Steady state relativistic stellar dynamics around a massive black hole

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The stellar dynamical “loss-cone” problem:

How do stars interact with, and fall into a massive black hole (MBH) in a galactic center, and at what rates?

Implications

Plunge processes:
Tidal disruption flares, GW flares

Inspiral processes:
EMRIs, tidal “squeezars”, accretion disk capture

MBH formation and evolution
Exotic stellar populations near MBHs

How do galactic nuclei randomize and relax?
**Non-coherent 2-body relaxation (NR)**

**Uncorrelated close gravitational collisions:**

\[
\Gamma_{\text{coll}} \sim n_* v_{\text{rel}} \Sigma_{\text{coll}} \\
\sim n_* \sigma \left( \frac{G M_*}{\sigma^2} \right)^2 \propto n_* M_*^2
\]

\[
t_{\text{coll}} = \Gamma_{\text{coll}}^{-1} \sim \frac{\sigma^3}{G^2 n_* M_*^2}
\]

**Far/close collisions contribute only logarithmically:**

\[
T_{\text{NR}} \sim \frac{\sigma^3}{G^2 n_* M_*^2 \log \Lambda} \left( \sim \frac{E}{E} \sim \frac{L_c}{L} \right)
\]

**Keplerian approximation near MBH:**

\[
\sigma^2(r) \sim \frac{G M_*}{r}
\]

\[
P(r) = 2\pi \sqrt{r^3/G M_*}
\]

\[
\Lambda \sim \frac{M_*}{M_*} \equiv Q
\]

**Non-resonant (2-body) relaxation:**

\[
T_{\text{NR}}(r) \sim Q^2 P(r) / N_*(r) \log Q
\]

1/\log Q: relaxation boost from close encounters
The classical loss-cone: Plunge vs inspiral

Loss primarily by by L-relaxation: \( T_L \sim \left[ L/L_c(E) \right]^2 T_E \)

\[
\Gamma_{\text{plunge}} \sim N_\ast \langle <r_h⟩ \rangle / \langle \log(L_c/L_{lc})T_E \rangle
\]

\[
\Gamma_{\text{inspiral}} \sim N_\ast \left[ <r_{\text{crit}}(t_E)⟩ \right] / \langle \log(L_c/L_{lc})T_E \rangle
\]

\[
\Gamma_{\text{inspiral}} \sim O(0.01)\Gamma_{\text{plunge}}
\]
Resonant relaxation in near-spherical systems (RR v1.0: Rauch & Tremaine 1996)

Coherent residual torques \((t < t_{\text{coh}})\):

\[
\dot{L} = R \times F \sim R \times \left( \sqrt{N_*}\frac{GM_*}{R^2} \right)
\]

\[
\frac{\Delta L(t)}{L_c} \sim \left( \frac{\Delta L}{L_c} \right) t \sim \left( \sqrt{N_*} / Q \right) \left( \frac{t}{P} \right)
\]

where \(L_c(R) = \max L(R) = \sqrt{GM_*R}\)

Coherence quenched \((t \sim t_{\text{coh}})\):

\[
\frac{\Delta L}{L_c} \sim \left( \frac{\Delta L}{L_c} \right)_{\text{coh}} \left( \sqrt{N_*} / Q \right) \left( \frac{t_{\text{coh}}}{P} \right)
\]

Fast relaxation \((t > t_{\text{coh}})\):

\[
\frac{\Delta L}{L_c} \sim \left( \frac{\Delta L}{L_c} \right)_{\text{coh}} \sqrt{\frac{t}{t_{\text{coh}}}}
\]

\[
\equiv \sqrt{t/T_{\text{RR}}}
\]

\[T_{\text{RR}} \sim \left[ \frac{Q^2}{N_*(r)} \right] P^2(r) / t_{\text{coh}}\]

Many variants of RR

- Depend on type of near-symmetry, coherence quenching, relaxed quantity

**Orbit-orbit interactions (extended objects)**

- **Perturbing stars**
- **Effect on perturbed star**

Rauch & Tremaine 1996

Stationary ellipses in point mass potential

Scalar resonant relaxation

Planar rosettes in spherical potential

Vector resonant relaxation

\[T_{\text{RR}} \sim \left[ \frac{Q^2}{N_*(r)} \right] P^2(r) / t_{\text{coh}}\]

\[1/t_{\text{coh}}: \text{relaxation boost from long coherence}\]
Why is resonant relaxation relevant?

\[ T_{NR} \sim \left( \frac{Q^2}{N* \log Q} \right) P \quad \quad T_{RR} \sim \left( \frac{Q^2}{N*} \right) P^2 / t_{coh} \]

In nearly symmetric potentials where \( t_{coh} \) is long

\[ \frac{T_{RR}}{T_{NR}} \sim \log Q \left( \frac{P}{t_{coh}} \right) \ll 1 \]

The potential near a MBH is nearly symmetric

The evolution of L near a MBH can be dominated by RR

Rapid \( L \rightarrow 0 \) evolution: Strong interaction with the MBH

But: GR precession can quench RR!
The danger of unquenched RR: No EMRIs

The “fortunate coincidence” conjecture:
(Hopman & Alexander 2006)

- Unquenched, RR drives all stars to plunge orbits (no EMRIs!).

- $\mathcal{O}(\beta^2 j^{-2})$ GR in-plane Schwarzschild precession becomes significant before $\mathcal{O}(\beta^{5/2} j^{-7} Q^{-1})$ GW dissipation.

- GR precession quenches RR and allows EMRIs to proceed unperturbed, decoupled from the background stars.
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- Correlated relaxation
- The “Schwarzschild Barrier”

**Gravitational wave source dynamics**

**The Schwarzschild Barrier**

A reflecting barrier in phase space? GR precession under an external fixed force?
Hamiltonian dynamics with correlated noise (RR v2.0: Bar-Or & Alexander 2014)

- A background noise model \( \eta(t) \): A 2-point correlated random vector in \( L \)-space.

- Stochastic EOMs to evolve test star (1st order \( \ell = 1 \) phase-averaged Hamiltonian):

\[
\begin{align*}
\dot{x} &= \nu_{\tau,x} (j, \psi, \cos \theta, \phi) \cdot \eta(t), \\
\dot{\psi} &= \nu_{\tau,\psi} (j, \psi, \cos \theta, \phi) \cdot \eta(t) + \nu_p(j).
\end{align*}
\]

where \( j = L/L_c = \sqrt{1 - e^2} \), \( x = (j, \phi, \cos \theta) \), \( \nu_{\tau} = \text{RR torque freq's} \)

- Effective diffusion to evolve probability density (Fokker-Planck equation for \( j \)):

\[
\begin{align*}
\frac{\partial}{\partial t} P(j, t) &= \frac{1}{2} \frac{\partial}{\partial j} \left\{ j D_2(j) \frac{\partial}{\partial j} \left[ \frac{1}{j} P(j, t) \right] \right\}, \\
D_2(j) &= |\nu_{\tau,j}|^2 \mathcal{F}_C(t)[\nu_p(j)], \quad (\mathcal{F}_C(t) : \text{Fourier transform of } \eta \text{ ACF}) \\
D_1(j) &= \frac{1}{2j} \frac{\partial}{\partial j} (j D_2).
\end{align*}
\]
Background stellar noise and adiabatic invariance (AI)

**Key issue:**

Does the background noise power have a high frequency cutoff $\nu_0$?

Stars with $\nu_{GR}(j) > \nu_0$ ($j < j_0$) decouple from the background stars.

Smooth noise has a cutoff. The physical noise is expected to be smooth.
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The relativistic loss-cone

Noise smoothness and the Schwarzschild Barrier

Smooth noise (Gaussian ACF)

Non-smooth noise (exponential ACF)

Adiabatic invariance: Fast oscillations protect against slow variations

But, 2-body relaxation erases the SB (The SB is relevant on short timescales only).

No NR

With NR (at \( t = T_{NR} \))

*Not a real reflective barrier: Max. entropy is reached in exponentially long time (i.e. never).
Effective RR diffusion that express correlated noise and secular precessions, together with NR diffusion and GW dissipation, provide a powerful scalable Monte Carlo tool for modeling long-term dynamics and loss-rates of galactic nuclei ($N \gg 1$ limit).
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The relativistic loss-cone

The steady state relativistic loss-cone

GR + mass precession + GW + NR + RR, Gaussian noise, mass quenching

\[
\log_{10}(a/\text{mpc}) \quad \log_{10}(j) \quad \log_{10}(d^2n/d\log j \, d\alpha) 
\]

-0.5 < -1.0 < -0.5
-1.5 < -1.0
-2.0 < -1.5
-2.5 < -2.0
-3.0 < -2.5
-3.5 < -3.0
<-3.5

LSO
GW
AI contour
DC contour
inspiral
plunge

Last Stable Orbit

\(F_{\text{RR}} j / F_{\text{NR}} j\)

(RR > NR)

GW

Bar-Or & Alexander, 2015 in prep.)
RR can matter
Example: Binary capture and red giant tidal disruption

(Bar-Or & Alexander, 2015 in prep.)
Summary

- **General conclusions**
  - NR, RR, GW dissipation and secular precession can be treated analytically as effective diffusion with correlated noise.
  - The steady state depends mostly on NR, which erases AI.
  - RR can be important in special cases.

- **Specific results / products**
  - Stochastic equations of motion for evolving test particles.
  - FP / MC evolution of the distribution function in phase space.
  - Models the relativistic loss-cone for galactic nuclei with $N_* \gg 1$.
  - Plunge / EMRI rates and branching ratios (e.g. as function of $M_\bullet$).
  - Quantitative models for “exotic” processes (e.g. the Schwarzschild Barrier, binary capture outcomes).