Standard Electroweak Interactions in Gravitational Theory with Chameleon Field and Torsion

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Torsion tensor $\mathcal{T}^{\alpha}_{\mu\nu}$, introduced as an antisymmetric part of the affine connection $\Gamma^{\alpha}_{\mu\nu}$

$$\mathcal{T}^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\nu\mu} - \Gamma^{\alpha}_{\mu\nu} = -\mathcal{T}^{\alpha}_{\nu\mu},$$

has 24 independent components, which can be given in the following general form

$$\mathcal{T}^{\alpha}_{\mu\nu} = \frac{1}{3}(g^{\alpha\mu}E^{\nu} - g^{\alpha\nu}E^{\mu}) - \frac{1}{2}\varepsilon^{\alpha\mu\beta}\mathcal{B}^{\beta} + M^{\alpha}_{\mu\nu},$$

where 4–vector $E^{\nu}$ and 4–axial vector $\mathcal{B}^{\alpha}$ are defined by

$$E^{\nu} = g^{\alpha\mu}\mathcal{T}^{\alpha}_{\mu\nu}, \quad \mathcal{B}^{\alpha} = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu}\mathcal{T}^{\beta}_{\mu\nu},$$

and tensor $M^{\alpha}_{\mu\nu} = -M^{\alpha}_{\nu\mu}$ with 16 independent components obeys the constraints

$$g^{\alpha\mu}M^{\alpha}_{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu}M^{\beta}_{\mu\nu} = 0$$
Dirac Action and Minimal Torsion-Fermion Coupling

For metric spacetime
\[
\tilde{g}_{\mu\nu;\beta} = \tilde{g}_{\mu\nu,\beta} - \tilde{\Gamma}^\lambda_{\mu\beta} \tilde{g}_{\lambda\nu} - \tilde{\Gamma}^\lambda_{\nu\beta} \tilde{g}_{\mu\lambda} = 0
\]
the affine connection \(\tilde{\Gamma}^\alpha_{\mu\nu}\) with torsion \(\tilde{T}^\alpha_{\mu\nu}\) is given by
\[
\tilde{\Gamma}^\alpha_{\mu\nu} = \{\alpha_{\mu\nu}\} - \frac{1}{2}(\tilde{T}^\alpha_{\mu\nu} - \tilde{T}^\alpha_{\mu\nu} - \tilde{T}^\alpha_{\nu\mu})
\]

Action for Dirac fermions (neutrons) coupled to the chameleon field in the curved spacetime with torsion
\[
S_\psi = \int d^4x \sqrt{-\tilde{g}} \left( i \frac{1}{2} \bar{\psi}(x) \tilde{\gamma}^\mu(x) \tilde{D}_\mu \psi(x) - m\bar{\psi}(x)\psi(x) \right)
\]
Covariant derivative of a fermion (neutron) field
\[
D_\mu = \partial_\mu - \frac{i}{4} \tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}} \sigma^{\hat{\alpha}\hat{\beta}}
\]
Spin connection \(\tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}\)
\[
\tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}(x) = -\eta^{\hat{\alpha}\hat{\phi}} \left( \partial_\mu \tilde{e}^{\hat{\phi}}_\nu(x) - \tilde{\Gamma}^\alpha_{\mu\nu}(x) \tilde{e}^{\hat{\phi}}_\alpha(x) \right) \tilde{e}^{\hat{\beta}}_\nu(x)
\]
Dirac equation for fermions with mass $m$ in curved spacetime with chameleon and torsion:

$$\left(i \bar{\psi}_\lambda^\mu(x) \gamma^\lambda D_\mu - \frac{1}{2} i \tilde{T}^\alpha_{\alpha\mu}(x) \bar{\psi}_\lambda^\mu(x) \gamma^\lambda - \frac{1}{2} i \tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}(x) \bar{\psi}_\lambda^\mu(x) \left( \eta^\hat{\lambda}^\hat{\beta} \gamma^\hat{\alpha} \right) + \frac{1}{4} i [\sigma^{\hat{\alpha}\hat{\beta}}, \gamma^\lambda] - m \right) \psi(x) = 0$$

Dirac Hamilton operator: $H = H_0 + \delta H = \gamma^0 m - i \gamma^0 \hat{\gamma} \cdot \vec{\nabla} + \delta H$

$$\delta H =$$

$$= (\bar{\psi}^0_0(x) - 1) \gamma^0 m - i (\bar{\psi}^0_0(x) - 1) \gamma^0 \gamma^j \frac{\partial}{\partial \chi^j} - i \tilde{\omega}^0_0(x)(\bar{\psi}^0_j(x) - \delta^0_j) \gamma^0 \gamma^j \frac{\partial}{\partial \chi^j}$$

$$+ \frac{1}{2} i \tilde{T}^\alpha_{\alpha\mu}(x) \bar{\psi}_\lambda^\mu(x) \gamma^0 \gamma^\lambda + \frac{1}{2} i \tilde{\omega}_{\mu\hat{\alpha}\hat{\beta}}(x) \bar{\psi}^0_0(x) \tilde{\psi}_\lambda^\mu(x) \gamma^0 \left( \eta^\hat{\lambda}^\hat{\beta} \gamma^\hat{\alpha} \right) + \frac{1}{4} i \{\sigma^{\hat{\alpha}\hat{\beta}}, \gamma^\lambda\}$$
Effective Low–Energy Potentials of Fermions

Schwarzschild metric $\tilde{g}_{\mu\nu}(x)$ in the weak gravitational field of the Earth approximation modified by the chamelelon field (Jordan–frame metric):

$$ ds^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = (1 + 2U_E) e^{2\beta\phi/M_{Pl}} (dt)^2 - (1 - 2U_E) e^{2\beta\phi/M_{Pl}} (d\vec{r})^2 $$

Schrödinger–Pauli equation:
(Foldy & Wouthuysen, 1950, Fischbach et al., 1981)

$$ i \frac{\partial \psi(\vec{r}, t)}{\partial t} = \left( -\frac{1}{2m} \Delta + mU_E + \Phi_{gr-ch} + \Phi_{tors} \right) \psi(\vec{r}, t) $$

Effective low–energy potentials $\Phi_{gr-ch}$ and $\Phi_{tors}$:

$$ \Phi_{gr-ch} = m(U_+ - U_E) - \frac{1}{2m} \nabla(U_+ + 2U_-) \cdot \nabla - \frac{1}{2m} (U_+ + 2U_-) \Delta $$

$$ - \frac{1}{8m} \Delta(U_+ + 2U_-) - \frac{i}{4m} \vec{\sigma} \cdot \left( \nabla(U_+ + 2U_-) \times \nabla \right) $$

$$ \Phi_{tors} = -\frac{1}{4} \vec{\sigma} \cdot \vec{B} + \frac{i}{4m} B_0 \vec{\sigma} \cdot \nabla + \frac{i}{8m} \vec{\sigma} \cdot \nabla B_0 $$
Upper bound on $B_0$ and $\vec{B}$:

(V. A. Kostelecky et al., PRL100, 111102 (2008))

\[ |B_0| < 8.7 \times 10^{-18} \text{ eV} \]
\[ |\vec{B}| < 3.0 \times 10^{-20} \text{ eV} \]

These upper bounds are in agreement with the estimates obtained by C. Lämmerzahl, PLA228, 223 (1997) and Yu. N. Obukhov et al., PRD90, 124068 (2014)

The upper bounds on the torsion–fermion axial 4–vector couplings were obtained from “null–result” in measurements of Lorentz invariance violation and Zeeman transition frequencies between neighbouring atomic levels.

What will happen if we set $B_0 = \vec{B} = 0$?
Spin–chameleon–fermion potential:
A. N. Ivanov & M. Pitschmann, PRD90, 045040 (2014)

\[ \Phi_{\text{spin–ch}} = \frac{1}{4m} \frac{\beta}{M_{\text{Pl}}} i \vec{\sigma} \cdot \left( \vec{\nabla} \phi \times \vec{\nabla} \right) \]

Spin–chameleon–fermion potential corresponds to the phenomenological Lagrangian of a torsion–fermion interaction:
V. A. Kostelecky et al., PRL100, 111102 (2008)

\[ \delta \mathcal{L}_T(x) = \frac{i}{2} g_T \mathcal{E}_\mu(x) \bar{\psi}(x) \sigma^{\mu\nu} \overset{\leftrightarrow}{\partial}_\nu \psi(x) \]

From comparison we obtain:
A. N. Ivanov & M. Pitschmann, PRD90, 045040 (2014)

\[ g_T \mathcal{E} = \frac{1}{4m} \frac{\beta}{M_{\text{Pl}}} \vec{\nabla} \phi \]
Covariant form of the torsion tensor field in terms of the chameleon field

\[ T_{\alpha \mu \nu} = \frac{\beta}{M_{Pl}} \left( g_{\alpha \nu} \partial_\mu \phi - g_{\alpha \mu} \partial_\nu \phi \right) \]

How can such a torsion tensor field be introduced?: S. Hojman et al., PRD17, 3141 (1976)

- Abstract: A formalism is given which makes it possible for a modified form of local gauge invariance and minimal coupling to be compatible with torsion ... The Lagrangian for interacting electromagnetic, gravitational, torsion, and complex scalar fields is presented ...

Extension to Standard Electroweak Model: A. N. Ivanov & M. Wellenzohn (submitted to PRD)

- The Lagrangian for interacting leptons, baryons, photons, W–, Z– and Higgs bosons with gravitational, chameleon and torsion fields is presented
Standard Electroweak Interactions in Gravitational Theory with Torsion as Gradient of Chameleon Action for particles
coupled to gravitational, chameleon and torsion fields:

\[ S_{g,ch,EW} = \int d^4x \sqrt{-g} \left( \frac{1}{2} M_{Pl}^2 R + \frac{1}{2} \phi,\mu \phi,\mu - V(\phi) \right) \]

\[ -\frac{1}{4} \int d^4x \sqrt{-\tilde{g}} \tilde{g}^\alpha\mu \tilde{g}^\beta\nu \mathcal{F}_{\alpha\beta} \mathcal{F}_{\mu\nu} + \int d^4x \sqrt{-\tilde{g}} \mathcal{L}_m[\tilde{g}_{\mu\nu}] = \]

\[ = \int d^4x \sqrt{-\tilde{g}} \left( \frac{1}{2} M_{Pl}^2 R + \frac{1}{2} \phi,\mu \phi,\mu - V(\phi) \right) \]

\[ -\frac{1}{4} \int d^4x \sqrt{-g} g^\alpha\mu g^\beta\nu \mathcal{F}_{\alpha\beta} \mathcal{F}_{\mu\nu} + \int d^4x \sqrt{-\tilde{g}} \mathcal{L}_m[\tilde{g}_{\mu\nu}] \]

\[ \tilde{g}_{\mu\nu} = g_{\mu\nu} f^2, \quad \tilde{g}^{\mu\nu} = g^{\mu\nu} f^{-2}, \quad \sqrt{-\tilde{g}} = f^4 \sqrt{-g} \]
Photon–Torsion (Chameleon) Interactions

Lagrangian of photon–torsion (chameleon) interactions

\[
\frac{\mathcal{L}_{\text{em-tors}}}{\sqrt{-g}} = -\frac{1}{2} \frac{\beta}{M_{\text{Pl}}} F_{\mu\nu}(A_{\mu} \phi,_{\nu} - A_{\nu} \phi,_{\mu})
\]

\[
-\frac{1}{4} \frac{\beta^2}{M_{\text{Pl}}^2} (A_{\mu} \phi,_{\nu} - A_{\nu} \phi,_{\mu})(A_{\mu} \phi,_{\nu} - A_{\nu} \phi,_{\mu}) - \frac{1}{2\xi} \left( A_{\mu} ;_{\mu} - 4 \frac{\beta}{M_{\text{Pl}}} \phi,_{\nu} A_{\nu} \right)^2,
\]

where the last term fixes gauge and \(\xi\) is a gauge parameter.
Torsion (Chameleon) Two–Photon Decay

Figure: Feynman diagram for the $\phi \rightarrow \gamma + \gamma$ decay

Amplitude of torsion (chameleon) two–photon decay

$$M(\phi \rightarrow \gamma \gamma) = -2 \frac{\beta}{M_{Pl}} \left( (\varepsilon_1^* \cdot \varepsilon_2^*)(k_1 \cdot k_2) - (\varepsilon_1^* \cdot k_2)(\varepsilon_2^* \cdot k_1) \right)$$

Partial width of torsion (chameleon) two–photon decay

$$\Gamma(\phi \rightarrow \gamma \gamma) = \frac{\beta^2}{M_{Pl}^2} \frac{m_{\phi}^3}{8\pi} , \quad m_{\phi}^2 = \frac{\partial^2 V(\phi)}{\partial \phi^2} \bigg|_{\phi=\phi_m}$$

$m_{\phi}$ is a chameleon mass:

Chameleon (Torsion)–Photon Scattering

Figure: Feynman diagrams for torsion (chameleon)-photon scattering

**Gauge invariance**

\[
\tilde{M}(\gamma \phi \rightarrow \phi \gamma) \bigg|_{\varepsilon_1 \rightarrow k_1} = \sum_{j=a,b,c} M^{(j)}(\gamma \phi \rightarrow \phi \gamma) \bigg|_{\varepsilon_1 \rightarrow k_1} = 0
\]

\[
\tilde{M}(\gamma \phi \rightarrow \phi \gamma) \bigg|_{\varepsilon_2^* \rightarrow k_2} = \sum_{j=a,b,c} M^{(j)}(\gamma \phi \rightarrow \phi \gamma) \bigg|_{\varepsilon_2^* \rightarrow k_2} = 0
\]

**Cross-section:** \(\sigma_{\gamma \phi \rightarrow \phi \gamma}(\omega) = \sigma_0 f(\omega)\)

\[
\sigma_0 = \frac{1}{16\pi} \frac{\beta^4}{M_{\text{Pl}}^4} < 2.5 \times 10^{-50} \text{ barn/MeV}^2, \quad \beta < 5.8 \times 10^{8}
\]
Chameleon–Photon Coupling Constant $g_{\text{eff}} = \frac{\beta_{\gamma}}{M_{\text{Pl}}}$

Since in our approach $\beta_{\gamma} = \beta$, one can estimate the constraints on astrophysical sources of the chameleon field using the following experimental data on $\beta$

$$g_{\text{eff}} < \begin{cases} 7.8 \times 10^{-12} \text{GeV}^{-1}, & \beta < 1.9 \times 10^7 \quad n = 1 \\ 2.0 \times 10^{-11} \text{GeV}^{-1}, & \beta < 5.8 \times 10^7 \quad n = 2 \\ 8.2 \times 10^{-11} \text{GeV}^{-1}, & \beta < 2.0 \times 10^8 \quad n = 3 \\ 2.0 \times 10^{-10} \text{GeV}^{-1}, & \beta < 4.8 \times 10^8 \quad n = 4 \end{cases}$$


$$g_{\text{eff}} < 2.4 \times 10^{-10} \text{GeV}^{-1}, \quad \beta < 5.8 \times 10^8, \quad 1 \leq n \leq 10$$


Neutron and Proton Charge Radii

Figure: Feynman diagram for the contribution of the torsion (chameleon) to charge radii of the neutron and the proton

\[ r_n^2 = -\frac{9}{4\pi^2} \frac{\beta^2}{m_e^2} \frac{m_e m_n}{M_{Pl}^2} \ell n \left( \frac{M_{Pl}}{m_e} \right), \quad (r^2)_{exp} = -0.1161(22) \text{ fm}^2 \]

\[ \delta r_p^2 = -\frac{9}{4\pi^2} \frac{\beta^2}{m_\phi^2} \frac{m_\mu m_p}{M_{Pl}^2} \ell n \left( \frac{M_{Pl}}{m_\mu} \right), \quad \delta E_{2s\to2p} = -5.180 \delta r_p^2 = 0.311 \text{ meV} \]

Figure: The lower bounds of the chameleon–matter coupling constant $\beta$ from the experiment data on the electric charge radii of the neutron (left) and the proton (right), respectively. The shaded area is excluded: $\beta < 10^{19}$ (left) and $\beta < 10^{17}$ (right)
Chameleon–Induced Neutron $\beta^-$ Decay

Figure: Feynman diagrams for the reaction $\phi + n \rightarrow p + e^- + \bar{\nu}_e$

**Decay rate of** $\phi + n \rightarrow p + e^- + \bar{\nu}_e$

$$\lambda_{\phi n} = \int_0^\infty dE_\phi \ \Phi_{\text{ch}}(E_\phi) \ \sigma_{\phi n \rightarrow p e^- \bar{\nu}_e}(E_\phi)$$

$$\sigma_{\phi n \rightarrow p e^- \bar{\nu}_e}(E_\phi) = (1 + 3\lambda^2) \ \frac{G_F^2 |V_{ud}|^2}{2\pi^3} \ \beta^2 \ \frac{(m_n + m_p)^2}{M_{\text{Pl}}^2} \ \frac{(E_0 + E_\phi)^5}{120E_\phi^3}$$

**Half–life:** $T_{1/2} = \ln 2 / \lambda_{\phi n} > 3 \times 10^{33} \text{ yr}$

**Proton half–life** $T_{1/2} > 5.9 \times 10^{33}$ (Super–Kamiokande)

- $\Phi_{\text{ch}}(E_\phi)$: Ph. Brax et al., PRD85, 043014 (2012)
Gauge invariance of the chameleon–photon interactions might be interpreted as unrenormalisability of the chameleon–matter coupling $\beta$ by any interactions. This may imply that a screening, caused by the Vainstein mechanism, should not be valid in such an approach.

The relation $\beta_\gamma = \beta$ gives stronger constraints on astrophysical sources of chameleons, coming from direct photon–chameleon transitions in magnetic fields.

For the chameleon–induced $\beta^-$–decay one may propose the reaction:

$$\phi + ^{112}_{48}\text{Cd} \rightarrow ^{112}_{49}\text{In} + e^- + \bar{\nu}_e \ , \ \ E_\phi > 0.7 \text{ MeV},$$

where $^{112}_{48}\text{Cd}$ is stable and $^{112}_{49}\text{In}$ is unstable under EC (56%) and $\beta^-$ (44%) decays with $T_{1/2} = 14.97(10)\text{ m}$. In principle the chameleon–induced electron spectrum can be distinguished from the $^{112}_{49}\text{In}$–decay electron spectrum. For comparison the half–life of the proton for specific modes is $T_{1/2} > 5.9 \times 10^{33}\text{ yr}$ (Super–Kamiokande).
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