time