Dark Matter Superfluidity

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JK, 1409.0012
L. Berezhiani & JK, to appear
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On these scales, only use the hydrodynamical limit of DM.

\[ T_{\mu\nu} = (\rho + P)u_\mu u_\nu + P g_{\mu\nu} \]

\[ \Rightarrow \text{Any perfect fluid with } P \approx 0 \text{ and } c_s \approx 0 \text{ does the job.} \]
Actual galaxies are remarkably regular.

- Baryonic Tully-Fisher relation
  
  McGaugh (2011)

\[ M_b \sim v_c^4 \]
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Hydro simulations

- Star formation model
- Mass and metal return
- Gas enrichment
- Stellar evolution
- SN rates
- Cooling/heating rates
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How can these feedback processes, which are inherently stochastic, result in tight correlation displayed in BTFR?
Small scale controversies

- Missing satellites
Small scale controversies

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- Too big to fail

Boylan-Kolchin et al., (2011)
Small scale controversies

- Missing satellites
- Vast planar structures

Too big to fail

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Lynden-Bell (1976); Pawlowski & Kroupa (2012); Ibata et al (2013)
Small scale controversies

- **Missing satellites**
  - MW, M31
  - Lynden-Bell (1976); Pawlowski & Kroupa (2012); Ibata et al. (2013)
- **Too big to fail**
  - Boylan-Kolchin et al., (2011)
- **Vast planar structures**
  - Lynden-Bell (1976); Pawlowski & Kroupa (2012); Ibata et al (2013)
Milgrom's relation

\[ a = \begin{cases} 
     a_N & a_N \gg a_0 \\
     \sqrt{a_N a_0} & a_N \ll a_0
\end{cases} \]

Milgrom (1983)
cf. talks by B. Famaey & L. Blanchet

\[ a_N = \frac{G_N M_D(r)}{r^2} \]
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Test mass orbiting galaxy in MOND regime,

\[ \frac{v^2}{r} = \sqrt{\frac{G_N M_b a_0}{r^2}} \]
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\[ \Rightarrow \quad \text{Flat rotation curve} \]

\[ \Rightarrow \quad \boxed{v^4 = G_N M_b a_0} \quad \text{BTFR} \]
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Test mass orbiting galaxy in MOND regime,

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\[ v^4 = G_N M_b a_0 \quad \implies \quad \text{BTFR} \]

\[ a_0 \approx \frac{1}{6} H_0 \approx 1.2 \times 10^{-8} \text{ cm/s}^2 \]

Milgrom (1983)

cf. talks by B. Famaey & L. Blanchet
The MOND regime is described by the effective theory:

\[
\mathcal{L}_{\text{MOND}} = -\frac{2M_{\text{Pl}}^2}{3a_0} \left( (\partial \phi)^2 \right)^{3/2} + \frac{\phi}{M_{\text{Pl}}} \rho_b
\]

For static, spherically-symmetric source, \( \mathbf{\nabla} \cdot \left( \frac{|\mathbf{\nabla} \phi|}{a_0} \mathbf{\nabla} \phi \right) = 4\pi G_N \rho \)

\[
\phi' = \sqrt{a_0 \frac{G_N M(r)}{r^2}} = \sqrt{a_0 a_N}
\]
Poor fit to galaxy clusters:
Poor fit to galaxy clusters:

Relativistic extension is rather frightening...

\[
\mathcal{L} = -\frac{1}{2} \left[ \sigma^2 h^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + \frac{G_N}{2\ell^2} \sigma^4 F(kG_N\sigma^2) \right] \\
- \frac{K}{32\pi G_N} \left[ g^{\alpha\beta} g^{\mu\nu} B_{\alpha\mu} B_{\beta\nu} + \frac{2\lambda}{K} (g^{\mu\nu} u_\mu u_\nu - 1) \right] \\
+ S_{\text{matter}} \left[ \psi_m, e^{2\phi} g^{\mu\nu} - 2u^{\mu} u^{\nu} \sinh(2\phi) \right]
\]

\[
B_{\mu\nu} = \partial_\mu u_\nu - \partial_\nu u_\mu \quad h_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta
\]

Bekenstein (2004)
DM-MOND hybrids

How about MOND and DM together?

Blanchet (2006); Bruneton et al. (2008); Ho, Minic & Ng (2009); JK (2014) ...
DM-MOND hybrids

How about MOND and DM together?

- Occam’s razor?
- Common origin?

Blanchet (2006); Bruneton et al. (2008); Ho, Minic & Ng (2009); JK (2014)…
DM-MOND hybrids

How about MOND and DM together?

- Occam’s razor?
- Common origin?
- Exclusion principle?

Blanchet (2006); Bruneton et al. (2008); Ho, Minic & Ng (2009); JK (2014)...

All DM, no MOND

Mostly DM

Mostly MOND

No MOND
Unified approach:

MOND phenomenon from DM superfluidity
Dark Matter Condensation

2 necessary conditions:
Dark Matter Condensation

2 necessary conditions:

- Overlapping de Broglie wavelength

\[ \lambda_{dB} \sim \frac{1}{mv} \geq \ell \sim \left( \frac{m}{\rho_{\text{vir}}} \right)^{1/3} \]

\[ \Rightarrow m \lesssim 2 \text{ eV} \]
Dark Matter Condensation

2 necessary conditions:

1. Overlapping de Broglie wavelength

\[ \lambda_{dB} \sim \frac{1}{mv} \gtrsim \ell \sim \left( \frac{m}{\rho_{vir}} \right)^{1/3} \]

\[ \implies m \lesssim 2 \text{ eV} \]

2. Thermal equilibrium

\[ \Gamma \sim v \sigma \frac{\rho_{vir}}{m} \gtrsim H_0 \]

\[ \implies \frac{\sigma}{m} \gtrsim 100 \text{ cm}^2 \text{ g}^{-1} \approx 200 \text{ GeV}^{-1} \]

DM is quite cold:

\[ T_c = 6.5 \left( \frac{\text{eV}}{m} \right)^{5/3} (1 + z_{vir})^2 \text{ mK} \]

\[ ( ^7 \text{Li atoms} \implies T_c \sim 0.2 \text{ mK} ) \]
Two-fluid model
Two-fluid model

Ignoring interactions,

\[ \frac{N_{\text{cond}}}{N} = 1 - \left( \frac{T}{T_c} \right)^{3/2} \]

Galaxies are mostly condensed

Galaxy clusters are in mixed phase
Phonons

Relevant low-energy degrees of freedom are phonons. At zero temperature and finite chemical potential,

\[ L = P_{T=0}(X); \quad X = \mu - V + \dot{\phi} - \frac{\left(\nabla \phi\right)^2}{2m} \]

Son and Wingate (2005)

Exact at lowest order in derivatives
Phonons

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\[ \mathcal{L} = P_{T=0}(X) ; \quad X = \mu - V + \dot{\phi} - \frac{(\vec{\nabla} \phi)^2}{2m} \]

Son and Wingate (2005)

Exact at lowest order in derivatives

Conjecture: DM superfluid phonons are governed by MOND action

\[ P_{\text{MOND}}(X) = \frac{2\Lambda (2m)^{3/2}}{3} X \sqrt{|X|} \]

Phonons couple to baryons:

\[ \mathcal{L}_{\text{coupling}} = -\frac{\Lambda}{M_{\text{Pl}}} \phi \rho_b \]

Match to MOND:

\[ \Lambda = \sqrt{a_0 M_{\text{Pl}}} \approx 0.8 \text{ meV} \]
Superfluid EFT

Weyl symmetry \textit{Milgrom (2008)}

\[ \mathcal{L}_{\text{MOND}} \sim \sqrt{h} \left( h^{ij} \partial_i \phi \partial_j \phi \right)^{3/2} \]

invariant under \( h_{ij} \rightarrow \Omega^2(x) h_{ij} \). Symmetry group is \( SO(4, 1) \).
Superfluid EFT

- **Weyl symmetry** [Milgrom (2008)]

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\mathcal{L}_{\text{MOND}} \sim \sqrt{h} \left( h_{ij} \partial_i \phi \partial_j \phi \right)^{3/2}
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invariant under \( h_{ij} \rightarrow \Omega^2(x) h_{ij} \). Symmetry group is \( SO(4, 1) \).

- **Unitary Fermi Gas** [Son & Wingate (2005)]

\[
\mathcal{L}_{\text{UFG}} \sim m^{3/2} X^{5/2}
\]

The MOND action \( \sim X^{3/2} \) corresponds to UFG in 2+1 dimensions.
Condensate properties

Action uniquely fixes properties of the condensate through standard thermodynamics

- **Pressure:**
  \[ P_{\text{cond}} = \frac{2\Lambda}{3}(2m\mu)^{3/2} \]

- **Number density:**
  \[ n_{\text{cond}} = \frac{\partial P_{\text{cond}}}{\partial \mu} = \Lambda(2m)^{3/2}\mu^{1/2} \]
Condensate properties

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- Number density:
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In the non-relativistic approx’n, \( \rho_{\text{cond}} = mn_{\text{cond}} \), therefore:

\[ P_{\text{cond}} = \frac{\rho_{\text{cond}}^3}{12\Lambda^2 m^6} \]

- Polytropic equation of state, with index \( n = 1/2 \)

- Different than BEC DM, where \( P_{\text{cond}} \sim \rho_{\text{cond}}^2 \)
  
Halo density profile

Assuming hydrostatic equilibrium,

\[
\frac{1}{\rho_{\text{cond}}(r)} \frac{dP_{\text{cond}}(r)}{dr} = -\frac{4\pi G_N}{r^2} \int_0^r dr' r'^2 \rho(r')
\]

Using equation of state \( P_{\text{cond}} \sim \rho_{\text{cond}}^3 \), find:

\[ R \approx \left( \frac{M_{\text{DM}}}{10^{12} M_\odot} \right)^{1/5} \left( \frac{\text{eV}}{m} \right)^{6/5} \left( \frac{\text{meV}}{\Lambda} \right)^{2/5} \]

Remarkably, have realistic-size halos with \( m \sim \text{eV} \) and \( \Lambda \sim \text{meV} \)!
Phonon-mediated force

Phonon equation of motion is not quite MOND:

\[ \vec{\nabla} \cdot \left( \sqrt{\frac{\phi'^2}{2m} - \mu(r)} + V(r) \vec{\nabla}\phi \right) = \frac{\rho_b(r)}{2\sqrt{2mM_{Pl}}} \]

\[ \phi'(r) = \left( m(\mu - V) + \sqrt{m^2(\mu - V)^2 + \left( \frac{M_b(r)}{8\pi M_{Pl}r^2} \right)^2} \right)^{1/2} \]

Note: At small distances, have \( \phi' \frac{r^2}{2m} \gg \mu \). Should we be worried?
The total acceleration on a test baryonic particle:

\[ a_{\text{tot}} = a_{\phi} + a_{\text{DM}} \]
Validity of effective theory: Landau's criterion

\[ v = \frac{\left| \nabla \phi \right|}{m} < c_s = \sqrt{\frac{2\mu}{m}} \]

Satisfied for \( r \gtrsim \text{kpc} \)

\[ \longrightarrow \]

Quasi-particle production (DM-like behavior) in inner regions of galaxies
Validity of effective theory: Landau's criterion

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Satisfied for \( r \gtrsim kpc \)

Quasi-particle production (DM-like behavior) in inner regions of galaxies

Solar system

A MOND scalar acc'n, \( \frac{\Delta a}{a_N} = \sqrt{\frac{a_0}{a_N}} \), albeit small in the solar system, is ruled out.

must we complicate the theory?
Validity of effective theory: Landau’s criterion

\[ v = \frac{|\nabla \phi|}{m} < c_s = \sqrt{\frac{2\mu}{m}} \]

Satisfied for \( r \gtrsim kpc \)

\[ a_N = \frac{\Delta a}{a_0} = \sqrt{\frac{a_0}{a_N}} \]

A MOND scalar acc’n, albeit small in the solar system, is ruled out.

No need to! Landau’s criterion is satisfied for \( r \gtrsim 300 \text{ AU} \)

Quasi-particle production (DM-like behavior) in inner regions of galaxies

Solar system

must we complicate the theory?

superfluid description breaks down in solar system.
DM behaves as ALPs.

Good news for ALP searches.
Gravitational Lensing

Claim: Conformal coupling $\tilde{g}_{\mu\nu} = e^{-2\phi} g_{\mu\nu}$ not enough.

Proof: Null geodesics are invariant under Weyl transf’ns, hence photons are oblivious to $\phi$. 
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Proof: Null geodesics are invariant under Weyl transf’ns, hence photons are oblivious to $\phi$.

TeVeS solution:
- Introduce unit, time-like vector
  $$A_\mu A^\mu = -1$$
- Couple to matter in a very specific way
  $$g_{\mu\nu}^{\text{TeVeS}} = e^{-2\phi} g_{\mu\nu} - 2 A_\mu A_\nu \sinh 2\phi$$

In weak-field limit, ignoring vector-field perturbations

$$ds^2_{\text{TeVeS}} = -\left(1 + 2(\Phi_N + \phi)\right) dt^2 + \left(1 - 2(\Phi_N + \phi)\right) dx^2$$

equality of potentials

$\implies$ Lensing mass estimates = Dynamical estimates
Gravitational Lensing (cont’d)

Our case is much simpler:

- Normal DM component already provides a unit vector
  \[ u_\mu u^\mu = -1 \]

- DM contributes to lensing, hence can detune metric
  \[ \tilde{g}_{\mu\nu} \simeq g_{\mu\nu} - 2\phi \left( \gamma g_{\mu\nu} + (1 + \gamma) u_\mu u_\nu \right) \]

\[ d\tilde{s}^2 = -\left(1 + 2(\Phi_N + \phi)\right)dt^2 + \left(1 - 2(\Phi_N + \gamma\phi)\right)d\tilde{x}^2 \]

Maybe even conformal coupling (\(\gamma = -1\)) is allowed?
Observational Consequences
The Bullet Cluster
The Bullet Cluster
The Bullet Cluster

\[ \frac{\sigma}{m} \lesssim 1.25 \ \text{cm}^2 \text{g}^{-1} \]

S. Randall et al. (2008)
Bullet Cluster (cont’d)

Superfluid cores should pass through each other with negligible dissipation if

\[ v_{\text{infall}} \ll c_s \]

We find:

- **Sub-cluster**, \( M_{\text{sub}} \sim 10^{14} M_\odot \) : \( c_s \sim 1100 \ \text{km/s} \)

- **Main cluster**, \( M_{\text{main}} \sim 10^{15} M_\odot \) : \( c_s \sim 2500 \ \text{km/s} \)

i.e., comparable to the infall velocity:

\[ v_{\text{infall}} \sim 2700 \ \text{km/s} \]

Springel & Farrar (2007)

\[ \Rightarrow \]

Dissipative processes between superfluid cores should be suppressed
The Counter-Bullet

Abell 520  Optical/X-ray/Lens
The Counter-Bullet

Abell 520

Optical/X-ray/Lens
Vortices

When spun faster than critical velocity, superfluid develops vortices.

\[ \omega_{cr} \sim \frac{1}{mR^2} \sim 10^{-41} \text{s}^{-1} \]

For a halo of density \( \rho \),

\[ \omega \sim \lambda \sqrt{G_N \rho} \sim 10^{-18} \lambda \text{ s}^{-1} ; \quad 0.01 < \lambda < 0.1 \]

\[ \implies \text{Vortex formation is unavoidable} \]

Line density:

\[ \sigma_v \sim m\omega \sim 10^2 \lambda \text{ AU}^{-2} \]

Observational consequences?
Galaxy mergers

- Force between galaxies same as in CDM (MOND confined to galaxies)

  "Encounter rate" as in CDM

What happens then?

- If $v_{\text{infall}} < c_s \sim 200 \text{ km/s}$, then negligible dynamical friction between superfluids

  Longer merger time scale + multiple encounters

- If $v_{\text{infall}} > c_s$, then encounter will excite DM particles out of the condensate, which will result in dynamical friction

  Merged halo thermalize and settle back to condensate
Vast planar structures

**Explanation:** Most satellites are tidal dwarfs resulting from early fly-by encounter between MW and M31

(Impossible in CDM because of dynamical friction)

Pawlowski & Kroupa (2013)
Zhao et al. (2013)
Vast planar structures

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- Arp 87: NASA, ESA, Weilbacher et al.
- Arp 302: NASA/STScI/NRAO/A. Evans et al.
- Tiret & Combes 2008

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**Pawlowski & Kroupa (2013)**
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Vast planar structures

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(Impossible in CDM because of dynamical friction)

Crazy (and probably ruled out) idea:
What if the entire Local Group is enveloped in superfluid?
Conclusions

Small scales present greatest challenge to $\Lambda$CDM

DM superfluidity:

- MOND arises from DM superfluid phase in galaxies
- All scales are comparable:
  \[ m \sim \text{eV} \quad \Lambda \sim \text{meV} \]

Open questions:

- Can we find a precise CM analogue?
- Can we distinguish superfluid idea from MOND and standard CDM?
- How does dark energy fit into this picture?
Tidal dwarfs galaxies

Bournaud et al., Science (2007)
Superfluid phonons

Expanding in small field,

\[ \mathcal{L}_{\text{quad}} = \frac{\Lambda (2m)^{3/2}}{4\mu^{1/2}} \left( \dot{\phi}^2 - \frac{2\mu}{m} (\vec{\nabla} \phi)^2 \right) \]

\[ \implies c_s^2 = \frac{2\mu}{m} \]

Schematic interaction term:

\[ \mathcal{L}_{\text{int}} \supset \Lambda m^{3/2} \mu^{3/2} - n \partial n \phi^n \]

\[ \implies \Lambda_s = \left( \Lambda m^{3/2} \mu^{3/2} \right)^{1/4} \sim \text{meV} \]

\[ \implies \text{Dark energy?} \]