Atomic Clocks on the Ground and in Space: toward Chronometric Geodesy and new tests of the Gravitational Redshift

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Gravitational redshift prediction

The flow of time, or the rate of a clock when compared to coordinate time, depends on the velocity of the clock and on the space-time metric (which depends on the mass/energy distribution). In the weak-field approximation:

\[
\frac{\Delta \tau}{\tau} = \frac{\Delta f}{f} = \frac{U_B - U_A}{c^2} + \frac{v_B^2 - v_A^2}{2c^2} + O(c^{-4})
\]

- U and v known → Δf prediction
  = Clock syntonization

- U, v and Δf known
  = Gravitational redshift test

- Δf known → ΔW prediction (W=U+v^2/2)
  = Chronometric geodesy
A brief introduction to chronometric geodesy
Microwave clocks: $10^{-16}$ accuracy (Fountains)
In space: microwave clocks with at best $10^{-14}$ stability at present (GNSS)
Best performance of optical clocks to date:
- Accuracy: Sr, $6.4 \times 10^{-18}$ (JILA); Stability: Yb, $1.6 \times 10^{-18}$ after 7 h averaging (NIST)
Research in highly accurate clocks is an active, innovative and competitive field
Principle of chronometric geodesy

- Clock frequency comparison → measure directly gravity potential differences

\[ \frac{\Delta f}{f} = \frac{W_B - W_A}{c^2} + O(c^{-4}) \]

\[ W = U + \frac{v^2}{2} \]

\[ 10^{-18} \leftrightarrow 0.1 \text{ m}^2\text{s}^{-2} \leftrightarrow 1 \text{ cm} \]


LINK

33 cm height increase

37 ± 15 cm
An isochronometric surface is a surface where all clocks beat at the same rate.

They are almost equivalent to equipotential surfaces of the gravity field (differences of the order of 2 mm)

\[
\frac{d\tau}{dt}\bigg|_S = \text{cst}
\]

Let \( t \) be the time given by a clock at infinity and at rest in the GCRS. Then the reference isochronometric surface (TT) defined by IAU is:

\[
\frac{d\tau}{dt} = \text{cst} = 1 - L_G
\]

where \( L_G = 6.969290134 \times 10^{-10} \) is a defining constant (IAU resolution B1.9, 2000)

From this definition we get a reference equipotential

\[
W_0 \equiv U + \frac{v^2}{2} = c^2 L_G + O(c^{-2})
\]

EGM2008 includes satellite data + gravimetric (ground) data → decomposition in spherical harmonics (up to degree 2100)
As a proof-of-principle, one can determine (roughly) $J_2$ with two clocks:

$$\frac{\Delta f}{f} = \frac{W_B - W_A}{c^2} + O(c^{-4}) , \quad W = U + \frac{v^2}{2}$$

$$U = \frac{GM_E}{r} \left[ 1 + \frac{J_2 R_E^2}{2r^2} \left( 1 - 3 \sin(\phi)^2 \right) \right]$$

A: INRIM CsF1 (Turin, Italy)
B: SYRTE FO2 (Paris, France)

$$J_2 = (1.097 \pm 0.016) \times 10^{-3}$$

- Error of ~1.4% compare to best known value
- However, ground clocks are sensitive to higher order multipoles
Chronometric geodesy with ACES

- Measure “absolute” altitude of clocks (referenced to the space clock)
- Measure ground-to-ground gravitational potential differences up to 1 m².s⁻² accuracy (10 cm, 10⁻¹⁷ relative frequency shift)
A coordinated programme of optical clock comparisons

Local optical frequency comparisons using femtosecond combs

Frequency comparisons using transportable optical clocks

Optical frequency comparisons using broad bandwidth TWSTFT

Absolute frequency measurements
- Determination of the static gravity potential at all clock locations
- Development of a refined European geoid model including new gravity observations around all relevant clock sites (IFE)
- Investigation of time-variable components of the gravity potential, e.g. due to tides.

SYRTE clocks leveling campaign (IGN SGN Travaux Spéciaux)
Aim: to demonstrate that optical clocks can be used to measure gravity potential differences over medium-long baselines with high temporal resolution.

- Height difference ~ 1000 m \(\rightarrow\) Gravitational redshift \(\sim 10^{-13}\)
- Target \(\rightarrow\) resolution of tens of cm in a few hours
Towards a high resolution geopotential model using chronometric geodesy

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Check the poster at coffee break!

Region of interest

- French Alps to the Mediterranean Sea
- Region from 42° to 47° latitude and 4.5° to 9° longitude

Topography [m]

Elevation range: 

-2000 to 4000
Proposal for a test of the gravitational redshift with Galileo satellites 5 and 6
Galileo satellites 5 and 6 were launched with a Soyuz rocket on 22 August 2014 on the wrong orbit due to a technical problem.

Launch failure was due to a temporary interruption of the joint hydrazine propellant supply to the thrusters, caused by freezing of the hydrazine, which resulted from the proximity of hydrazine and cold helium feed lines.

Next launch of Galileo satellites 7 and 8 scheduled on Friday 27 March.
The eccentricity correction

- For a Keplerian orbit one shows that:

\[ \tau(t) = \left(1 - \frac{3Gm}{2ac^2}\right)t - \frac{2\sqrt{Gma}}{c^2}e \sin E(t) + \text{Cste} \]

- One need an **accurate clock** to measure the constant frequency bias.

- The eccentricity correction is a periodic term → **use the stability of the clock to “average” the random noise**.

- Limitations are due to mismodeled **systematics effects**.
- RMS deviation between theory and experiment is $\sim 2.2\%$
- Evidence of **systematic bias** during some particular passes

Figure 5: *Comparison of predicted and measured eccentricity effect for SV nr. 13.*
- Test of the redshift on a **single parabola**
- Continuous **two-way microwave link** between a spaceborne hydrogen maser clock and ground hydrogen masers
- Frequency shift verified to $7 \times 10^{-5}$
- Gravitational redshift verified to $1.4 \times 10^{-4}$

R. Vessot et al.,
GRG 1979, PRL 1980, AdSR 1989
Simulation of:

1. Galileo 5 and 6 orbits
2. Realistic onboard clock noise
3. Gravitational Redshift Signal (including a Local Position Invariance violation, random noise and systematic effects)

Analysis of the simulated signal with two different methods:

1. Matched Filtering in the frequency domain
2. Linear Least-Square + Monte-Carlo in the time domain
Wrong orbit due to a technical problem: **eccentric orbit** (~0.16 today, ~0.23 initially)

- Two-Lines Elements (TLE) + Kepler equation for a duration of 2 years

\[ \tilde{E} - \text{mean}(\tilde{E}) \]

\[ \tilde{E} = \frac{v^2}{2} - \frac{GM}{r} \]
Simulation of the signal

- Simple phenomenological model for LPI violation (C. Will, LRR 2014)
- Alpha is 0 in GR
- GP-A limit: $\alpha < 1.4 \times 10^{-4}$

$$\tilde{y}(\alpha) = -(1 + \alpha) \frac{GM}{c^2 r_s}$$
Simulation of onboard clock noise

L. Prange et al., IAG Potsdam Proceedings, 2014, accepted for publication
Simulation of onboard clock noise

MDEV of the simulated clock noise

- White noise $\sim 5 \times 10^{-14}$ @ 1000s
- Flicker noise $\sim 8 \times 10^{-15}$

L. Prange et al., IAG Potsdam Proceedings, 2014, accepted for publication
**Matched filtering method**

Sensitivity is the inverse of the signal-to-noise (SNR) ratio $\rho$, which is maximized with **matched filtering**

$$\rho^2 = \int_{-\infty}^{+\infty} \frac{|\tilde{X}(f)|^2}{S_N(f)} \, df$$

- $\tilde{X}(f)$: Fourier transform of the (ideal) signal
- $S_N(f)$: PSD of the random noise

**Linear least-square method**

Find the minimum of the merit function $\chi^2$ with respect to alpha

$$\chi^2 = \sum_{i=1}^{N} \left[ (y(t_i) + \epsilon_i + \epsilon_{sys}) - (\tilde{y}(\alpha; t_i) + A) \right]^2$$
The best actual limit on grav. redshift (GP-A) is reached after ~2 weeks with Galileo 5

After one year of integration the sensitivity is \( \sim 3 \times 10^{-5} \rightarrow \) a factor of 5 better than GP-A, which was a dedicated experiment (expected sensitivity of ACES-PHARAO is \( 2-3 \times 10^{-6} \))

- The two very different methods agree on the sensitivity of the test
- We proved mathematically that \( \sigma_\alpha = \rho^{-1} \)
- Problem: all systematic effects that mimic the gravitational redshift signal will induce a bias in the estimation of alpha → fake violation of LPI
Systematic effects

Bump in the MDEV $\rightarrow$ systematic effect at orbital frequency due to a radial error in the estimated orbit (Montenbruck et al., J.Geo., 2014) $\rightarrow$ mimick a grav. redshift violation!
Systematic effect shows a dependency with the sun elevation angle, i.e., the direction of Sun w.r.t. the satellite orbital plane (Montenbruck et al., J. Geo., 2014).

At least 75% of this effect due to mismodeling of Solar Radiation Pressure (SRP), other effects could be due onboard temperature variations.
Systematic effects

- Systematic effect due to mismodelling of the SRP:
  - Effect at orbital frequency with a frequency shift (1/year) (linked to the direction of the Sun)
  - Amplitude modulation at frequency (1/year)

\[ \epsilon_{\text{sys}} = A \sin((n_{\text{sat}} + \omega_{\text{year}})t + \phi_1)(1 + B(\cos(\omega_{\text{year}}t + \phi_2) - 1)), \quad \omega_{\text{year}} = 2\pi/\text{year} \]
Systematic effects

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$$\epsilon_{\text{sys}} = A \sin((n_{\text{sat}} + \omega_{\text{year}})t + \phi_1)(1 + B(\cos(\omega_{\text{year}}t + \phi_2) - 1)), \quad \omega_{\text{year}} = 2\pi/\text{year}$$
Full fit of SRP parameters + alpha

\[ \epsilon_{\text{sys}} = A \sin((n_{\text{sat}} + \omega_{\text{year}})t + \phi_1)(1 + B(\cos(\omega_{\text{year}}t + \phi_2) - 1)), \quad \omega_{\text{year}} = \frac{2\pi}{\text{year}} \]

- Decorrelation between fit parameters occurs for 1 year integration time $\rightarrow$ alpha bias is zero if the SRP effect is well modeled
- Atomic clocks are rapidly improving in accuracy and stability

- **Chronometric Geodesy**: directly measure gravity potential differences with clock comparisons ($\sim 0.6 \text{ m}^2\text{s}^{-2}$, $\sim 6 \text{ cm}$); and variations of gravitational potential differences ($\sim 0.1 \text{ m}^2\text{s}^{-2} @ 7\text{h}$, $\sim 1 \text{ cm} @ 7\text{h}$)

- Several projects linked to chronometric geodesy: ACES, ITOC, applications to geophysics (see poster by Guillaume Lion)

- it will be possible, with Galileo satellites 5 and 6, and at least one year of data, to improve on the GP-A (1976) limit on the gravitational redshift test, down to an accuracy around $3-5\times10^{-5}$