Hulluilla on Halvat Huvit

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Disformal vectors and anisotropies on a warped brane. Hulluilla on Halvat Huvit

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Disformal what?
The disformal metric

\[ g_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi) \phi,\mu \phi,\nu. \]

This is the most general physically consistent relation with these ingredients. 

The disformal metric

Have a scalar: $\phi(x)$. 
The disformal metric

Have a scalar: $\phi(x)$. And a metric: $g_{\mu\nu}(x)$. 

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What would you do?

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What would you do?
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\[
C(\phi) \, g_{\mu\nu}
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$$C(\phi) g_{\mu\nu} + \phi,_{\mu} \phi,_{\nu}$$
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How to lose your audience 101

D-branes and disformal metrics

• Take a 10D spacetime compactified into a 6D Calabi-Yau and 4D noncompact space
• The geometry is warped due to the internal 6D fluxes
• Now throw in a “probe” D3-brane: it will wander about in search of the bottom of the valley
• The induced metric on the D3 is: $g_{\mu\nu} = h(\phi)^{-1/2}g_{\mu\nu} + h(\phi)^{1/2}\phi_{,\mu}\phi_{,\nu}$

$h$ is the warp factor, $\phi$ is the compact direction along which the D3 moves

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\( h \) is the warp factor, \( \phi \) is the compact direction along which the D3 moves

*T. Koivisto, D. Wills and I. Zavala, JCAP 1406, 036 (2014)*
Got it?
Good!
Good!

Now,
Good!

Now, forget about it!

😊
The point being...

Look at this Lagrangian:

$$\mathcal{L} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\alpha\beta} R_{\alpha\beta} + p(\phi, X) \right]$$

We need anisotropic metric (Bianchi I)

Want to see some equations?
The point being...

- Look at this Lagrangian:

\[
S_\phi = \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\alpha\beta} R_{\alpha\beta} + p(\phi, X) \right]
\]
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YOU SAID NO LAGRANGIANS!!!
The point being...

- Look at this Lagrangian:

\[ S_A = - \int d^4x \sqrt{-g} \left[ \frac{1}{4} g^{\alpha\beta} g^{\gamma\delta} F_{\alpha\gamma} F_{\beta\delta} + \frac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right] \]
The point being...

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- Everything else instead:

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Want to see some equations?
Beautification

\[
\ddot{A} + (\dot{\alpha} + 4\dot{\sigma})\dot{A} + \gamma^{-2}M^2A + \gamma^2A\dot{\phi}\left(h\dot{\phi} + \frac{1}{2}h'\dot{\phi}^2\right) = 0
\]

\[
\left(\ddot{\phi} + 3\dot{\alpha}\dot{\phi}\right) \rho, x + \ddot{\phi}^2 \rho, xx + \dot{\phi}^2 \rho, \phi x - \rho, \phi
\]

\[
+ \frac{1}{2} \gamma^3 e^{-2\alpha + 4\sigma} \dot{A}^2 \left[h \left(\gamma^2 \ddot{\phi} - (\dot{\alpha} + 4\dot{\sigma})\dot{\phi}\right) + \frac{1}{2} \gamma^2 h' \dot{\phi}^2\right]
\]

\[
+ \frac{1}{2} \gamma e^{-2\alpha + 4\sigma} A^2 \left[hM^2 \left(\gamma^2 \ddot{\phi} + (\dot{\alpha} + 4\dot{\sigma})\dot{\phi}\right) + \frac{1}{2} \gamma^2 h' M^2 \dot{\phi}^2 + 2MM'\right] = 0
\]

\[
3\ddot{\sigma} + 9\dot{\alpha}\dot{\sigma} - \gamma e^{-2\alpha + 4\sigma} \dot{A}^2 + \gamma^{-1} e^{-2\alpha + 4\sigma} A^2 M^2 = 0
\]

\[
6\ddot{\alpha} + 9\dot{\alpha}^2 + 9\dot{\sigma}^2 + 3\rho + \frac{1}{2} \gamma e^{-2\alpha + 4\sigma} \dot{A}^2 - \frac{1}{2} \gamma^{-1} e^{-2\alpha + 4\sigma} A^2 M^2 = 0
\]

\[
3\ddot{\sigma}^2 - 3\dot{\sigma}^2 - \rho, x\dot{\phi}^2 + p - \frac{1}{2} \gamma^3 e^{-2\alpha + 4\sigma} \dot{A}^2 - \frac{1}{2} \gamma e^{-2\alpha + 4\sigma} A^2 M^2 = 0
\]
\[
\ddot{A} + (\ddot{\alpha} + 4\dot{\sigma}) \dot{A} + \gamma^{-2} M^2 A + \gamma^2 \dot{A} \dot{\phi} \left(h \ddot{\phi} + \frac{1}{2} h' \dot{\phi}^2\right) = 0
\]

\[
\left(\ddot{\phi} + 3\dot{\alpha} \dot{\phi}\right) p,x + \dot{\phi}^2 p,xx + \dot{\phi}^2 p,\phi x - p,\phi
\]

\[
+ \frac{1}{2} \gamma^3 e^{-2\alpha + 4\sigma} \dot{A}^2 \left[h \left(\gamma^2 \ddot{\phi} - (\ddot{\alpha} + 4\dot{\sigma}) \dot{\phi}\right) + \frac{1}{2} \gamma^2 h' \dot{\phi}^2\right]
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\]

\[
3\dot{\alpha}^2 - 3\dot{\sigma}^2 - p,\phi \dot{\phi}^2 + p - \frac{1}{2} \gamma^3 e^{-2\alpha + 4\sigma} \dot{A}^2 - \frac{1}{2} \gamma e^{-2\alpha + 4\sigma} A^2 M^2 = 0
\]
Beautification

\[ \dddot{A} + (\dot{\alpha} + 4\dot{\sigma})\ddot{A} + \gamma^{-2}M^2A + \gamma^2\dot{A}\dot{\phi}\left(h\ddot{\phi} + \frac{1}{2}h'\dot{\phi}^2\right) = 0 \]

\[ 2\tilde{\gamma}^2 x,_{\alpha} = 3\dot{\alpha}(1 + 2\tilde{\gamma})(1 - \tilde{\gamma}) \left[ \sqrt{\star}\lambda h x^2 - \epsilon x \right] + \Upsilon \]

\[ u,_{\alpha} = (2\Sigma - 1) u + \tilde{\gamma} v \]

\[ v,_{\alpha} = (\epsilon - 2\Sigma - 2) v - \tilde{\gamma} M^2 u \]

\[ \Sigma,_{\alpha} = (\epsilon - 3) \Sigma + 2\tilde{\gamma} v^2 - 2\tilde{\gamma} M^2 u^2 \]

\[ M,_{\alpha} = \sqrt{\star}\lambda M M x + \epsilon M \]

\[ 3\ddot{\sigma} + 9\dot{\sigma} - \gamma e^{-2\alpha+4\sigma} \dot{A}^2 + \gamma^{-1} e^{-2\alpha+4\sigma} A^2 M^2 = 0 \]

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\[ 3\dot{\alpha}^2 - 3\dot{\sigma}^2 - p,\phi^2 + p - \frac{1}{2}\gamma^3 e^{-2\alpha+4\sigma} \dot{A}^2 - \frac{1}{2}\gamma e^{-2\alpha+4\sigma} A^2 M^2 = 0 \]
Isotropy and vectors

Non-zero vector $\text{vev} \neq \text{anisotropic expansion}$

The shear is zero on average due to the vector oscillations
Semi-stability VS stability

Here we have anisotropic expansion, but this is not asymptotically stable

However, this is a semi-stable, that is, bounded solution (as before).
Summarising

- We studied the most generic vector action coupled to a disformal metric.

1. de Sitter or kinetic isotropic expansion (stable)
2. Anisotropic expansion with zero vector vev (semi-stable)
3. Vector-driven anisotropic stiff solutions (semi-stable)

Most are not cosmologically viable (no acceleration or large anisotropy)

1: vector fields can be compatible with isotropic expansion
2: asymptotic stability is not everything
3: our parametrisation for these systems is awesome!
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Thank you!