Non-local infrared modifications of gravity and dark energy

Michele Maggiore

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the general idea: modify GR in the infrared using non-local terms

• motivation: explaining DE
  IR modification $\Rightarrow$ mass term?

• (local) massive gravity: Fierz-Pauli, dRGT
  – significant progresses (ghost-free), still open issues
    (see Tolley's talk)

• our approach: mass term as coefficient of non-local terms
some sources of inspiration:

- \( \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2_A A_\mu A^\mu \) is equivalent to

  \[
  \mathcal{L} = -\frac{1}{4} F_{\mu\nu} \left( 1 - \frac{m^2_A}{\Box} \right) F^{\mu\nu}
  \]

  duality between locality and gauge-invariance for massive theories

- degravitation

  \[
  \left( 1 - \frac{m^2}{\Box} \right) G_{\mu\nu} = 8\pi G T_{\mu\nu}
  \]

  (Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

we can introduce a mass parameter without breaking the gauge-invariance of the theory
different possible implementations of the idea

- \( G_{\mu\nu} - m^2 (\Box^{-1} G_{\mu\nu})^T = 8\pi G T_{\mu\nu} \) (M. Jaccard, MM, E. Mitsou 2013)

however, instabilities in the cosmological evolution

- \( G_{\mu\nu} - m^2 (g_{\mu\nu} \Box^{-1} R)^T = 8\pi G T_{\mu\nu} \) (S. Foffa, MM, E. Mitsou 2013)

nice cosmological properties (\( w_{\text{DE}} = -1.04 \)). Not clear if there is an action

- last twist \( S_{\text{NL}} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left[ R - m^2 R \frac{1}{\Box^2} R \right] \) (MM and M. Mancarella 2014)
Conceptual aspects

• **degrees of freedom**
  – the graviton remain massless
  – only the non-radiative sector of GR is affected
  – is an effective classical theory

• **the successes of GR at solar system and lab scale are reproduced**
  – no vDVZ discontinuity
  – for static source, no classical strong coupling regime and no breakdown of the linear expansion below a Vainshtein radius

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MM 2013; S. Foffa, MM and E. Mitsou 2013

A. Kehagias and MM, 2014
Cosmological consequences

• define \( U = -\Box^{-1} R, \quad S = -\Box^{-1} U \)

• in FRW we have 3 variables: \( H(t), U(t), \ W(t) = H^2(t)S(t) \).
  define \( x = \ln a(t), \ h(x) = H(x)/H_0 \)

\[
h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(U, U', W, W')
\]

\[
U'' + (3 + \zeta)U' = 6(2 + \zeta)
\]

\[
W'' + 3(1 - \zeta)W' - 2(\zeta' + 3\zeta - \zeta^2)W = U
\]

\[
\gamma = m^2/(9H_0^2) \quad \zeta = h'/h
\]
• there is an effective DE term, with

\[ \rho_{DE}(x) = \rho_0 \gamma Y(x) \quad \rho_0 = 3H_0^2/(8\pi G) \]

• define \( w_{DE} \) from

\[ \dot{\rho}_{DE} + 3(1 + w_{DE})H \rho_{DE} = 0 \]

• the model has the same number of parameters as \( \Lambda \)CDM, with \( \Omega_\Lambda \leftrightarrow \gamma \).
• results:

• Fixing $\gamma = 0.0089..$ (m=0.28 $H_0$) we reproduce $\Omega_{DE}=0.68$
• having fixed $\gamma$ we get a pure prediction for the EOS:

$$w(a) = w_0 + (1-a) w_a$$

in the region $0 < z < 1.6$

$$w_0 = -1.144, \quad w_a = 0.084$$

on the phantom side!

general consequence of

$$\dot{\rho}_{DE} + 3(1 + w_{DE}) H \rho_{DE} = 0$$

together with $\rho > 0$ and $d\rho/dt > 0$
Cosmological perturbations

- well-behaved?

- consistent with structure formation?
  - Deser-Woodard nonlocal model ruled out at the 8σ level by the comparison with structure formation

Dodelson and Park 1310.4329

- Bayesian model comparison with \( \Lambda \)CDM
• the perturbations are well-behaved and differ from $\Lambda$CDM at a few percent level

\[
\Psi = [1 + \mu(a; k)]\Psi_{GR} \\
\Psi - \Phi = [1 + \Sigma(a; k)](\Psi - \Phi)_{GR}
\]

• deviations at $z=0.5$ of order 4%

• consistent with data: CFHTLenS gives $\Delta\Psi/\Psi=0.05\pm0.25$

(Simpson et al 1212.3339)
Lensing: again deviations at 4% level

growth index:
• **linear power spectrum**

DE clusters but is its linear power spectrum is small compared to that of matter.

Matter power spectrum compared to $\Lambda$CDM.
Comparison with $\Lambda$CDM

• A caveat: this is not wCDM!

• for the model $G_{\mu\nu} - m^2 (g_{\mu\nu} \Box^{-1} R)^T = 8\pi GT_{\mu\nu}$ (MM 2013)

  the perturbations have been recently computed and compared them to CMB, BAO, SNIa and growth rate data

  Nesseris and Tsujikawa 1402.4613

  – If $h_0 > 0.70$ the data strongly support this nonlocal model over $\Lambda$CDM

  – If $0.67 < h_0 < 0.70$ the two models are statistically comparable

(however, CMB studied using the shift parameter, rather than a full Boltzmann code)
• for the model
\[ S_{NL} = \frac{1}{16\pi G} \int d^4 x \sqrt{-g} \left( R - m^2 R \frac{1}{\Box^2} R \right) \]

we find that

• structure formation: statistically equivalent to $\Lambda$CDM with present data

• SNIa: fit to the JLA data gives equivalent $\chi^2$

• CMB: full Boltzmann code analysis under way
Conclusions

• we have an interesting IR modification of GR
• and testable predictions
  - \( w = -1.14 \) + a full prediction for \( w(z) \)
  - DES \( \Delta w = 0.03 \) (stage IV+Planck \( \Delta w = 0.01 \))
  - EUCLID \( \Delta w = 0.01 \)

- \( \mu(a) = \mu_s a^s \rightarrow \mu_s = 0.09, s = 2 \)
  - Forecast for EUCLID, \( \Delta \mu = 0.01 \)

- \( \Sigma(z) \): lensing deviations at a few %
- \( \gamma = 0.53 \)
Thank you!
Absence of ghosts

• define \( U = -\Box^{-1} R, \quad S = -\Box^{-1} U \)
the eqs. \( \Box U = -R, \quad \Box S = -U \)
do not describe radiative d.o.f!

\[-\Box^{-1} R = U_{\text{hom}}(x) - \int d^4x' \sqrt{-g(x')} G(x; x') R(x')\]

The homogeneous solution is fixed by the definition of \( \Box^{-1} \) i.e. by the def of the non-local theory. It is not a free Klein-Gordon field!
linearize the full eqs of motion. Scalar sector:

\[ h_{00} = 2\Psi, \quad h_{0i} = 0, \quad h_{ij} = 2\Phi \delta_{ij} \]

\[ \nabla^2 \left[ \Phi - \left( \frac{m^2}{6} \right) S \right] = -4\pi G \rho \]

\[ \Phi - \Psi - \left( \frac{m^2}{3} \right) S = -8\pi G \sigma \]

\[ (\Box + m^2)U = -8\pi G (\rho - 3P) \]

\[ \Box S = -U \]

\( \Phi \) and \( \Psi \) remain non-radiative!

In contrast, in massive gravity with FP mass term \( (\Box - m^2)\Phi = 0 \) and with generic mass there is a \( (\Box \Phi)^2 \) in the action (ghost)

\( U \) and \( S \) are non-radiative despite the KG operator.

No radiative d.o.f. in the scalar sector!
beyond the scalar sector: linearizing the eq of motion

\[ \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{d} m^2 P^{\mu\nu} P^{\rho\sigma} h_{\rho\sigma} = -16\pi G T^{\mu\nu} \]

\[ P^{\mu\nu} \equiv \eta^{\mu\nu} - \frac{\partial^{\mu} \partial^{\nu}}{\Box} \]

the corresponding matter-matter interaction is

\[ \tilde{T}_{\mu\nu}(-k) \frac{1}{2k^2} \left( \eta^{\mu\rho} \eta^{\nu\sigma} + \eta^{\mu\sigma} \eta^{\nu\rho} - \eta^{\mu\nu} \eta^{\rho\sigma} \right) \tilde{T}_{\rho\sigma}(k) \]

\[ + \frac{1}{6} \tilde{T}(-k) \left( \frac{1}{k^2} - \frac{1}{k^2 - m^2} \right) \tilde{T}(k) \]
• no vDVZ discontinuity!

• For \( m = O(H_0) \), solar system test easily passed. Corrections are \( O(m^2/k^2) = 10^{-30} \) for \( k = (1 \text{ a.u})^{-1} \).

• massless graviton + extra contribution to \( \tilde{T}_{\mu\nu}(-k) \tilde{D}_{\mu\nu\rho\sigma}(k) \tilde{T}_{\rho\sigma}(k) \)

\[
\frac{1}{d(d-1)} \tilde{T}(-k) \left[-\frac{i}{k^2} - \frac{i}{(-k^2 + m^2)} \right] \tilde{T}(k)
\]

these are the contribution of U and S and do not correspond to a radiative dof. In a quantum treatment there are no creation/annihilation operators associated to them
A fake ghost in massless GR

\[ S_{\text{EH}}^{(2)} = \frac{1}{2} \int d^{d+1}x \, h_{\mu\nu} \mathcal{E}^{\mu\nu,\rho\sigma} h_{\rho\sigma} \]

\[ h_{\mu\nu} = h_{\mu\nu}^{\text{TT}} + \frac{1}{2} (\partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) + \frac{1}{d} \eta_{\mu\nu} s \]

\[ S_{\text{EH}}^{(2)} = \frac{1}{2} \int d^{d+1}x \left[ h_{\mu\nu}^{\text{TT}} \square (h_{\mu\nu}^{\text{TT}}) - \frac{d-1}{d} s \square s \right] \]

\[ S_{\text{int}} = \frac{\kappa}{2} \int d^{d+1}x \, h_{\mu\nu} T^{\mu\nu} = \frac{\kappa}{2} \int d^{d+1}x \left[ h_{\mu\nu}^{\text{TT}} (T_{\mu\nu}^{\text{TT}}) + \frac{1}{d} s T \right] \]

\[ \square h_{\mu\nu}^{\text{TT}} = -\frac{\kappa}{2} T_{\mu\nu}^{\text{TT}} \], \quad \square s = \frac{\kappa}{2(d-1)} T \]

It looks as if there are many more propagating d.o.f
Furthermore s seems a ghost!
• the origin of the problem is that $s$ is a non-local function of $h_{\mu\nu}$:

$$s = \left( \eta^{\mu\nu} - \frac{1}{\Box} \partial^\mu \partial^\nu \right) h_{\mu\nu} = P^{\mu\nu} h_{\mu\nu}$$

• example: $\nabla^2 \phi = \rho$

$$\tilde{\phi} \equiv \Box^{-1} \phi \quad \Box \tilde{\phi} = \nabla^{-2} \rho \equiv \tilde{\rho}$$

it looks as if we have generated a dynamical dof!

However, the solution of the homogeneous eq are spurious!

the same happens for $s$: $s$ is non-radiative, and we must discard the solutions of the homogeneous eq $\Box s = 0$

• at the quantum level, no annihilation/creation operators associated to it; $s$ cannot be put on the external lines (otherwise, the vacuum in GR would decay!)
• the same happens in our non-local theory. The extra term in

\[ \mathcal{L}_2 = \frac{1}{2} h_{\mu\nu} \mathcal{E}_{\mu\nu,\rho\sigma} h_{\rho\sigma} - \frac{d-1}{2d} m^2 (P_{\mu\nu} h_{\mu\nu})^2 \]

\[ = \frac{1}{2} \left[ h_{\mu\nu}^{TT} \Box (h_{\mu\nu}^{TT})^{TT} - \frac{d-1}{d} s (\Box + m^2) s \right] \]

is just a mass term for \( s \) ! However, it remains a non-radiative field, as in GR, and we must discard the plane-wave solutions of

\[ (\Box + m^2) s = \frac{\kappa}{2(d-1)} T, \]

again, no propagating dof associated to \( s \), and no issue of quantum vacuum decay!
Non-local QFT or classical effective equations?

- we have $\nabla^{-1}_{\text{ret}}$ directly in the EoM (rather than in the solution). This EoM cannot come from the variation of a Lagrangian. E.g.

$$\frac{\delta}{\delta \phi(x)} \int dx' \phi(x')(\nabla^{-1}_{\text{ret}} \phi)(x') = \frac{\delta}{\delta \phi(x)} \int dx'dx'' \phi(x')G(x', x'')\phi(x'')$$

$$= \int dx'[G(x, x') + G(x', x)]\phi(x')$$

- we can replace $\nabla^{-1}$ → $\nabla^{-1}_{\text{ret}}$ after the variation, as a formal trick to get the EoM from a Lagrangian. Deser-Waldron 2007, Barvinski 2012

However, any connection to the QFT described by this Lagrangian is lost.
Absence of vDVZ discontinuity and Vainshtein strong coupling regime

- write the eqs of motion of the non-local theory in spherical symmetry: $U(r)$, $S(r)$, plus

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

- for $mr \ll 1$: low-mass expansion

- for $r \gg r_S$: Newtonian limit (perturbation over Minowski)

- match the solutions for $r_S \ll r \ll m^{-1}$ (this fixes all coefficients)
\[
A(r) = 1 - \frac{r_s}{r} \left[ 1 + \frac{1}{3} (1 - \cos mr) \right]
\]
\[
B(r) = 1 + \frac{r_s}{r} \left[ 1 - \frac{1}{3} (1 - \cos mr - mr \sin mr) \right]
\]

for \( r_s \ll r \ll m^{-1} \)

\[
A(r) \simeq 1 - \frac{r_s}{r} \left( 1 + \frac{m^2 r^2}{6} \right)
\]

the limit \( m \to 0 \) is smooth!

By comparison, in massive gravity the same computation gives

\[
A(r) = 1 - \frac{4}{3} \frac{r_s}{r} \left( 1 - \frac{r_s}{12m^4 r^5} \right)
\]

vDVZ discontinuity breakdown of linearity below

\( r_v = (r_s/m^4)^{1/5} \)
EoMs involving $\Box_{\text{ret}}^{-1}$ emerge from a classical or a quantum averaging of a more fundamental (local) QFT

- **Classically**, when separating long and short wavelength and integrating out the short wave-length (e.g. cosmological perturbation theory, or GWs)

- **In QFT**, when computing the effective action that includes the effect of radiative corrections. This provides effective non-local field eqs for $\langle 0|\hat{\phi}|0\rangle$, $\langle 0|\hat{g}_{\mu\nu}|0\rangle$

- The in-in matrix elements satisfy non-local and retarded equations

  Jordan 1986, Calzetta-Hu 1987
• So, we interpret our non-local eqs as a classical, effective equation, derived from a more fundamental local theory by a classical or quantum averaging.

• any problem of quantum vacuum stability can only be addressed in this fundamental theory.

• in any case, the apparent ghost that we found has nothing to do with quantum vacuum decay in our model. It can however trigger classical cosmological instabilities.